Comparison of Ply-wise Stress-Strain results for graphite/epoxy laminated plate subjected to in-plane normal loads using CLT and ANSYS® ACP PrepPost

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Abstract- In this paper, comparison of stress and strain results in each ply, [A][B][D] matrix (stiffness matrix) and laminate in-plane as well as flexural properties in 3 layered graphite/epoxy plate having 15mm thickness were made between CLT (Classical Laminate Theory) analysis and ANSYS®ACP PrepPost for the in-plane tensile loading on laminated plate.

Keywords: FRP, ANSYS ACP PrepPost, CLT, [A][B][D] matrix, tensile loading, stress and strain

I. INTRODUCTION

Fiber reinforced Plastic (FRP) represents a new class of construction materials. In the past two decades, their use has spread from the aerospace industry to civil infrastructure which presents a new set of challenges. FRP are competitive with more traditional construction materials because of its significant advantages, such as resistance to corrosion, high strength to weight ratio, light weight and ease of installation. FRP can be made into structural shapes by pultrusion, a process that combines extrusion and pulling of molten or curable resin and continuous fibers usually arranged in unidirectional layers, or plies, through a die of a desired structural shape of constant cross-section.

During past decades, analysis for the FRP plates have been investigated by several researchers through experiments and by numerical simulation to check the possibilities of FRP in place of traditional isotropic materials. Zdravko Vnucec [1] et al. studied the analysis of the laminated composite plates under combined loads and prepared the graph for laminate properties for angle-ply symmetric laminated plates (+α,-α,-α,+α) and ply wise stress and strain vs lamination angles. A. T. Nettles [2] prepared the technical report on basic mechanics of laminated composite plates for NASA in which generalized Hook's law for non isotropic materials, laminate in plane properties, stress and strain within lamina for symmetrical and unsymmetrical laminate were prepared. G Restivo [3] et al. studied the 3D strain analysis of single lap bolted joints in thick composite using fiber optic gauges and the Finite Element Method and found that the largest difference between the axial strains obtained from the experiments and the FEM analysis was approximately 16%. Lotfi Toubal [4] et al. studied the stress concentration in a circular hole in composite plate and plotted graph for stress distribution of woven fabric composite plate subjected to tensile load.

Most commonly used analytical base for laminate analysis is CLT (Classical Laminate Theory). There are several softwares which provides module for composite modelling and analysis. ANSYS® has also launched sub-module specially for composite modelling and analysis. In the present study, an attempt to model a simple laminated composite plate subjected to in-plane loads using ANSYS® ACP PrepPost has made. Further, ply-wise stress and strain have also been calculated from CLT and the comparison is carried out.

II. CLASSICAL LAMINATE THEORY(CLT):

Classical Laminate Theory is based on some assumptions that each lamina is homogeneous, orthotropic and elastic. Also, the line straight and perpendicular to the middle surface remains straight and perpendicular to middle surface during deformation($\nu_{xz} = \nu_{yz} = 0$). There is no slip occurs between lamina interface. So, it can be used to find out ply wise (lamina wise) stresses and strains when the external actions on laminate is known.

Strain-Stress relationship for orthotropic material with the plane stress assumption can be obtained as follows:-

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$ i.e $\{\varepsilon\} = \{S\}\ \{\sigma\}$, where $\{S\}$ is compliance matrix of lamina

The components of $\{S\}$ matrix can be obtained using $S_{11} = 1/E_1$; $S_{12} = -\gamma_{12}/E_1$; $S_{22} = 1/E_2$ and $S_{66} = 1/G_{12}$

Transformed reduced stiffness matrix $[\bar{Q}] = [T]^{-1}[Q]$

Where, Reduced stiffness matrix $[Q] = [S]^{-1}$ and transformation matrix $[T]^{-1} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}$

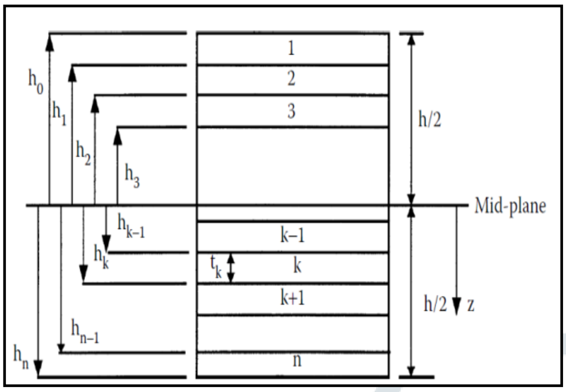
c = cosθ and s = sinθ (where, θ is the angle of fiber orientation with global horizontal axis)

$A_{ij} = \sum_{k=1}^{3} [\bar{Q}ij]_k(h_k - h_{k-1})$;

$B_{ij} = \frac{1}{2}\sum_{k=1}^{3} [\bar{Q}ij]_k(h_k^2 - h_{k-1}^2)$;

$D_{ij} = \frac{1}{3}\sum_{k=1}^{3} [\bar{Q}ij]_k(h_k^3 - h_{k-1}^3)$

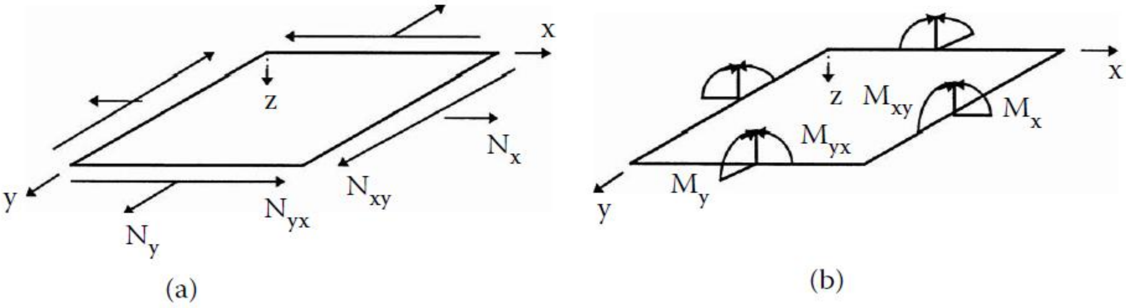
Where, [*A*], [*B*], and [*D*] matrices are called the extensional, coupling, and bending stiffness matrices



where, t_k= Lamina thickness
h_{k-1} = Distance from mid surface to inner surface of kth lamina.
h_k = Corresponding distance from mid surface to outer surface of the kth lamina.

Fig.1 - Coordinate locations of plies (laminae) in laminate

Relationships developed for a plate under in-plane loads such as shear and axial forces, and bending and twisting moments can be obtained as follows:



(a) (b)

Fig.2 - Laminated plate subjected to In-plane and Out of plane actions

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} A_{12} A_{16} B_{11} B_{12} B_{16} \\ A_{12} A_{22} A_{26} B_{12} B_{22} B_{26} \\ A_{16} A_{26} A_{66} B_{16} B_{26} B_{66} \\ B_{11} B_{12} B_{16} D_{11} D_{12} D_{16} \\ B_{12} B_{22} B_{26} D_{12} D_{22} D_{26} \\ B_{16} B_{26} B_{66} D_{16} D_{26} D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Where, ε_x^0 , ε_y^0 and γ_{xy}^0 are laminate mid-plane strain and κ_x , κ_y and κ_{xy} are mid-plane curvatures for the laminate.

So, matrix [*A*] relates the resultant in-plane forces to the in-plane strains, matrix [*D*] relates the resultant bending moments to the plate curvatures. and matrix [*B*] couples the force and moment terms to the mid-plane strains and mid-plane curvatures.

Lamiate global strain $\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + h \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$

Local strain $\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$ where, Transformation matrix [T] = $\begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -sc \\ -2sc & sc & c^2 - s^2 \end{bmatrix}$

Global and local stress strain relationship can be obtained for each lamina level as follows:

Global stress $\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = [Q]\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}$ and

Local stress $\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = [Q]\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix}$

Laminate engineering constants are as follows :

Effective in-plane longitudinal modulus $E_x = \frac{1}{hA^*_{11}}$;

Effective in-plane Transverse modulus $E_y = \frac{1}{hA^*_{22}}$;

Effective in-plane Shear modulus $G_{xy} = \frac{1}{hA^*_{66}}$;

Effective in-plane poison's ratio $\nu_{xy} = -\frac{A^*_{12}}{A^*_{11}}$ and $\nu_{yx} = -\frac{A^*_{12}}{A^*_{22}}$

(Where, A^*_{ij} is the element of inverse A matrix and h is the laminate thickness)

Effective flexural longitudinal modulus $E^f_x = \frac{12}{h^3 D^*_{11}}$;

Effective flexural Transverse modulus $E^f_y = \frac{12}{h^3 D^*_{22}}$;

Effective flexural shear modulus $G^f_{xy} = \frac{12}{h^3 D^*_{66}}$;

Effective flexural poison's ratio $\nu^f_{xy} = -\frac{D^*_{12}}{D^*_{11}}$ and $\nu^f_{yx} = -\frac{D^*_{12}}{D^*_{22}}$

(Where, D^*_{ij} is the element of inverse D matrix)

III. ANSYS® ACP PrepPost:

Engineering layered composites involves complex definitions that include numerous layers, materials, thicknesses and orientations. The engineering challenge is to predict how well the finished product will perform under real-world working conditions. This involves considering stresses and deformations as well as a range of failure criteria. ANSYS® Composite PrepPost provides all necessary functionalities for the analysis of layered composite structures.ACP has a pre- and post-processing mode. In the pre-processing mode, all composite definitions can be created and are mapped to the geometry (FE mesh). These composite definitions are transferred to the FE model and the solver input file. In the post-processing mode, after a completed solution and the import of the result file(s), post-processing results (failure, safety, strains and stresses) can be evaluated and visualized.

Square plate of 100mm x 100mm having 3 layer of Graphite/Epoxy with total thickness of 15 mm was analysed using ACP Pre as Pre processor, ANSYS static structure as solver and ACP Post as Post Processor to get the stiffness matrix, in-plane as well as flexural properties and ply wise stress-strain results. Material properties for the Graphite/Epoxy lamina are listed in Table-1. Tensile line pressure of 1N/mm was given along all the faces of square plate.

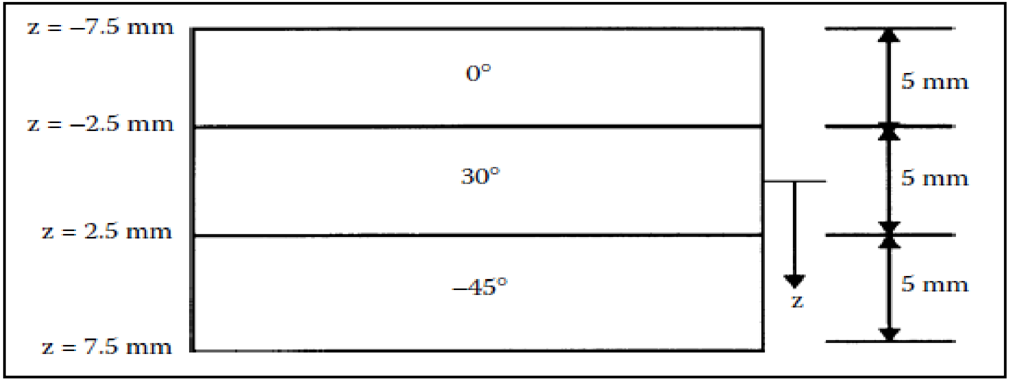


Fig.3 - Laminate definition in ANSYS ACP Pre

Dimensions of square plate = 100mm*100mm*15mm

Material properties:

Table 1 Material properties for Graphite/Epoxy Lamina

Property	Symbol	Graphite/Epoxy
Fiber volume fraction	V_f	0.7
Longitudinal elastic modulus	E_1	181 GPa
Transverse elastic modulus	E_2	10.30 GPa
Major poison's ratio	v_{12}	0.28
Shear modulus	G_{12}	7.17 GPa

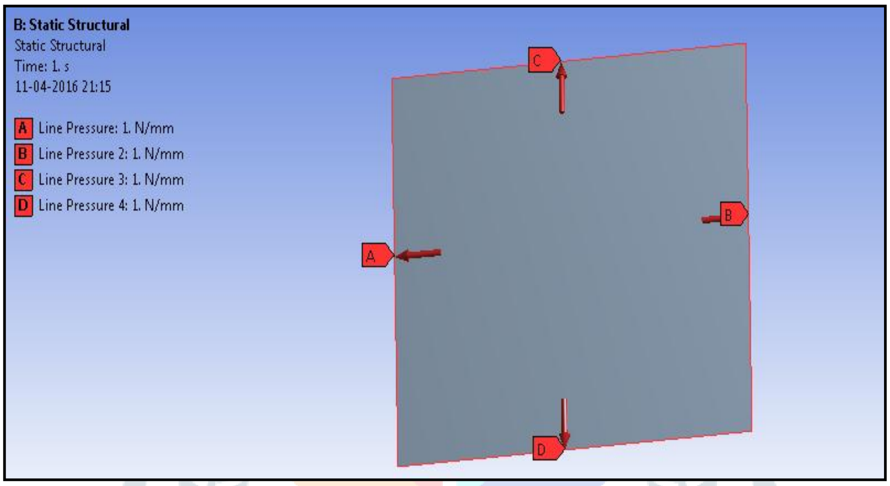


Fig.4 - Line pressure along boundary of square plate

In ACP (Pre) several steps should be followed to define the laminate setup. First of all fabric should be defined in which laminae properties with their thicknesses are added . Then after stack up sequences are defined in which fibers orientation with global coordinate system are added. In case of hybrid composite having two or more materials, sub laminates should also be defined properly. Rosettes are the coordinate systems that used to set the reference direction of Oriented Element Sets. In other words, rosettes define the 0° direction for the composite layup. Rosettes direction are shown in Fig. 6

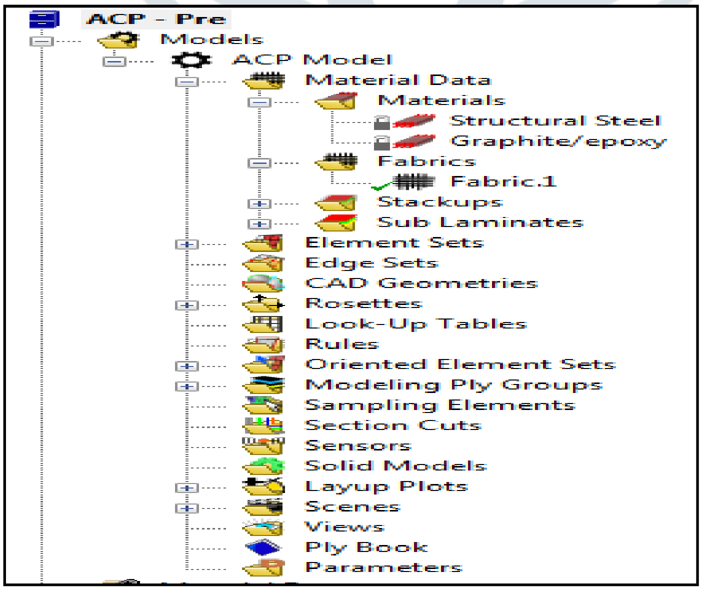


Fig.5 - Tree of ACP Pre

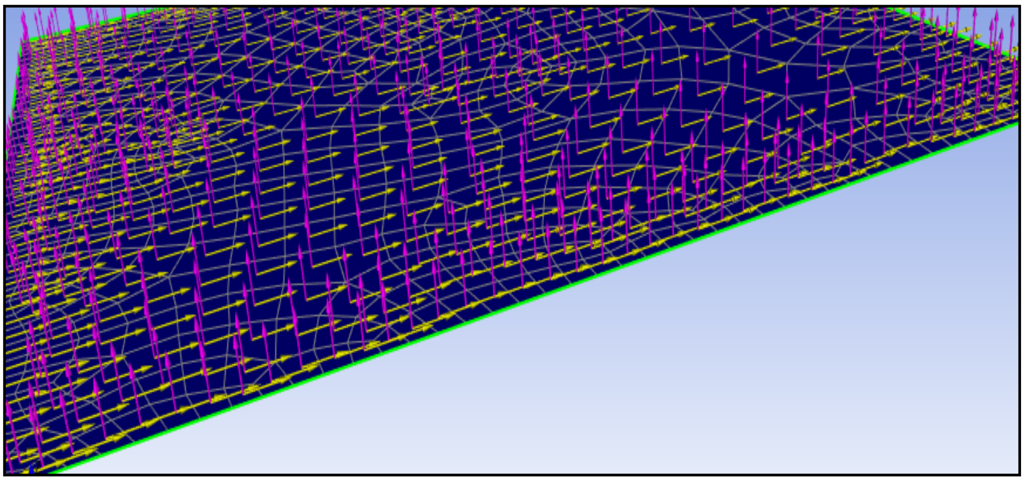
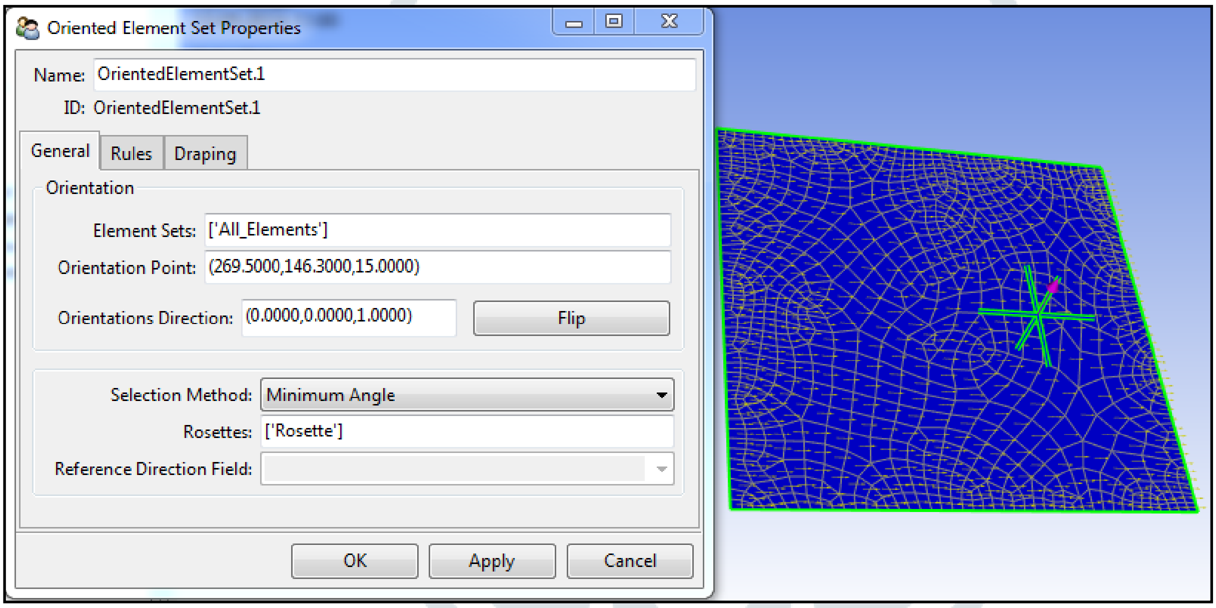


Fig. 6 Reference direction for fiber orientation



Name: OrientedElementSet.1
ID: OrientedElementSet.1
Element Sets: ['All_Elements']
Orientation Point: (269.5000,146.3000,15.0000)
Orientations Direction: (0.0000,0.0000,1.0000)
Selection Method: Minimum Angle
Rosettes: ['Rosette']
Reference Direction Field:

Fig. 7 Orientation point for lamina layups

For deciding the lamina [illegible] lement on the model is selected and orientation direction of the lamina with respect to that element is decided i.e upward or downward or at any angle with respect to that element.(Fig. 7)

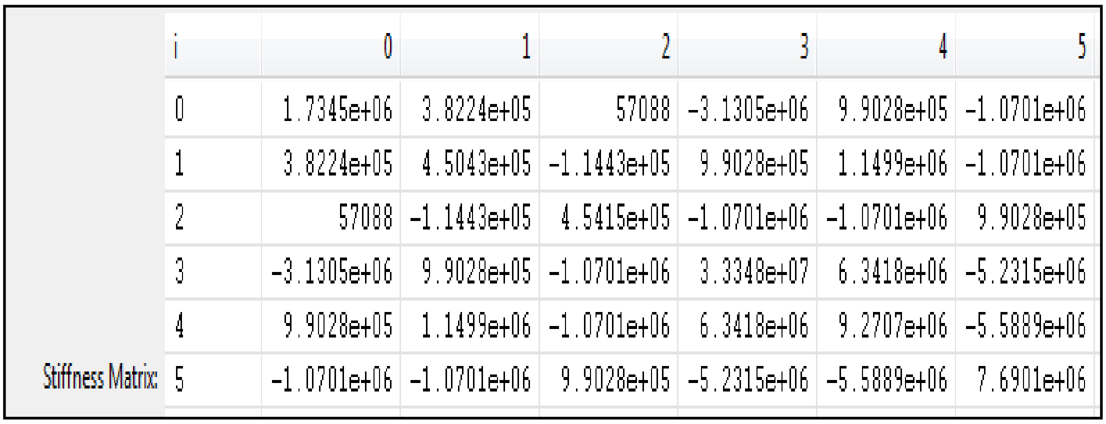
IV. RESULT COMPARISON:

(A) Comparison of Stiffness matrix

Stiffness matrix ([A][B][D] matrix) by CLT analysis:

$$\begin{bmatrix} 1.739*10^6 & 3.884*10^5 & 5.663*10^4 & -3.129*10^6 & 9.855*10^5 & -1.072*10^6 \\ 3.884*10^5 & 4.533*10^5 & -1.141*10^5 & 9.855*10^5 & 1.158*10^6 & -1.072*10^6 \\ 5.663*10^4 & -1.141*10^5 & 4.525*10^5 & -1.072*10^6 & -1.072*10^6 & 9.855*10^5 \\ -3.129*10^6 & 9.855*10^5 & -1.072*10^6 & 3.343*10^7 & 6.461*6 & -5.24*10^6 \\ 9.855*10^5 & 1.158*10^6 & -1.072*10^6 & 6.461*10^6 & 9.32*10^6 & -5.596*10^6 \\ -1.072*10^6 & -1.072*10^6 & 9.855*10^5 & -5.24*10^6 & -5.596*10^6 & 7.663*10^6 \end{bmatrix}$$

Stiffness matrix ([A][B][D] matrix) by ANSYS ACP Prep Post:



Stiffness Matrix:

i	0	1	2	3	4	5
0	1.7345e+06	3.8224e+05	57088	-3.1305e+06	9.9028e+05	-1.0701e+06
1	3.8224e+05	4.5043e+05	-1.1443e+05	9.9028e+05	1.1499e+06	-1.0701e+06
2	57088	-1.1443e+05	4.5415e+05	-1.0701e+06	-1.0701e+06	9.9028e+05
3	-3.1305e+06	9.9028e+05	-1.0701e+06	3.3348e+07	6.3418e+06	-5.2315e+06
4	9.9028e+05	1.1499e+06	-1.0701e+06	6.3418e+06	9.2707e+06	-5.5889e+06
5	-1.0701e+06	-1.0701e+06	9.9028e+05	-5.2315e+06	-5.5889e+06	7.6901e+06

Where,[A] is in MPa-mm, [B] is in MPa-mm^2 and [C] is in MPa-mm^3

(B) Comparison of Laminate Properties

Table 2 Comparison of Laminate Properties

Laminate properties	CLT analysis	ANSYS ACP Prep Post	Error(%)
In-plane longitudinal modulus	51.619 GPa	55.43GPa	6.87
In-plane transverse modulus	15.02 GPa	14.82 GPa	1.33
In-plane shear modulus	17.96 GPa	18.005 GPa	0.25
Flexural longitudinal modulus	59.62 GPa	59.56 GPa	0.1
Flexural transverse modulus	14.39 GPa	14.37 GPa	0.14
Flexural shear modulus	11.36 GPa	11.39 GPa	0.26

(C) Comparison of Ply-wise Stress and Strain results

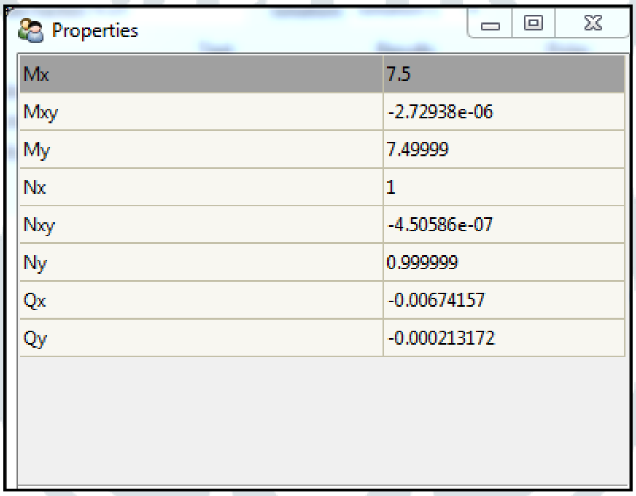
Table 3 Comparison of Ply-wise Global and local Strain results

Global Strain		CLT analysis			ANSYS ACP Prep Post		
Ply no.	Position	ε_x(x 10^{-6})	ε_y(x 10^{-6})	γ_{xy}(x 10^{-6})	ε_x(x 10^{-6})	ε_y(x 10^{-6})	γ_{xy}(x 10^{-6})
1(0°)	Top	0.106	5.97	-3.85	0.316	11.2	-1.76
	Middle	0.178	5.15	-2.82	0.813	16.1	-0.838
	Bottom	0.25	4.33	-1.79	1.3	21	0.315
2(30°)	Top	0.25	4.33	-1.79	0.004	3.4	-2.66
	Middle	0.322	3.51	-0.761	-1.14	10.61	-1.72
	Bottom	0.394	2.68	0.267	-2.38	17.84	-4.73
3(-45°)	Top	0.394	2.68	0.267	1.74	-11.9	-2.74
	Middle	0.466	1.86	1.30	-4.60	-9.68	2.51
	Bottom	0.538	1.04	2.32	-1.68	2.37	-1.82
Local Strain		CLT analysis			ANSYS ACP Prep Post		
Ply no.	Position	ε_1 (x 10^{-6})	ε_2(x 10^{-6})	γ_{12}(x 10^{-6})	ε_1(x 10^{-6})	ε_2(x 10^{-6})	γ_{12}(x 10^{-6})
1(0°)	Top	0.106	5.97	-3.85	0.316	11.2	-1.76
	Middle	0.178	5.15	-2.82	0.813	16.1	-0.838
	Bottom	0.25	4.33	-1.79	1.3	21	0.315
2(30°)	Top	0.495	4.08	2.64	-1.45	4.85	0.139
	Middle	0.789	3.04	2.38	0.308	9.16	4.23
	Bottom	1.08	2	2.12	2.27	13.5	8.52
3(-45°)	Top	1.41	1.67	-2.29	-2.33	-7.8	6.8
	Middle	0.516	1.81	-1.40	-9.65	-4.63	2.54
	Bottom	-0.373	1.95	-0.502	2.16	-1.47	-2.02

Table 4 Comparison of Ply-wise Global and local Stress results

Global Stress		CLT analysis			ANSYS ACP Prep Post		
Ply no.	Position	σ_x(x 10^{-5}) GPa	σ_y(x 10^{-5}) GPa	τ_{xy}(x 10^{-5}) GPa	σ_x(x 10^{-5}) GPa	σ_y(x 10^{-5}) GPa	τ_{xy}(x 10^{-5}) GPa

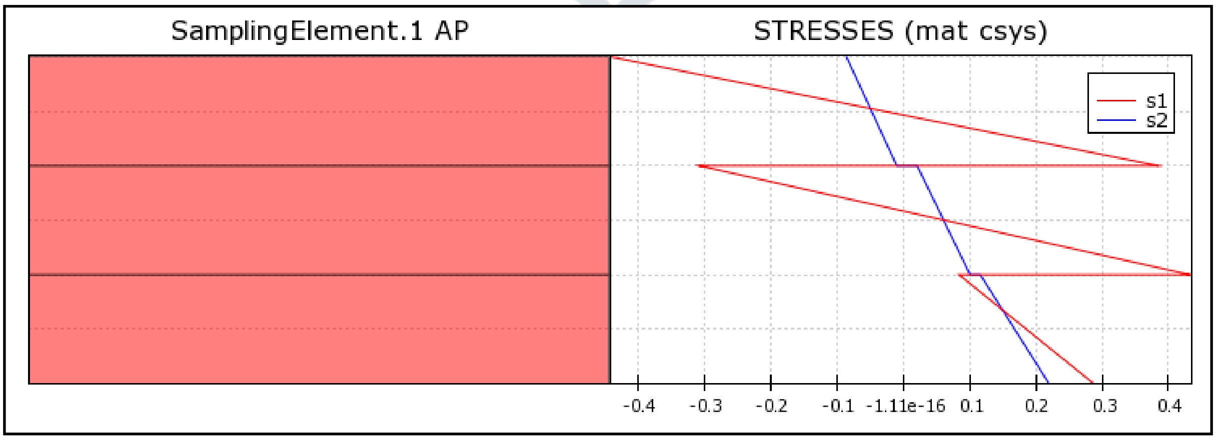
1(0°)	Top	3.67	6.21	-2.76	8.4	12	-1.3
	Middle	4.73	5.38	-2.02	18.6	17	-0.6
	Bottom	5.81	4.55	-1.28	28.7	22	0.22
2(30°)	Top	7.33	7.48	3.46	-10.9	-10.9	15.8
	Middle	11	7.83	5.98	11.1	5.1	1.5
	Bottom	14.6	8.19	8.50	33.8	21.6	-15.7
3(-45°)	Top	12.6	15.8	-12	-21.6	-31.4	17.9
	Middle	5.10	7	-3.94	-2	-5.6	-1
	Bottom	-2.44	-1.76	4.06	18	21	-20.5
Local Stress		CLT analysis			ANSYS ACP Prep Post		
Ply no.	Position	σ_1(x 10^{-5}) GPa	σ_2(x 10^{-5}) GPa	τ_{12}(x 10^{-5}) GPa	σ_1(x 10^{-5}) GPa	σ_2(x 10^{-5}) GPa	τ_{12}(x 10^{-5}) GPa
1(0°)	Top	3.67	6.21	-2.76	8.4	12	-1.3
	Middle	4.74	5.38	-2.02	18.6	17	-0.6
	Bottom	5.81	4.55	-1.28	28.7	22	0.22
2(30°)	Top	10.4	4.44	1.8	-26.7	4.9	0
	Middle	15.4	3.44	1.63	6.6	9.6	3
	Bottom	20.4	2.43	1.46	43.3	12	6.1
3(-45°)	Top	26.2	2.25	-1.56	-44.3	-8.6	4.9
	Middle	10	2.11	-0.95	-2.8	-4.8	1.8
	Bottom	-6.16	1.97	-0.34	40	-1	-1.5



Mx	7.5
Mxy	-2.72938e-06
My	7.49999
Nx	1
Nxy	-4.50586e-07
Ny	0.999999
Qx	-0.00674157
Qy	-0.000213172

Fig. 8 Resultant in-plane forces and moments

Here, applied external actions are [illegible] gnificant moment along X-axis too with in-plane forces N_x and $N_{y.}$ Because of this extra moment M_x, result of ply wise stress and strain differs significantly with CLT analysis. Comparison of these ply-wise stress and strain results are given below.



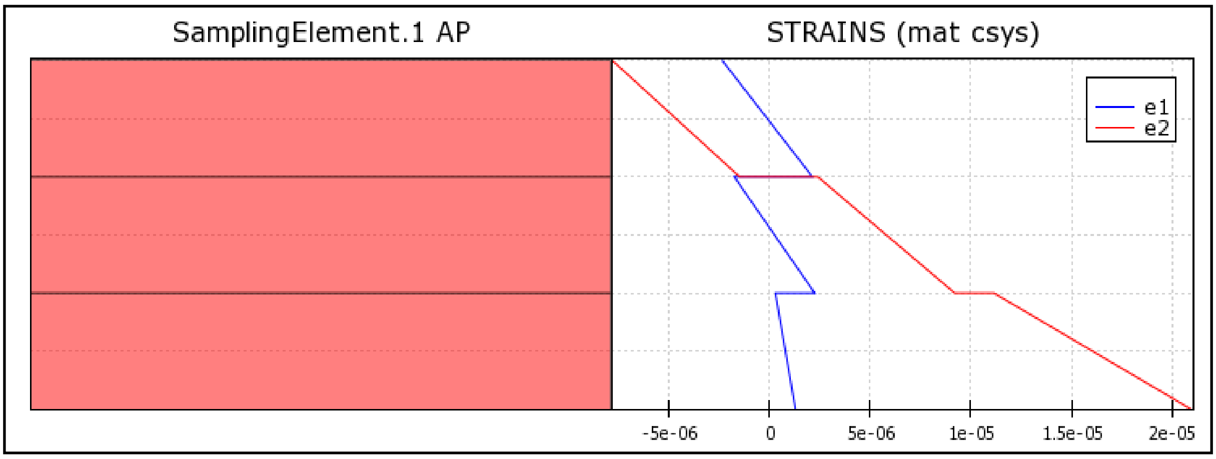


Fig. 9 Graph for Ply-wise stress and strain results

V. CONCLUSION

It can be said that ANSYS® ACP PrepPost gives the result for laminate properties and stiffness matrix with fair degree of accuracy but in case of ply wise stress and strain result, results are not matching at all. The probable reason is the extra moment M_x in addition to applied in-plane forces N_x and N_y..

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