Feature Extraction and Classification of Electroencephalogram Signals for implementation of Brain Computer Communication interface

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Abstract: Brain Computer Interface establishes a communication channel between human brain and external world. Brain Computer Interface is capable of controlling numerous applications and assistive devices such as controlling a cursor on computer screen, movement of a robotic arm or a wheel chair. The efficiency ofa Brain Computer Interface system completely relies on efficient preprocessing and classification algorithms. In present work, a methodology of feature extraction and classification of Electroencephalogram signals is proposed for implementation of Brain Computer Interface for physically disabled. The EEG signal under consideration has been recorded from seven different subjects performing five different mental tasks. Time Frequency Energy Distribution spectrum is computed from the coefficient obtained from Hilbert Huang transform of Electroencephalogram signals. Four statistical parameters are calculated from the TFED of signals as features. For classification, Support Vector Machine classifier model is employed. The results of the classification show efficacy of present methodology of feature extraction and classification for implementation of Brain Computer Interface.

***Index Terms*-- Brain Computer Interface (BCI), Electroencephalogram (EEG), Hilbert Huang Transform (HHT), Support Vector Machine (SVM), Classification**

I. INTRODUCTION

Electrical activity of human brain for different metal states and corresponding to different mental tasks, for reality or imagination can be efficiently recorded simply by placing electrodes on human scalp. Recording of the electrical signals is called Electroencephalograph, which in turn is useful to establish an artificial man machine interface (MMI) or Brain Computer Interface (BCI). A Brain Computer Interface (BCI) is an alternate pathway for communication between human brain and the outside world. BCI capabilities are largely helpful to the patient suffering from severe motor disorder to communicate and control assistive appliances and applications using brain power [1]. A BCI permits an individual with severe motor disabilities to achieve a good control to an external device via the thoughts and works as new mode of human-computer interaction. It also works as a significant diagnostic tool that facilitated the study and analysis of brain abnormalities.

Major breakthrough in the field of BCI research was marked after invention of non-invasive techniques for Electroencephalogram (EEG) signals recording. Previously, electrodes and sensors were implanted inside the brain using invasive techniques. EEG signals recorded by using invasive techniques are less noisy due to less contamination from surroundings. But, user has to go under surgery for implantation [2]. This makes invasive techniques an expensive and cumbersome approach. Various patterns of rhythmic or periodic activity exhibit by EEG signals can be divided in the frequency bands likewise 0.1- 3.5Hz (delta), 4-7.5Hz (theta), 8-13Hz (alpha), 13-30Hz (beta) and >30Hz (gamma) [3-4]. In this work, we have used EEG signals, recorded using non-invasive methods only. The essential steps for BCI realization are recording of brain electrical activity, preprocessing, feature extraction, classification and finally command generation. Successful working of a BCI is determined by careful measures taken during each step. Preprocessing the EEG signals combines filtration and noise removal from raw EEG data. Signals recorded using non-invasive techniques are more prone to noise. Common Spatial Pattern (CSP), Independent Component Analysis (ICA), Principle Component Analysis (PCA) and Multiscale PCA (MSPCA) are very common and mostly used algorithms for EEG signals processing [5]. MSPCA has been applied on different biomedical signals and images for preprocessing and de-noising, like Sharma et al. [6] have proposed MSPCA for denoising multichannel Electrocardiogram (ECG) signals. Zhang and Zhang [7] employed Gabor Wavelet based MSPCA and SVM for face recognition. Many researchers have also applied MSPCA model for analyzing and preprocessing EEG data. Xie and Krishnan [8] has proposed a model of MSPCA by combining Discrete Wavelet Transform (DWT) and PCA for de-noising and decomposing EEG signals in both spatial and temporal domain.Neuromechanism employed for EEG signals recording affects largely the efficiency of BCI in terms of time and classification. Most commonly used neuromechanism are sensorimotor activity [9-11], Visual Evoked Potential (VEP) [12], Slow Cortical Potential (SCP) [13] and P300. EEG signals employed in present work are recorded for imagined motor activity using sensorimotor activity as neuromechanism for left fist blink, right blink and both leg movement.

It is evident from the previous studies that EEG signal are non-stationary in nature [14]. Hence, the EEG signal processing techniques should consider non-stationary characteristics of EEG signalseffectively. Among variousnon-stationary techniques available, the Hilbert Huang Transform (HHT) technique has been extensively used in recent years for non-stationary signal analysis in diverse domains such as in speech signal processing [15], biomedical engineering [16] and fault feature identification [17].

In present work, EEG signals recorded for five different mental tasks are i.e. baseline tasks, figure rotation task, letter composition task, Maths task and visual counting task are considered for BCI implementation. Firstly, signals are passed through MSPCA model for reconstructing simplified EEG signals. Further, HHT based method is applied for the successful implementation of the BCI. Empirical Mode Decomposition (EMD) is performed on EEG signals to obtain series of mono-component modes with local characteristic time scales called Intrinsic Mode Functions (IMF). Further, Instantaneous Frequency of each IMF was represented through Hilbert transform.The methodology proposed in present work is useful for extracting features, classifying mental task based EEG data using extracted features, and generating command with higher classification efficiency. EEG signal shows different classification performance for different set of subjects. Proposed methodology is used for analyzing feasibility of EEG signals recorded corresponding to different mental tasks, for higher classification efficiency.Fig.1 shows overall methodology presented in this work.

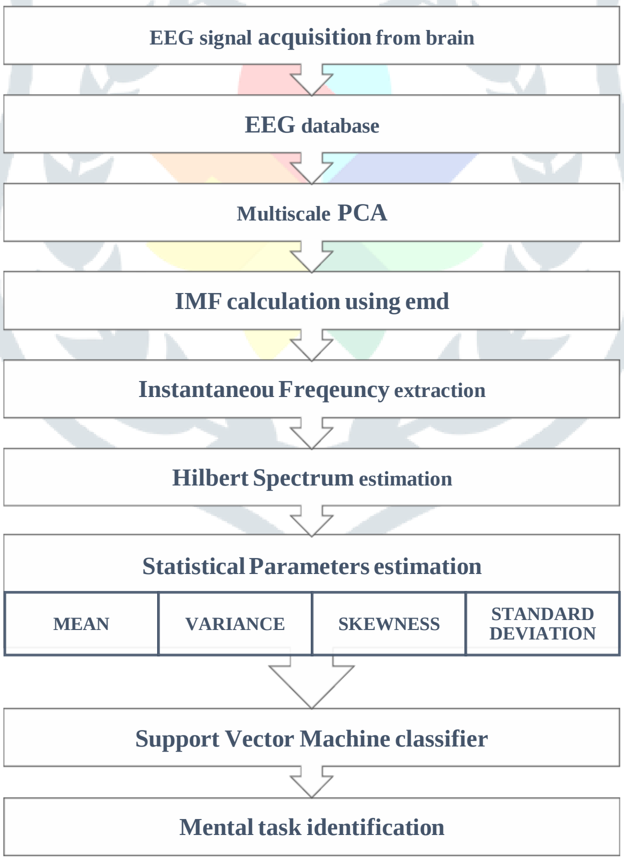


Fig. 1 Methodology proposed for EEG signals feature extraction and classification

II. EEG DATABASE

The EEG database used in present study is taken from Keirnand Aunon study [18]. The subjects were seated in a sound controlled compartment with shadowyenvironment and noise-less ventilation. To record EEG signals, elastic electrode were used and placed at the six different positions on scalp viz. C3, C4, P3, P4, O1 and

O2. The electrodes wereplace as per theinternational 10 -20 system of electrode placement [19]. However, the reference is taken as A1 and A2 to electrically linked mastoids. Theelectrodes are connected through a bank of amplifiers (Grass7P511), whose band-pass analog filters are set at 0.1 to100 Hz. The data are sampled at 250 Hz with a Lab Master 12-bit A/D converter mounted on a computer. Signals are recorded for 10 sec during one trial andeach task is performed for generating 10 such trials. In this paper, EEG signals from seven subjects performingfive different mental tasks are considered for analysis. The five mental tasks are described as follows:

1. Baseline task: The subjects were asked to relax without any thinking in mind in particular. This task is used as a reference and as a baseline measure of the EEG signals.

2. Math task: The subjects wereasked to solve multiplication problems, such as 13times 17 without vocalizing or making any other physical overt movements. The tasks are non-repeating and difficult so that an immediate answer is not obvious. The subjects verified at the end of the task for the solution. It was ensured that no subject completed the task before the 10s duration recording time.

3. Figure rotation task: The subjects were given to observe the drawing of a three dimensional block object, after which the drawing was removed and the subjects were asked to visualize the object being rotated about an axis.

4. Letter task: The subjects were asked to mentally compose a letter to a person without vocalizing. The task was repeated several times and the subjects was asked to continue with the letter from where they left off in the next period.

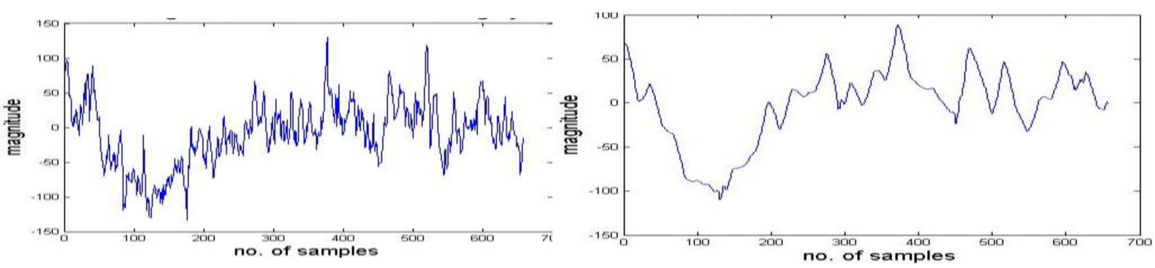
5. Visual counting task: The subjects were asked to imagine a blackboard and to visualize numbers being written on the board sequentially, with the previous number being erased before the next number was written. The subjects were instructed not to verbalize the numbers but to visualize them.

III. OVERVIEW OF TECHNIQUES

3.1 MULTISCALE PCA

When the behavior of the events associated with various phenomenon changes rapidly with time and frequency, MSPCA is found to be suitable modelling technique. The modelling of single scale data can be efficiently performed by PCA alone, as algorithm considers relationship of data for same time-frequency localization. However all practical data sources generate data which is multiscale in nature. Hence, it is feasible to use PCA with multiscale capabilities. MSPCA performs computation of PCA on wavelet coefficients at each scale and combines results at significant scales [20]. In MSPCA algorithm, first the data is decomposed in time-frequency using certain wavelets. Then the PCA is applied for the coefficients of each scale irrelevant to one another. Combination of the models corresponding to important scales takes place to obtain a unified model [20].

In present work, we have applied MSPCA to simplify the EEG data matrix. For decomposition of EEG data, Symlets wavelet is selected and decomposition level was set to *5*. However, Kaiser's rule is followed for retaining Principle Components (PCs). According to Kaiser's rule, only those components are retained which are having Eigen values (Variance) greater than the mean of all Eigen values. Fig.2 (a-b) shows the EEG data plots (in µV) before applying MSPCA and after application of MSPCA.



(a) (b)

Fig. 2 (a) EEG signals before applying MSPCA (b) Simplified EEG signals after applying MSPCA

3.2SUPPORT VECTOR MACHINE

SVM is the statistical learning classification algorithm, which can be applied for classification of two or multi classes of data. SVM model was developed by Vapnik in 1995 [21]. SVM is being used by many researchers because of its attractive features such as Empirical Risk Minimisation (ERM) principle and Structural Risk Minimisation (SRM) principle. This makes SVM a strong classification tool [22]. In SVM a hyperplane is created which maintainsa large gap between the two classes of data. Support Vectors are closest point to this hyperplane and SVM model ensures maximum gap for closest support vectors of both the classes from hyperplane [23].

Let data set be (x_i, y_i),with L training points, where each input x_i has D attributes, and belongs only one of two classes $y_i = -1$ or $+1$.If w is normal to the hyperplane and b/w is perpendicular distance from the hyperplane tothe origin, expression for hyperplane can be written as:

$$w * x + b = 0 \tag{1}$$

Fig.3 represents hyperplane, separating two linearly separable classes. *L1* and *L2* is the maximum distance of support vectors from hyperplane.

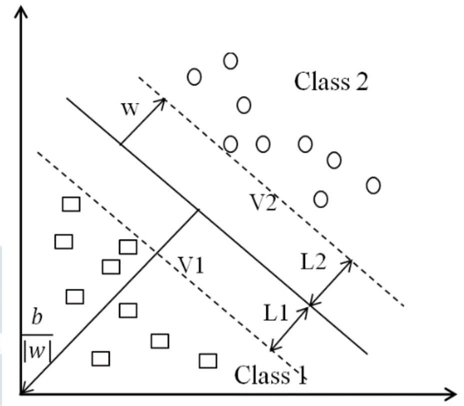


Fig.3 Hyperplane separating two classes of data.

3.3 Hilbert Huang Transform (HHT)

It is an effective tool for the analysis of nonlinear and non-stationary signals. It provides a good description of local oscillating frequency components present in the signal. HHT is the combination two basic steps: Empirical Mode Decomposition (EMD) and Hilbert Spectral Analysis.

3.3.1 Empirical Mode Decomposition (EMD)

If we consider any arbitrary signal as composed of a series of different intrinsic oscillation modes, then the EMD [24] can be used for decomposition the incoming signals into its different Intrinsic Mode Functions (IMF).The algorithm of generating IMFs is given in the following steps:

Step1: Determine the local extremes (maxima and minima) of the signal $x(t)$.

Step2: Generate the lower envelope $e_l(t)$ and upper envelope $e_m(t)$ by connecting all local minima and local maxima respectively using spline interpolation function.

Step3: Determine the local mean as $m(t = (e_l(t)+e_m(t))/2$.

Step4: Subtract the local mean from the signal, $h_i(t)=x(t)–m(t)$.

Step5: Check whether $h_i(t)$ is an IMF or not by checking the basic conditions mentioned above. If $h(t)$ satisfies the criteria, then set $IMF_i = h_i(t)$, and assign $c_i(t) = IMF_i(t)$.Else $x(t)=h_i(t)$ and repeat steps *1-4*.

Step6: To determine rest of IMFs, let $r_i(t)=x(t)–IMF_i$. Repeat the steps *1-5* assuming $r_i(t)$ as new $x(t)$ until $r_i(t)$ becomes a monotonic residual signal.At the end of the decomposition, $x(t)$ can be written as:

$$X(t) = \sum_{n=1}^{M} c_n + r_M \tag{2}$$

Where, N is the number of intrinsic mode functions, $c_n(t)$ is the n^{th} IMF and $r_m(t)$ is the final residual monotonic function.

However, an IMF is a function that satisfies following two conditions:

1. In the entire signal, the number of extremes and the zero-crossings must be equal or differ at most by one.
2. At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima must be zero.

3.3.2 Hilbert spectral analysis

IMFs generated using EMD generates signals which contain one frequency component at one time. Further, Hilbert transform is applied on every IMF obtained by EMD to compute IFs. Hilbert transform cannot be directly applied on the non-stationary and nonlinear signal which are having more than one frequency component at a given time. For signal $x(t)$ of L_p class [25], Hilbert transform is:

$$Y(t) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x(\tau)}{(t-\tau)} d\tau \tag{3}$$

where, PV denotes the Cauchy's principle integral value. Using this, analytic function $Z(t)$ can be computed as:

$$Z(t) = x(t) + j.y(t) = a(t)e^{j\phi(t)} \tag{4}$$

Computation of instantaneous frequencies can be given as below.

$$F(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} \tag{5}$$

3.4 Time frequency energy distribution (TFED)

The Time-Frequency–Energy Distribution (TFED) can represent the characteristics of signal in bothtime and frequency domains. More subtle features can be obtained from TFED [26,27].It is much better than the traditional approaches like Short-Time Fourier Transform (STFT), Wigner–Ville, Choi–Williams Distribution [28]. As, thesetransforms are not able to decompose thesignals adaptively and are also constrained by theuncertain principle which results in the conflictionbetween time resolution and frequency resolution [28, 29]. The Hilbert spectrum is calculated as:

$$H(w,t) = Re\left(\sum_{i=1}^{n} a_i(t)e^{j\int Wi(t)dt}\right) \tag{6}$$

The Hilbert Spectrum (HS) represents the TFED.The overall HS is expressed as the superposition of the individual IMFs' HSs defined as:

$$H(k,t) = \sum H^{(b)}(k,t)\ ;\ b=1, 2,\ldots\ldots,B.$$

Hence, each element $H(k,t)$ of HS is defined as the weighted sum of theinstantaneous amplitudes of all the IMFs atk^{th}frequencybin.

IV. BCI IMPLEMENTATION

4.1 FEATURE EXTRACTION AND CLASSIFICATION

HHT based features are extracted from preprocessed EEG signals and feature vector is prepared to train classifier. The feature extraction is carried out in three steps. In the first step, HHT is performed. Which is followed by the calculation of Time-Frequency Energy Distribution (TFED) also known as Hilbert spectrum, in the second step. HHTis a self-adaptive signal analysis method.The Hilbert Spectrum based on HHT offers amplitude distribution with the change of time and frequency i.e. it provides a complete TFED which is more accurate than the other time– frequency analysis methods including wavelets [30-32]. Therefore, it is suitable to obtain the TFED of a signal using HHT. At last, the estimation of time-frequency parameters is applied for feature extraction.In this work, four statistical parameters i.e. **Mean**, **Standard Deviation**, **Skewness** and **Variance** have been computed from Hilbert spectrum to prepare the final feature vectors.

In present work, SVM classifier is employed for classification or identification of the class of EEG signals. Once the SVM classifier is trained, it can be used for recognition of EEG signals and consecutively generate the external command. The generated external command can be used for controlling various assistive devices and applications.

V. RESULTS

Statistical features obtained from Hilbert spectrum of calculated of brain signals recorded during performing five different mental tasks are used to prepare input feature vector for training and validation of classification algorithm SVM. The signals have been recorded from seven different subjects while performing five tasks in certain number of trials to attain a better accuracy. The IMFs of EEG signal corresponding to each task has been computed. After that, IF vectors has been computed for each IMFs using Hilbert transform which is further used to calculate the Hilbert spectrum and statistical features. The feature vectors are passed through the SVM classifier to have high accuracy.The confusion matrix prepared using the SVM is prepared for all the five tasks for each seven subjects. Table 1-7 shows the confusion matrix prepared for the tasks for subject 1 to subject 7 respectively. Table 8represents the classification accuracy obtained after considering only two tasks (out of five tasks) at a time, for all the seven subjects.

Table (1) Confusion matrix for Subject-1

BAS	COU	LET	MUL	ROT	CLASSIFIED AS
65	0	3	1	1	Baseline (BASE)
0	66	1	0	3	Counting (COU)
2	3	63	1	1	Letter (LET)
0	0	0	70	0	Multiplication (MUL)
1	1	1	0	67	Rotation (ROT)

Table (2) Confusion matrix for Subject-2

BAS	COU	LET	MUL	ROT	CLASSIFIED AS
34	0	0	1	0	Baseline (BASE)
0	23	9	0	3	Counting (COU)
0	10	20	1	4	Letter (LET)
1	1	0	32	1	Multiplication (MUL)
0	6	6	1	15	Rotation (ROT)

Table (3) Confusion matrix for Subject-3

BAS	COU	LET	MUL	ROT	CLASSIFIED AS
66	2	1	0	1	Baseline (BASE)
1	63	5	0	1	Counting (COU)
1	2	61	2	4	Letter (LET)
0	0	2	68	0	Multiplication (MUL)
1	2	6	1	60	Rotation (ROT)

Table (4) Confusion matrix for Subject-4

BAS	COU	LET	MUL	ROT	CLASSIFIED AS
64	1	4	0	1	Baseline (BASE)
0	69	1	0	0	Counting (COU)
1	3	65	1	0	Letter (LET)
2	0	0	66	2	Multiplication (MUL)
4	0	0	8	51	Rotation (ROT)

Table (5) Confusion matrix for Subject-5

BAS	COU	LET	MUL	ROT	CLASSIFIED AS
66	1	1	2	0	Baseline (BASE)
1	68	1	00	0	Counting (COU)
0	0	67	2	1	Letter (LET)
2	0	1	66	1	Multiplication (MUL)
0	0	0	1	69	Rotation (ROT)

Table (6) Confusion matrix for Subject-6

BAS	COU	LET	MUL	ROT	CLASSIFIED AS
59	3	4	1	3	Baseline (BASE)
4	60	3	3	0	Counting (COU)
7	1	55	6	1	Letter (LET)
2	1	5	66	1	Multiplication (MUL)
1	1	0	0	68	Rotation (ROT)

Table (7) Confusion matrix for Subject-7

BAS	COU	LET	MUL	ROT	CLASSIFIED AS
30	2	0	2	1	Baseline (BASE)
4	26	3	2	0	Counting (COU)
0	7	25	3	0	Letter (LET)
1	2	1	29	2	Multiplication (MUL)
10	0	0	1	33	Rotation (ROT)

Table (8) Classification accuracy for one-versus-one mental tasks for all seven subjects using SVM classifier

TASKS	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**
BAS_CON	97.85%	100%	97.14	98.57	97.14	89.28	94.28
BAS_LET	97.85%	100%	97.85%	96.99	97.14	87.85	94.28
BAS_MUL	98.57%	100%	97.14	96.42	94.28	93.57	90
BAS_ROT	98.57%	97.14%	98.57	98.57	98.57	92.85	95.71
COU_LET	96.42%	65.71%	90.71	98.49	98.57	92.14	77.14
COU_MUL	100%	76.19%	98.57	96.42	100	96.42	84.28
COU_ROT	97.14%	100%	96.43%	97.14	100	96.42	100
LET_MUL	98.57%	74.60%	96.42	98.49	97.85	92.14	88.57
LET_ROT	97.85%	95.71%	92.14	91.72	97.85	95	88.57
MUL_ROT	100%	96.82%	97.85	94.28	97.85	97.85	91.42
All tasks	94.57%	73.80%	90.85	91.83%	96%	86.57	81.71

The classification results obtained with the methodology presented in this paper are shown in Table 1-8. The classification accuracy has been calculated considering two certain tasks at a time. In this way, all relative classification accuracy between every two tasks has been compared. Finally, an average accuracy has been calculated considering all the tasks at a time and shown in Table 8. It can be perceived from Table 8that the classification accuracies for the most of the tasks combinations is greater than 80%, with many task combinations having a classification accuracy of 100%.

VI. CONCLUSION

In this work a feature extraction method is proposed for the processing of EEG signals recorded for performing mental tasks and classification of those mental tasks. The proposed methodology helps in successful implementation of the Brain Computer Communication channel. It is based on the HHT and the estimation of statistical parameters such as standard deviation, variance, mean and skewness for Hilbert spectrum. The efficacy of the present methodology can be confirmed by the high classification accuracy for one-versus-one mental task classification using a simple SVM classifier. The average classification accuracy obtained is around above than 90%. So, This work suggests a simple feature extraction technique, which can be effectively used for the implementation of the Brain Computer Communication channel for controlling assistive devices and the assistive applications.

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