GAME THEORY – NASH EQUILIBRIUM AND ITS APPLICATIONS

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ABSTRACT:

Game theory is the mathematical study of strategic decision making in situations of conflict. In game theory, a single interaction is defined as a game, and those involved in the decision-making are called the players, who are assumed to act rationally. We will explore the basic ideas of game theory, including the ideas of payoffs and outcomes; classification of games as zero-sum or nonzero-sum; types of strategies and counterstrategies and the idea of dominance. Game theory has many applications in subjects such as economics, international relations and politics, and psychology as it can be used to analyze and predict the behavior and decisions of the players. Let's look how game theory is used in the game – CRICKET.

Keywords: Game theory, payoffs, zero game, nash equilibrium, game- cricket.

. I. INTRODUCTION:

Game theory is the mathematical study of strategic decision making. It can be used to analyze the options, motivations, and rewards involved in a decision. This paper will discuss some of the basic concepts and modelling tools of game theory. We will also give examples of famous and common applications of game theory. Game theory is widely applied in the real world. Major areas of application include economics, diplomacy, and military strategy. Game theory can also be applied in fields such as psychology, biology, political science, computer science, sociology, and moreatical study of strategic decision making. It is used to find the optimal outcome from a set of choices by analyzing the costs and benefits to each independent party as they compete with each other. According to game theory, one always lose, and another player always wins.

1.1BASIC CONCEPTS OF GAME THEORY:

ZERO SUM GAME:

Game theory provides a mathematical framework for analyzing the decision-making processes and strategies of adversaries (or *players*) in different types of competitive situations. The simplest types of competitive situations are two-person, zero-sum games. These games involve only two players; they are called *zero-sum* games because one player wins whatever the other player loses.

Example: Odds and Evens

Consider the simple game called odds and evens. Suppose that player 1 takes evens and player 2 takes odds. Then, each player simultaneously shows either one finger or two fingers. If the number of fingers matches, then the result is *even*, and player 1 wins the bet ($2). If the number of fingers does not match, then the result is *odd*, and player 2 wins the bet ($2). Each player has two possible strategies: show one finger or show two fingers. The *payoff matrix* shown below represents the payoff to player 1.

1.2N- PERSON GAME:

N-person game theory provides a logical framework for analyzing contests in which there are more than two players or sets of conflicting interests-anything from a hand of poker to the tangled web of international relations. In this sequel to his *Two-Person Game Theory,* Dr. Rapoport provides a fascinating and lucid introduction to the theory, geared towards readers with little mathematical background but with an appetite for rigorous analysis.

1.3ZERO - SUM GAME:

A zero sum game is a term used in game theory to describe both real games and situations of all kinds, usually between two players or participants, where the gain of one player is offset by the loss of another player, equaling the sum of zero. For instance, if a person plays a single game of chess with someone else, one person will lose and one person will win. The win (+1) added to the loss (-1) equals zero.

1.4PLAYER:

A person or thing that plays . A person who takes part or is skilled in some game or sport.

1.5STRATEGY:

A player's strategy set defines what strategies are available for them to play.

A player has a finite strategy set if they have a number of discrete strategies available to them. For instance, a game of rock-paper-scissors comprises a single move by each player - and each player's move is made without knowledge of the other's, not as a response - so each player has the finite strategy set {rock, paper, and scissors}.

1.6PURE STATEGY:

A pure strategy provides a complete definition of how a player will play a game. In particular, it determines the move a player will make for any situation they could face. A player's strategy set is the set of pure strategies available to that player.

1.7MIXED STRATEGY:

A mixed strategy is an assignment of a probability to each pure strategy. This allows for a player to randomly select a pure strategy. (See the following section for an illustration.) Since probabilities are continuous, there are infinitely many mixed strategies available to a player.

Of course, one can regard a pure strategy as a degenerate case of a mixed strategy, in which that particular pure strategy is selected with probability *1* and every other strategy with probability *0*.

A totally mixed strategy is a mixed strategy in which the player assigns a strictly positive probability to every pure strategy.

	A	B
A	1, 1	0, 0
B	0, 0	1, 1

Table 1:payoff matrix

Consider the payoff matrix pictured to the right. Here one player chooses the row and the other chooses a column. The row player receives the first payoff, the column player the second. If row opts to play A with probability 1 (i.e. play A for sure), then he is said to be playing a pure strategy. If column opts to flip a coin and play A if the coin lands heads and B if the coin lands tails, then he is said to be playing a mixed strategy, and not a pure strategy.

1.8OPTIMAL STRATEGY:

One of the pair of mixed strategies carried out by the two players of a matrix game when each player adjusts stra tegy soas to minimize the maximum loss that an opponent can inflict.

1.9MINIMAX PRINCIPLE:

A principle for decision-making by which, when presented with two various and conflicting strategies, one should, by the use of logic, determine and use the strategy that will minimize the maximum losses that could occur. This financial and business strategy strives to attain results that will cause the least amount of regret, should the strategy fail.

1.10 MAXIMIN PRINCIPLE:

The maximin principle is a principle for making choices when one is not sure of the outcome that will result from one's choice. The principle says to evaluate each option in terms of the worst possible outcome that could result from choosing that option, and to pick the option that offers the best worst outcome (the maximum minimum or maximin). Rational choice theory generally divides situations in which agents do not know for sure the outcome

Nash Equilibrium:

It is a concept that determines the optimal solution in a non-cooperative game in which each player lacks any incentive to change his/her initial strategy. Under the Nash equilibrium, a player does not gain anything from deviating from the initially chosen, assuming the other players keep their strategies unchanged. A game may include multiple Nash equilibria or none of them.

Nash equilibrium in a payoff matrix:

The rule goes as follows: if the first payoff number, in the payoff pair of the cell, is the maximum of the column of the cell and if the second number is the maximum of the row of the cell - then the cell represents a Nash equilibrium.

Player 2 Player 1	Option A	Option B	Option C
Option A	0, 0	25, 40	5, 10
Option B	40, 25	0, 0	5, 15
Option C	10, 5	15, 5	10, 10

A payoff matrix – Nash equilibria in bold

Game theory is the mathematical study of strategic decision making. It is used to find the optimal outcome from a set of choices by analyzing the costs and benefits to each independent party as they compete with each other. According to game theory, one always lose, and another player always wins.

Some relevant examples of game theory used in everyday life.

1. *Chess*

Figure1: game theory in chess.

We all have played the game chess once or more in our life. It depends upon the players, how they use the moves to win the game. The rules of the game are known to both the players and have remained unchanged which makes it a game of perfect information. So, chess is an example of game theory as both players know the possible moves and the effects of those moves.

2. *War Strategies*

Figure2: game theory in war strategies.

India's muscular riposte to Pakistan's perfidy post-Uri, post-Pulwama has underpinnings in game theory. After the jihadi attack on the Uri army camp in 2016, India launched its "surgical strike." This year, after a suicide bomber rammed a Central Reserve Police Force convoy and killed 40 jawans, the Indian Air Force struck a terrorist camp in Balakot deep inside Pakistan, in Khyber Pakhtunkhwa province. These war strategies and military decisions are examples of game theory. In general, the military head or commander select the course of action which offers the most significant promise of success in view of the enemy's capabilities of opposing him.

3. Rock, Paper and Scissor Game

Figure3: game theory in rock, paper, scissors.

Have you ever got into disputes with your friend and you couldn't decide who is right or wrong? Then the game rock, paper, and scissors remain the only option and the one who wins; wins the dispute. In this game like chess, we know the consequences but are not aware of another player is going to do.

4. Poker Card Game

Figure4: game theory in poker card game.

Most of us have seen people losing huge amount of money in poker clubs in movies as well as in real. Poker card game exemplifies the game theory correctly because one wins exactly the amount one's opponents lose.

5. *Evolution*

Figure5: game theory in evolution.

Humans usually imitate other people in living and survival. Evolution is a popular application of game theory; for example, people follow the trends and strategies for survival. Survival not only depends upon fitness but instead also depends upon evaluating how others in the same community are faring based on their actions. Because of this, it is important to figure out how certain survival strategies come to be adopted.

6. *Market Shares and Stockholders*

Figure6: game theory in market shares and stockholders.

By investing in the stock market, you become a player. You have invested your money in a company knowing that you will either make money or lose money, but you don't know what will happen. The company needs your investment to thrive. The decisions the company makes will either drive the price of its stock up or down, which determines its future success. The stockholder does not know what decisions the company will make, and the company does not know what decision the stockholder will make

II. CRICKET RULES:

Figure7: cricket ball and bat.

- Bowling and Fielding Strategy and Tactics. Field Placement. Choosing Bowlers. Taking the New Ball. Bowling the Over. Enforcing the Follow-On.
- Batting Strategy and Tactics. Batting Order. Batting Shot Selection. Sharing the Strike. Declaring the Innings Closed.

. The article considers decision making in cricket, specifically for batsmen facing fast bowlers – defined as bowlers who can consistently bowl at speeds above 90 miles an hour. It starts off with a specific situation in a game in 1981 and then goes on to, in general terms, comment on how and why certain batsmen can play fast bowlers effectively and others cannot.

The concept of game theory, taught in INFO 2040, is applied to a specific scenario in cricket. The game in this case would be the batsman facing the fast bowler, where the players are the batsman and the fast bowler. Each of the players has his own strategies. On a simplified level, the batsman's and the bowler's strategies are mentioned below.

For the batsman, his strategies are to:

- Attack the ball
- Defend the ball
- Take evasive action For the

bowler, his strategies are to:

- Bowl at the batsman's body
- Bowl towards outside the wickets
- Bowl towards the batsman's head (the article makes mention of aiming for the batsman's chin – such a ball is called a bouncer)

2.1STRATEGY OF BATTING:

Figure8: Batsman batting in field.

Just as Pitching controls the game of baseball, BATTING *controls the game in cricket.* So, you must start with batting to understand cricket strategy.

Recall that, in cricket, the batter (or BATSMAN) can *hit in every direction, all around him.* He uses many different kinds of "strokes" to do this...."driving" with a full golf-like swing to hit straight ahead or slightly to left or right, "pulling" or "hooking" to hit the ball across his body and to his left, "late- cutting" or "glancing" at the delivery, to just deflect a fastball past the catcher (wicket-keeper), and so on.

The BATSMAN'S STRATEGY, and *how* he executes his hits, is illustrated in the diagram below.

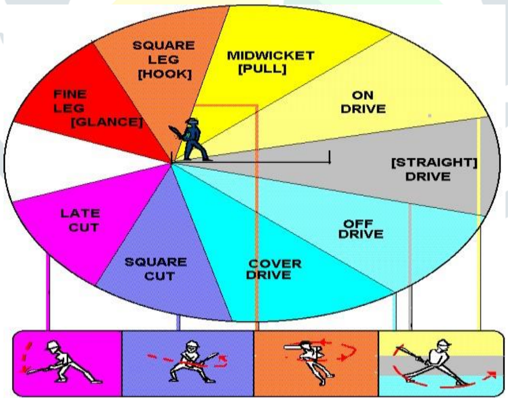


Figure9: batsman strategy.

2.2*BOWLER'S STRATEGY:*

Figure10: bowler bowling.

In cricket, the PITCHER (called the BOWLER) can *bounce the ball* on the ground if, but ONLY if, he wants to. That means *two* things. First, he can bounce the ball at *different distances* from the batter, getting him to mis-step in deciding how to deal with the pitch. Second, he can *do more things with the ball* .... not only move it in the air, like baseball pitchers do, but also "break", i.e. change directions after bouncing off the ground. By combining movement in the air with "breaks" off the ground, and *also* varying his length at the same time, he can throw some very complicated pitches!

All these options available to the cricket PITCHER, or BOWLER, are shown in the diagrams:

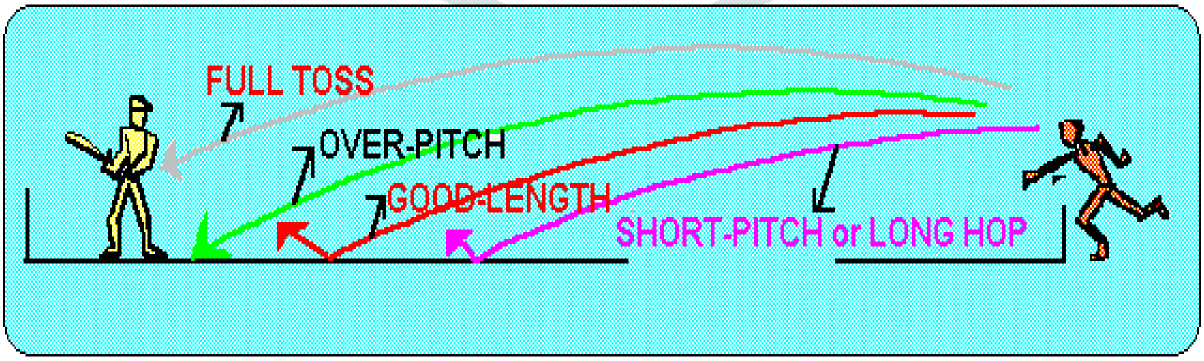


Figure11: options available for pitcher and bowler.

2.3<u>GAME THEORY IN CRICKET:</u>

ASSUMPTIONS:

- Consider that the attack or defend is between only one batsman and one bowler.
- Focusing only one ball that is bowled.

NASH EQUILIBRIUM:

The situation where none of the player can dominate the other one. The equilibrium occurs when each player's strategy is optimal, knowing the strategy of the other player.

MIXED NASH EQUILIBRIUM:

The strategy that is used in cricket is the mixed nash equilibrium.

The ACTIONS OF THE BATSMAN AND THE BOWLER decides the strategy such as attacking or defending.

ACTION OF THE BATSMAN:

Figure12: action of the batsman.

ACTION OF THE BOWLER:

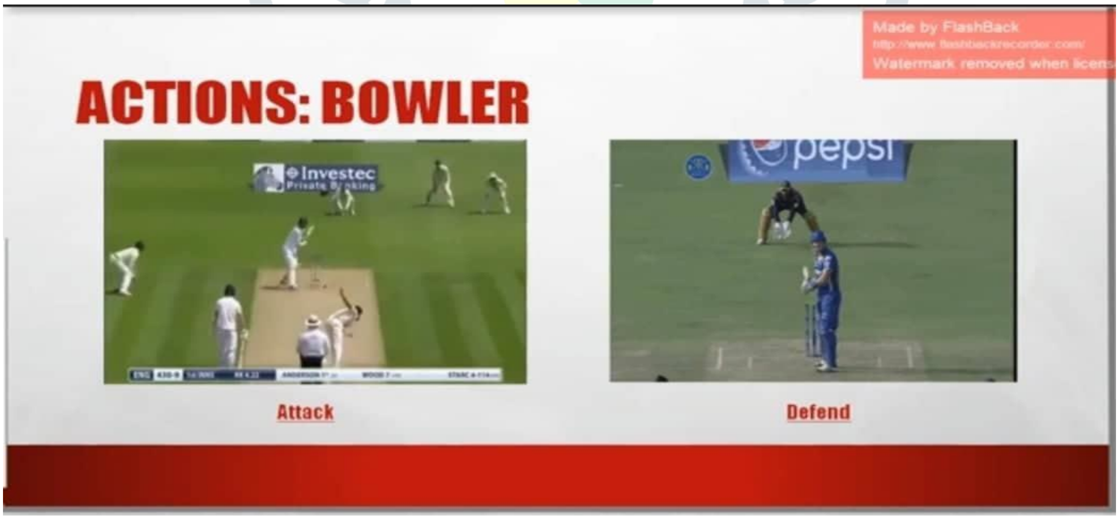
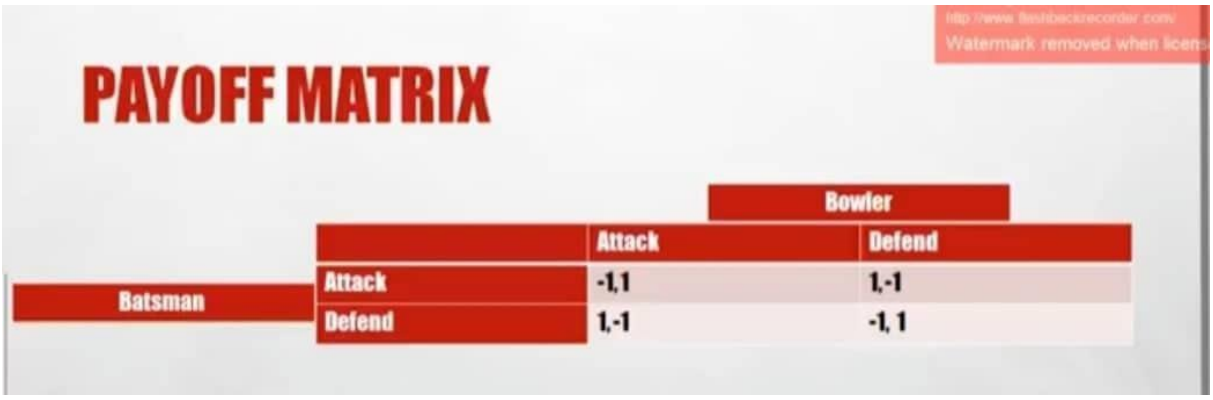


Figure13: action of the bowler.

NOW,

The payoff matrix obtained, when the batsman attacks whereas bowler defends and when bowler attacks whereas batsman defends, is shown below.



		Bowler	
		Attack	Defend
Batsman	Attack	-1,1	1,-1
	Defend	1,-1	-1, 1

Figure14: payoff matrix.

MIXED NASH EQUILIBRIUM IF APPLIED:

Concerned about the batsman:

		Bowler	
		Attack (p)	Defend (1-p)
Batsman	Attack (q)	-1,1	1,-1
	Defend (1-q)	1,-1	-1, 1

Figure15: mixed nash equilibrium. Batsman's

expected payoff from playing attack: p . (-1) + (1-p) . 1 = 1-2p. Batsman's

expected payoff from playing defending: p . (1) + (1-p) .(-1) = 2p-1.

If we set 1-2p = 2p-1, then p is obtained as 0.5.

We can observe that:

- If the bowler plays attack with the probability 0.5 then the batsman will play defend with the probability 0.5.
- If the bowler plays defending with the probability 0.5 then the batsman will play attack with probability 0.5.
- Both the batsman and the bowler will be indifferent in playing attack and defend.

III. CONCLUSION.

Thus the CONCLUSION is that:

- The mixed strategy profile would be (0.5A\0.5D , 0.5A\0.5D)
- Both the batsman and the bowler are equally as likely to play each actions.
- Therefore, in equilibrium both the batsman and bowler should randomize their

actions. This is an illustration for 2*2 matrix.

Let’s have a general look of how 3*3 matrix is used in game theory:

This the general payoff matrix of a batsman facing 3 kinds of ball, and the expected runs as per the strategy of the batsman.

	Batsman		
	Remain still	Come down the pitch	Scoop Shot
Yorker	0	6	4
Bouncer	4	0	2
Slower Ball	2	4	2

Figure16: general payoff matrix.

But the strategy differs ad the bowler has his own strategy. Thus the payoff matrix may be in other form, this is according to the bowler:

	Batsman		
	Remain still	Come down the pitch	Scoop Shot
Yorker	6, 0	0, 6	2, 4
Bouncer	2, 4	6, 0	4, 2
Slower Ball	4, 2	2, 4	4, 2

Batsmen Utility = runs
Bowler Utility = (6-runs)

Figure17: utility of the batsman and bowler.

Expected Batsman Payoffs:

Remain still:
=0p+4q+2(1-p-q)
=2+2q-2p

Come down the pitch:
=6p+0q+4(1-p-q)
=4+2p-4q

Scoop shot:
=4p+2q+2(1-p-q)
=2+2p

Batsman

Bowler	Remain still *a*	Come down the pitch *b*	Scoop Shot *1-a-b*
p Yorker	6, 0	0, 6	2, 4
q Bouncer	2, 4	6, 0	4, 2
1-p-q Slower Ball	4, 2	2, 4	4, 2

Indifferent when expected payoffs equal:
2+2q-2p=4+2p-4q=2+2p

Solving this set of simultaneous equations gives

$$p = \frac{1}{4} \qquad q = \frac{1}{2}$$

Figure18: expected batsman payoffs.

Expected Bowler Payoffs:

Yorker:
=6a+0b+2(1-a-b)
=2+4a-2b

Bouncer:
=2a+6b+4(1-a-b)
=4-2a+2b

Slower Ball:
=4a+2b+4(1-a-b)
=4-2b

Batsman

Bowler	Remain still *a*	Come down the pitch *b*	Scoop Shot *1-a-b*
p Yorker	6, 0	0, 6	2, 4
q Bouncer	2, 4	6, 0	4, 2
1-p-q Slower Ball	4, 2	2, 4	4, 2

Indifferent when expected payoffs equal:
2+4a-2b=4-2a+2b=4-2b

Solving this set of simultaneous equations gives:

$$a = \frac{1}{2} \qquad b = \frac{1}{4}$$

Figure19: expected bowler payoffs.

<u>*CONCLUSION:*</u>

Shabab Ahamed as already introduced a paper normal sum game in cricket using Nash Equilibrium but |I have introduced in this paper game theory used in the game cricket using Nash Equilibrium and Mixed Nash Equilibrium .In two person game, the players are independent. There will be only one winner and one looser
.but in Nash equilibrium, the players are dependent. There won't be any looser or winner, only the team wins both or loses not the players. The batsman and the bowler have their own strategy and its indifferent

to each other.

The equilibrium is maintained thus both the bowler and the batsman have their randomized actions.

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