Computational Method for finding Heat Transfer with Viscous Dissipation in Couette-Poiseuille Flow under Asymmetric Wall Heat Fluxes

***Renuka M Budihal. Asst Professor, Govt First Grade College. Davanagere.**

Abstract –

The heat transfer characteristics of a laminar Coutte –Poiseulle flow is analyzed taking into account viscous dissipation. Some interesting result in terms of Brinkman number and heat flux ratio was observed. The case of lower plate being fixed and the upper plate moving with constant velocity and both being imposed to different but constant heat fluxes is considered.The energy equation is solved using Scilab 5.4.1 leading to expression in temperature profiles and it is found that Brinkman number and speed of the moving upper plate has a great impact on thermal development. **Keywords: Coutte-Poiseulle flow, heat flux, viscous dissipation.**

INTRODUCTION

Flow of Newtonian fluids through various channels is of practical importance and heat transfer is dependent on flow conditions such as flow geometry and physical properties. Investigations in heat transfer behavior through various channels showed that the effect of viscous dissipation cannot be neglected for some applications, such as flow through micro-channels, small conduits and extrusion at high speeds[1].

However, the moving boundary deform the fluid velocity profile, and shears the fluid layer near the boundary, and thus, results in local changes in velocity gradient. Hence, the viscous dissipation effects cannot be neglected in heat transfer analysis of system associated with moving boundaries[2,3].

The thermal development of forced convection through infinitely long fixed parallel plates, both plates having specified constant heat flux had been investigated[4,5]. For the same but filled by a saturated porous medium, heat transfer analysis was done where the walls were kept at uniform wall temperature with the effect of viscous dissipation and axial conduction take into account [6]. In [7], it was concluded that in a porous medium, the absence of viscous dissipation effect can have great impact.

This study is necessary specifically in the design of special heat exchanges and other devices where the dimensions have to be kept very small. Hence, the case of lower plate being fixed and the upper plate moving with constant velocity and both being imposed to different but constant heat fluxes is considered [8]. The energy equation is solved using **Scilab 5.4.1** leading to expression in temperature profiles , which could be useful to industrial applications.

The heat transfer characteristics of a laminar Coutte –Poiseulle flow is analyzed taking into account viscous dissipation. Some interesting result in terms of Brinkman number and heat flux ratio was observed.

STATEMENT OF THE PROBLEM

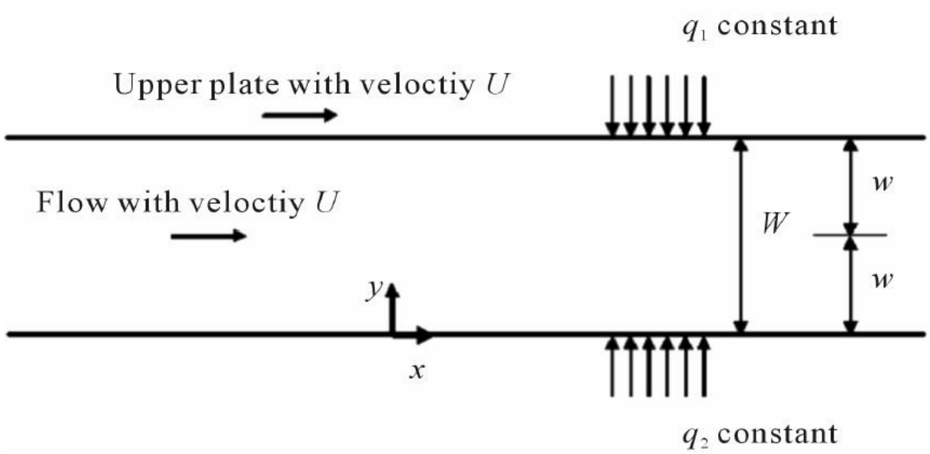

Figure 1. Notation to the Problem

Consider two flat infinitely long parallel plates [9] distance W or 2W apart, where the upper plate is moving with constant velocity U and the lower plate is fixed. The coordinate system chosen is shown in figure 1. The flow through the plates is considered at a sufficient distance from the entrance such that it is both hydro-dynamically and thermally fully developed. The axial heat conduction in the fluid and through the wall is assumed to be negligible.

The fluid is assumed to be Newtonian and with constant properties. The thermal boundary conditions are, the upper plate is kept at constant heat flux while the lower plate at different constant heat fluxes.

MATHEMATICAL FORMULATION

The momentum equation in the x-direction is described as

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dP}{dx} \qquad (1)$$

Where u is the velocity of the fluid and μ is the dynamic viscosity, P is the pressure.

The velocity boundary conditions are u=0 when y=0 and u=U when y=W.

Using the following dimensionless parameters:

$$u^* = u/m \quad U^* = U/m \quad Y = y/W \qquad (2)$$

The well-known velocity-distribution is,

$$u^* = (3U^* - 6)(Y^2 - Y) + U^*Y \tag{3}$$

where the mean velocity is given by

$$U_m = \frac{1}{W}\int_0^W u\,dy \tag{4}$$

For the above equation, expression for u is obtained by solving the momentum equation (1).

The energy equation, including the effect of viscous dissipation, is given by

$$u\frac{\partial t}{\partial x} = \frac{\gamma}{P_r}\frac{\partial^2 t}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{5}$$

where the second term on the right-hand side is the viscous dissipative term.

It is assumed that for a thermally fully developed flow with uniformly heated boundary walls, the longitudinal conduction term is neglected in the energy equation.[10].

Following this the temperature gradient along the axial direction is independent of transverse direction and is given by

$$\frac{\partial T}{\partial x} = \frac{\partial T1}{\partial x} = \frac{\partial T2}{\partial x} \tag{6}$$

Where T1 and T2 are upper and lower wall temperatures respectively.

By taking

$\alpha = k/\rho cp$, introducing the non-dimensional quantity,

$$\theta = T - T_1/(q_1 W/k) \tag{7}$$

and defining a dimensionless constant β,

$$\beta = \frac{P_r\,\mu_m\,kW}{\gamma q_1}\frac{dT1}{dx} \tag{8}$$

and modified Brinkman as,

$$Br_{q1} = \frac{\mu u^2}{2Wq1} \; \underline{m} \quad (9)$$

Equation (5) can be written as

$$\frac{d^2\theta}{dy^2} = \beta([3U^8 - 6][Y^2 - Y]\{U^*Y\]) - 2Br_{q1}([3U^8 - 6][2Y - 1]U^*)^2 \quad (10)$$

The thermal boundary conditions are,

$k\partial T\partial y/= q_1$ at y=W, or $\partial\theta\partial Y/= 1$ at Y=1, T=T_1 at y=W , or θ = 0 at Y=1. (11)

The solution of the above thermal boundary conditions can be obtained.

To evaluate β in the above equation, a third boundary condition is required,

$$k\frac{\partial T}{\partial y} = q1 \; at \; y = 0 \; or \quad \frac{\partial\theta}{\partial y} = 1 \text{ at y = 1} \quad (12)$$

Using (11) and (12) in (10) an expression for β can be obtained as

$$\beta = 1 + \frac{q2}{q1} + 8\,U^{*2}Br_{q1} - 24U^*Br_{q1} + 24Br_{q1} \quad (13)$$

The main equation (10) is solved using Scilab and the behavior of the Coutte Poiseuille flow is analysed and the temperature profile is plotted with variations of parameter to indicate the heated region.

GRAPHICAL RESULT AND DISCUSSIONS

For the purpose of discussion on the behavior of the Couette-Poiseuille flow, two types of graphs based on the analytical solutions are made. The temperature profile in the channel is plotted with variations of various parameters to indicate the heated region,.

Temperature profiles against the channel Width for Various parameters:

Figures 2,3,4 show the dimensionless temperature profile of θ versus Y, where the lower plate is insulated at five dimensionless velocities U* = -1.0, -0.5, 0.0, 0.5 and 1.0, and at three selected

$Brq1$ values -0.01, 0.10 and 0.5 and also for q2/q1 = 0. The temperature distribution have similar pattern but different shapes, and all the curves converge at Y = 1, θ = 0(by definition).

At Y = 0, the curves are vertical to satisfy the insulated condition.

As expected, generally the motion of the upper plate tends to impart more heat into the fluid layer that are dragged along, unless off-set by the viscous dissipation effects.

It is observed in figures 2,3 that when $Brq1$ = -0.01, and -0.1 the temperature distribution is negative which implies there is decrease in heat transfer.

When $Brq1$ = 0.5, figure 4, θ manifests in a different way such that θ takes both negative and positive values.

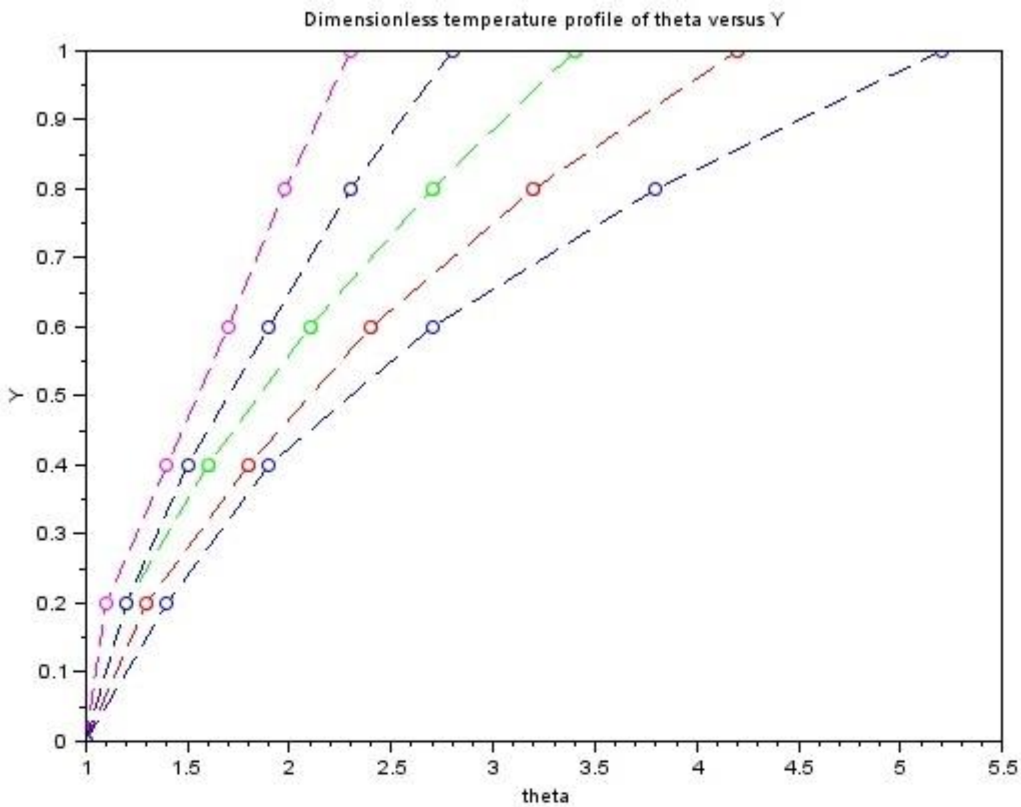

Figure 2. Temperature Profile at U^* = -0.1,-0.5,0.0,0.5 and 1.0,Brq1 = -0.01

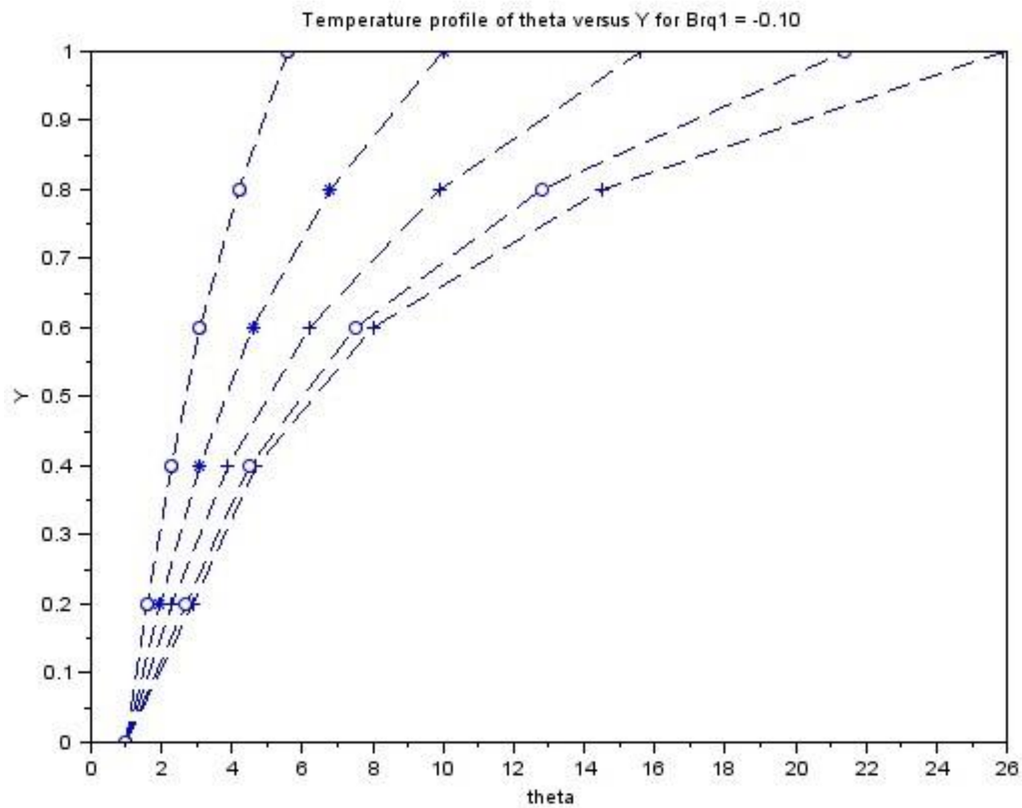

Fig3. Temperature Profile at U^* = -0.1,-0.5,0.0,0.5 and 1.0 ,Br_{q1} = -0.10

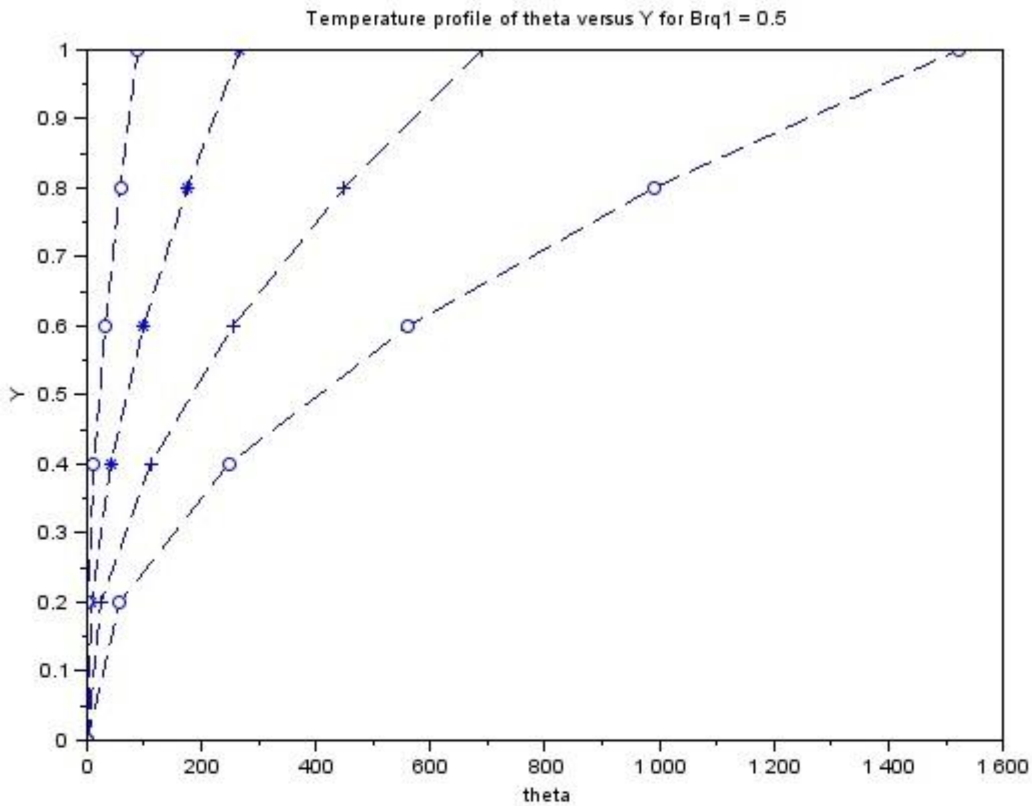

Figure 4. Temperature Profile at U^* = -0.1,-0.5,0.0,0.5 and 1.0 ,Br_{q1} = 0.5

Conclusion

Heat transfer with the effect of viscous dissipation has been analyzed. For fully developed Newtonian fluid flow between infinitely long parallel plates, where the lower plate is fixed and the upper plate is moving with constant velocity and when both plates are kept at different constant heat fluxes, the dimensionless temperature distribution given by equation (10) is solved using Scilab 5.4.1 The modified Brinkman numbers -0.01, -0.1, 0.5 are considered in the analysis. The behavior of the temperature distribution against these parameters are discussed. The Brinkman number, the speed of the moving plate have significant impact in the thermal development.

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