Hydro Magnetic Viscous Incompressible Fluid Flow past an Infinite Flat Plate

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Abstract : In the present paper hydro magnetic viscous incompressible fluid flow past an infinite flat plate has been considered with time dependent suction in the presence of transverse magnetic field. Expressions for velocity and skin friction have been calculated in the following cases :

(i) When velocity is an exponentially increasing function of time.

(ii) When velocity is an exponentially decreasing function of time.

Tabulated values for various components have been given. In both the above cases exponentially increasing or exponentially decreasing no back flows are possible even for higher values of parameter *n*.

Keywords - Magnetic viscous, Incompressible fluid flow, Suction, Transverse magnetic field, Velocity, Skin friction.

I. Introduction

Equations of Motion

Let us consider two dimensional viscous incompressible electrically conducting fluid flow past in infinite flat plate with time dependent suction, in the presence of transverse magnetic field. Taking the flow along x'-axis, and transverse magnetic field normal to it, the equations of motion and continuity reduce to,

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=-\frac{1}{\rho}\frac{\partial p'}{\partial x'}+v\frac{\partial^2 u'}{\partial y^2}-\frac{\sigma}{\rho'}B'^2_0u' \quad \text{.................. (1)}$$

$$\frac{\partial v'}{\partial t}=\frac{1}{\rho}\frac{\partial p'}{\partial y'} \quad \text{.................. (2)}$$

$$\frac{\partial v'}{\partial t}=0 \quad \text{.................. (3)}$$

Where σ is electrical conductivity of the fluid and is the strength of the magnetic field. All the electro magnetic quantities in Eqn. (1) have been measured in electro magnetic system of units.

Since the suction of free stream velocities fluctuate only in magnitude but not in direction, we take,

$$V'=-V'_0(1+Ae^{n't'}) \quad \text{.................. (4)}$$

and

$$U'=U'_0(1+\varepsilon Ae^{n't'}) \quad \text{................. (5)}$$

Where V'_0 is always positive and ε and εA are less than unity. Outside the boundary layer equations (1) gives

$$\frac{dU'}{dt'}=-\frac{1}{\rho'}\frac{\partial p'}{\partial x'}-\frac{\sigma}{\rho'}B'^2_0U' \quad \text{.................. (6)}$$

Now from equation (1) and (6) we get,

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=v\frac{\partial^2 u'}{\partial y^2}+\frac{dU'}{dt'}+\frac{\sigma}{\rho'}B'^2_0\left(U'-u'\right) \quad \text{..................(7)}$$

Introducing the non-dimensional quantities defined by,

$$u = \frac{u'}{u'_0},\ U = \frac{U'}{U'_0},\quad n = \frac{4\nu n'}{v'^2_0}$$

$$t = \frac{V'^2_0 t'}{4v},\ y = \frac{y'v'0}{v},\quad M = \frac{B'_0}{v'_0}\sqrt{\frac{v\sigma}{\rho'}}$$

Where M is the Hartmann number : equation (7) reduces to non-dimensional form as;

$$\frac{\partial^2 u}{\partial y^2} + \left(1 + \varepsilon A e^{nt}\right)\frac{\partial u}{\partial y} - \frac{1}{4}\frac{\partial u}{\partial t} - \frac{1}{4}\frac{dU}{dt} - M^2(U - u) \quad \text{.................. (8)}$$

with the boundary condition,

$$u = 0, \qquad y = 0,$$

$$u \to U, \text{ as } \qquad y \to \infty$$

Equation (8) constitute the basic equation of them problem which is to be solved subject to the boundary conduction on *u*.

II. Velocity Field and Skin Friction

Case-I Exponentially increasing

Let the free stream velocity be of the form,

$$u = 1 + \varepsilon e^{nt} \quad \text{................. (9)}$$

and let the solution of the Eqn. (8) be,

$$u = 1 + e e^{nt} - f_1 - \varepsilon e^{nt}\ f_2 - \varepsilon^2 e^{2nt} f_{3-e^3} e^{3nt} f_4 \quad \text{............... (10)}$$

f_1, f_2, f_3, f_4 are function by *y*, alone.

Now we have to solve the Eqn. (8) with the help of Eqn. (9) and (10) under the boundary condition.

$$u = 0, \text{ at } y = 0$$

$$u = 1 + \varepsilon e^{nt} \text{ at } y ......\infty$$

Substituting the value of *u* and U from (9) and (10) in Eqn. (8) and comparing the harmonic term we get,

$$f''_1 + f'_1 - M^2 f_1 = 0 \quad \text{.............. (11)}$$

$$f''_2 + f'_2 - \left(M^2 + \frac{n}{4}\right) f_2 = -Af'_1 \quad \text{.............. (12)}$$

$$f''_3 + f'_3 - \left(M^2 - \frac{n}{4}\right) f_3 = -Af'_2 \quad \text{................(13)}$$

$$f''_4 + f'_4 - \left(M + \frac{3n}{4}\right) f_4 = -Af'_3 \quad \text{............... (14)}$$

Here the dashes denote the differentiation with respect to *y*, the boundary condition of f_1, f_2, f_3, f_4 are :

$$\text{at } y = 0, f_1 = f_2 = 1; f_3 = f_4 = 0$$

$$\text{at } y = \infty, f_1 = f_2 = f_3 = f_4 = 0 \quad \text{.................(15)}$$

The solutions of the Eqns. (11) to (14) subject to the boundary condition (15) are :

$$f_1 = e^{-gy} \quad \text{................ (16)}$$

$$f_2 = s e^{-hy} + (1 - s)e^{-gy} \quad \text{................ (17)}$$

$$f_3 = s_1 e^{-h_1 y} - \frac{4Aghe^{-hy}}{n} - \frac{2Ag(1-s)e^{-gy}}{n} \quad \text{.................(18)}$$

$$f_4 = s_2 e^{-h_2 y} - \frac{4Ashe^{-h_1 y}}{n} + \frac{8A^2 sh^2 e^{-h}}{n^2} + \frac{8A^2 g^2 (1-s) e^{-gy}}{3n^2} \qquad \text{................ (19)}$$

Where

$$h = \frac{1+\sqrt{1+n+4M^2}}{2}$$

$$h_1 = \frac{1+\sqrt{1+2n+4M^2}}{2}$$

$$h_2 = \frac{1+\gamma\sqrt{1+3n+4M^2}}{2}$$

$$g = \frac{1+\sqrt{1+4M^2}}{2}$$

$$s = 1 + \frac{4Ag}{n}$$

$$s_1 = \frac{4Ash}{n} + \frac{2Ag(1-s)}{n}$$

$$s_2 = \frac{4As_1 h_1}{n} - \frac{8As^2 sh^2}{n^2} - \frac{8A^2 g^2 (1-s)}{3n^2}$$

Hence velocity field and skin friction are given by :

$$u = 1 + \varepsilon e^{nt} - f_1 - \varepsilon e^{nt} f_2 - \varepsilon^2 e^{2nt} f_3 - \varepsilon^3 e^{3nt} f_4 \qquad \text{................ (20)}$$

and

$$= g + \varepsilon e^{nt}\{sh + g(1-s)\} - \varepsilon^2 e^{2nt}\left\{-s_1 h_1 + \frac{4Ash^2}{n} + \frac{2Ag^2(1-s)}{2}\right\} - \varepsilon^3 e^{3nt}$$

$$\left\{-s_2 h_2 + \frac{4As_1 h_1^2}{n} - \frac{8A^2 sh^3}{n^2} - \frac{8A^2 g^3 (1-s)}{3n^2}\right\} - \varepsilon^3 e^{3nt}\left\{-s_2 h_2 + \frac{4As_1 h_1^2}{n}\right\} \qquad \text{............... (21)}$$

Where the values of f_1, f_2, f_3 and f_4 are given in the equations (11), (12), (13) and (14).

The variation of u, for different values of M, n and $\varepsilon A;$ is noted in the Table 1.

Table-1

M = 0, n = 50, nt = $\pi/2$, ε = 05							M = 2, n = 50, nt = $\pi/2$, ε = .5					
y	0	.02	.04	.06	.08	.1	0	.02	.04	.06	.08	.1
u	0	.269	.517	.739	.943	1.13	0	.211	.508	.9	1.061	1.301
ε = .284							ε = .284					
u	0	.148	.285	.412	.527	.636	0	.157	.338	.503	.643	.797
M = 0, n = 300, nt = $\pi/2$, ε = .5							M = 2, n = 300, nt = $\pi/2$, ε = .5					
y	0	.02	.04	.06	.08	.08	.1	0	.02	.04	.06	.1
u	0	.473	.883	1.162	1.499	1.499	1.674	0	.508	.508	1.286	1.03
ε = .284							ε = .284					
u	0	.263	.493	.664	.831	.97	0	.294	.551	.767	.947	1.096

It is clear from the table-1, that velocity field increases as the value of Hartmann number m increases. It is also clear from the table that u is always positive even for the higher values of *n*.

Case-II Exponentially Decreasing

If we consider the free stream velocity of the form U = 1 + εe^{-nt} ; we get the boundary condition on u again as;

$$u=0 \quad \text{at } y=0 \qquad \text{................ (22)}$$

$$u=1+\varepsilon e^{-nt} \quad \text{as } y=\infty,$$

Let the solution of the equation (8) in this case be,

$$u=1+\varepsilon e^{-nt}-f_1-\varepsilon e^{-nt} f_2-\varepsilon^2 e^{-2nt} f_3-\varepsilon^3 e^{-3nt} f_4 \qquad \text{................ (23)}$$

Now we have to solve the equation (8) under the boundary condition (22) with the help of equation (23). Substituting the value of u in (8) from (23) and comparing the harmonic terms we get,

$$f_1''+f_1'-M^2 f_1=0 \qquad \text{............... (24)}$$

$$f_2''+f_2'-\left(M^2-\frac{n}{4}\right)f_2=-Af_1' \qquad \text{............... (25)}$$

$$f_3''+f_3'-\left(M-\frac{n}{2}\right)f_3=-Af_2' \qquad \text{................ (26)}$$

$$f_4''+f_4'-\left(M+\frac{3n}{2}\right)f_4=-Af_3' \qquad \text{................ (27)}$$

The boundary condition of f_1, f_2, f_3 and f_4.

$$f_1=f_2=1 \text{ and } f_3=f_4=0 \text{ at } y=0$$

$$f_1=f_2=f_3=f_4=0 \quad \text{at } y=\infty \qquad \text{................ (28)}$$

The solution of the equations (24) to (27) under the boundary condition (28) are given by-

$$f_1=e^{-gy} \qquad \text{................ (29)}$$

$$f_2=se^{-hy}+(1-s)e^{-gy} \qquad \text{................ (30)}$$

$$f_3=s_1e^{-h_1y}+\frac{4Ashe^{-hy}}{n}+\frac{2A(1-s)ge^{-gy}}{n} \qquad \text{................ (31)}$$

$$f_4=s_2e^{-h_2y}+\frac{4As_1h_1e^{-h_1y}}{n}+\frac{SA^2h^2se^{-hy}}{n^2}+\frac{8A^2g^2(1-s)e^{-gy}}{3n^2} \qquad \text{................ (32)}$$

Where

$$h=\frac{1+\sqrt{1-n+4M^2}}{2}$$

$$h_1=\frac{1+\sqrt{\left(1-2n+4M^2\right)}}{2}$$

$$h_2=\frac{1+\sqrt{\left(1-3n+4M^2\right)}}{2}$$

$$g=\frac{1+\sqrt{\left(1+4M^2\right)}}{2}$$

$$s=\left(1-\frac{4Ag}{n}\right)$$

$$s_1 = -\left[\frac{4Ash}{n} + \frac{2Ag(1-s)}{n}\right]$$

$$s_2 = -\left[\frac{4As_1h_1}{n} + \frac{8A^2h^2s}{n^2} + \frac{8A^2g^2(1-s)}{3n^2}\right]$$

Hence the velocity field and skin-friction are given by.

$$u = 1 + \varepsilon e^{-nt} - f_1 - \varepsilon e^{-nt} f_2 - \varepsilon^2 e^{-2nt} f_3 - \varepsilon^3 e^{-3nt} f_4 \quad \text{................ (33)}$$

and

$$\tau_0 = g + \varepsilon e^{-nt}\{s + (1-s)g\} + \varepsilon^2 e^{-2nt}\left\{s_1h_1 + \frac{4Ash^2}{n} + \frac{2Ag^2(1-s)}{2}\right\} + \varepsilon^3 e^{3nt}$$

$$\left\{s_2h_2 + \frac{4As_1h_1^2}{n} - \frac{8A^2sh^3s}{n^2} - \frac{8A^2g^3(1-s)}{3n^2}\right\} \quad \text{............. (34)}$$

where the value of f_1, f_2, f_3 and f_4 are given by the Eqns. (29), (30), (31) and (32).

The variation of velocity field for different values of M and n is noted in table-II (a)

Table - II (a)

M = 0, n = 50, nt = π/2, ε = .5, A = 1						
y	0	.02	.04	.06	.08	.1
u	0	.021	.043	.065	.088	.11
For, ε = .284						
u	0	0.020	0.041	0.03	.084	.106
M = 0, n = 50, nt = π/2, ε = .5, A = 1						
y	0	.02	.04	.06	.08	.1
u	0	.05	.108	.146	.191	.233
For, ε = .284						
u	0	.050	.09	.143	.187	.228

Table - II (b)

M = 0, n = 300, nt = π/2, ε = .5, A = 1						
y	0	.02	.04	.06	.08	.1
u	0	.023	.047	.075	.107	.137
For, ε = .284						
u	0	0.021	0.44	0.068	.095	.120
M = 2, n = 300, nt = π/2, ε = .5, A = 1						
y	0	.02	.04	.06	.08	.1
u	0	.052	.103	.155	.207	.257
For, ε = .284						
u	0	.050	.10	.148	.196	.242

It is clear from the table no. II(a) and (b) that velocity field increases as the Hartmann number M increases but the variation is very low as compared to exponentially increasing case (i) shown in table I.

The point of interest lies here in the fact that the effect of magnetic field in the presence of suction is to accelerate the flow and to prevent the reverse flow, even for the higher values of constant parameter n, in both the cases.

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