REVIEW OF STABILITY OF EQUILIBRIUM POINT ON INEXTENSIBLE CABLE-CONNECTED SATELLITES SYSTEM IN CIRCULAR ORBIT OF CENTRE OF MASS

Dr Uday Narayan Jha

Assistant Professor (Guest), Department of Mathematics,

Dr Jannath Mishra College, B.R.A. Bihar University, Muzafarpur-842001.

ABSTRACT: ***This paper deals with the study of motion and stability of the Centre of mass of a system of two satellites connected by inextensible cable under the influence of air resistance, magnetic force and the shadow of the oblate earth due to solar pressure. We get an equilibrium point which has been shown to be stable in sense of Liapunov.*** **It is well known that in addition to the gravitational forces, the dissipative and disturbing forces are also present in nature. Though these forces are small, in comparison to the gravitational forces, it is expected that they may exert considerable effect on the oscillation of the system.** ***This work is the generalization of works done by Singh; R.B. Sharma; B. Das; S.K,. Singh; C.P. and Sinha S.K.***

KEYWORDS: **Magnetic force, Perturbative forces, Solar Pressure, Equilibrium Point, Lagrange's equations, etc.**

1. INTRODUCTION

In this paper we have studied the motion and stability of two satellites connected by a light, flexible and inextensible cable under the influence of air resistance, magnetic force and the shadow of the oblate earth due to solar pressure in the circular orbit of the centre of mass. First of all we have derived differential equations of motion of the system in circular orbit of the centre of mass under the above mentioned perturbative forces by u sing Lagrange's equations of motion of first kind. Then Jacobian integral of the problem has been obtained and with the help of this Jacobian integral we have obtained an equilibrium point. This equilibrium point is shown to be stable. This work is the generalization of works done by Singh; R.B. Sharma; B. Das; S.K,. Singh; C.P. and Sinha S.K.

2. MATHEMATICAL ANALYSIS

The equation of motion of one of the two satellites when the centre of mass of the system moves along Keplerian elliptical orbit in NECHVILL's coordinate system have been derived by using Lagrange's equation of motion of first kind in the form :

$$\left.\begin{aligned}
&x''-2y'-3x\rho+\frac{4Ax}{\rho}+f\rho\rho'+\rho^3B\psi_1\operatorname{Cos}\in\operatorname{Cos}(v-\alpha)\\
&=-\frac{C\operatorname{Cos}i}{\rho}+\rho^4\lambda_\alpha x\\
&y''+2x'\frac{Ay}{\rho}+f\rho^2-\rho^3B\psi_1\operatorname{Cos}\in\operatorname{Sin}(v-\alpha)\\
&=\rho^4\lambda_\alpha y-\frac{C\rho'\operatorname{Cos}i}{\rho^2}\\
&z''+z-\frac{Az}{\rho}-\rho^3B\psi_1\operatorname{Sin}\in=\rho^4\lambda_\alpha z\\
&-\frac{c}{\rho}\left[\frac{\rho}{\rho'}\operatorname{Cos}(v+w)+\frac{1}{\mu_E}\left(3p^3\rho^3-\mu_E\right)\operatorname{Sin}(v+w)\right]\operatorname{Sin}i
\end{aligned}\right\}\qquad(1)$$

Where A, B, C, f and □□ are oblateness, solar pressure, magnetic force, air resistance and shadow function parameters respectively and dashes denote differentiation which respect to true anomaly v. This condition of constraint is given by:

$$x^2 + y^2 + z^2 \leq \frac{1}{\rho^2} \qquad \text{...(2)}$$

Since the general solution of the system of equation (1) is beyond our reach, so we restrict to the case of circular orbit of the centre of mass. For this, we have

$$\rho = \frac{1}{1 + e\,\mathrm{Cos}\,v} = 1 \quad \text{as } e = 0$$

$$\therefore \rho' = 0$$

For equation plane, we have i=0. Thus on putting □=1, □'=0 and i=c, we from (1) and (2) as

$$\left.\begin{aligned} & x'' - 2y' - 3x + 4Ax + B\psi_1 \,\mathrm{Cos} \in \mathrm{Cos}(v-\alpha) = \lambda_\alpha x - c \\ & y'' + 2x' - Ay - B\psi_1 \,\mathrm{Cos} \in \mathrm{Sin}(v-\alpha) + f = \lambda_\alpha y \\ & \text{and } z'' + z' - az - B\psi_1 \,\mathrm{Sin} \in = \lambda_\alpha z \end{aligned}\right\} \qquad \text{.......(3)}$$

$$x^2 + y^2 + z^2 \leq 1 \qquad \text{....(4)}$$

If inequality sign holds in (4), then the motion takes place with loose string and in this case motion is called free motion. If equality sign holds in (4), then the motion takes place with tight string and the motion is called constrained motion.

$$x^2 + y^2 + z^2 = 1 \qquad \text{........(5)}$$

Differentiating (5) w.r.t. to v, we get

$$xx' + yy' + zz' = 0 \qquad \text{........(6)}$$

For managing the periodic terms present in (3), we use the following :

$$\frac{1}{2\pi}\left[\int_{\substack{-\theta \\ \psi_1=0}}^{\theta} B\operatorname{Cos}\in\operatorname{Cos}(v-\alpha)\,dv+\int_{\substack{\theta \\ \psi_1=0}}^{2\pi-\theta} B\operatorname{Cos}\in\operatorname{Cos}(v-\alpha)\,dv\right]=\frac{-B\operatorname{Cos}\in\operatorname{Cos}\alpha\operatorname{Sin}\alpha}{\pi}$$

$$\frac{1}{2\pi}\left[\int_{\substack{-\theta \\ \psi_1=0}}^{\theta} B\operatorname{Cos}\in\operatorname{Sin}(v-\alpha)\,dv+\int_{\substack{\theta \\ \psi_1=0}}^{2\pi-\theta} B\operatorname{Cos}\in\operatorname{Sin}(v-\alpha)\,dv\right]=\frac{B\operatorname{Cos}\in\operatorname{Sin}\alpha\operatorname{Sin}\theta}{\pi} \tag{7}$$

Using (7) in (3) and putting $\in=0$, We get

$$\left.\begin{aligned} x''-2y'-3x+4Ax-\frac{B\operatorname{Cos}\alpha\operatorname{Sin}\theta}{\pi}&=\lambda_\alpha.x-c \\ y''+2x''-Ay+f-\frac{B\operatorname{Sin}\alpha\operatorname{Sin}\theta}{\pi}&=\lambda_\alpha.y \end{aligned}\right\} \quad \ldots(8)$$

$$z''+z-Az=\lambda_\alpha.z$$

Multiplying 1st, 2nd and 3rd equations of (8) by $2x', 2y'$ and $2z'$ respectively and adding, we get after integrating and using (6)

$$\begin{aligned} &x'^2+y'^2+z'^2-3x^2+z^2+4Ax^2-Ay^2-Az^2 \\ &=\hbar-2cx-2fy-\frac{2B\operatorname{Sin}\theta}{\pi}\left(x\operatorname{Cos}\alpha+y\operatorname{Sin}\alpha\right) \end{aligned} \quad \ldots(9)$$

Where $\hbar$ the constant of integration and (9) is is known as Jacobian integral and can be interpreted as energy equation with modified potential v given by

$$V=-\frac{1}{2}\left(3x^2-z^2\right)+\frac{A}{2}\left(4x^2-y^2-z^2\right)+cx+fy+\frac{B\operatorname{Sin}\theta}{\pi}\left(x\operatorname{Cos}\alpha+y\operatorname{Sin}\alpha\right) \tag{10}$$

Differentiating (6) with respect to v, we get

$$x'^2+y'^2+z'^2=-\left(xx''+yy''+zz''\right) \quad ..(11)$$

Multiplying 1st, 2nd and 3rd equations of (8) by x, y and z respectively and adding we get after using (5), (6) and (11)

$$\begin{aligned} -\lambda_\alpha=&\,x'^2+y'^2+z'^2+2\left(xy'-x'y\right)-\left(5x^2-1\right)A-cx-fy \\ &-\frac{B\operatorname{Sin}\theta}{\pi}\left(x\operatorname{Cos}\alpha+y\operatorname{Sin}\alpha\right) \end{aligned} \tag{12}$$

Now, we use spherical polar coordinates on the unit sphere given by :

$$\left.\begin{array}{l} x = \mathrm{Cos}\,\varphi\,\mathrm{Cos}\,\psi \\ \\ y = \mathrm{Cos}\,\varphi\,\mathrm{Sin}\,\psi \\ \\ c = \mathrm{Sin}\,\varphi \end{array}\right\} \qquad \text{…(13)}$$

Putting the value of x,y,z from (13) and their derivatives x,y,z in (9) we get –

$$\phi'^2 + \psi'^2\,\mathrm{Cos}^2\,\phi = \left[3\mathrm{Cos}^2\,\psi + 1\right]\mathrm{Cos}^2\,\phi - 2c\,\mathrm{Cos}\,\phi\,\mathrm{Cos}\,\psi$$

$$-2f\,\mathrm{Cos}\,\phi\,\mathrm{Sin}\,\psi + 5A\,\mathrm{Cos}^2\,\psi\,\mathrm{Cos}^2\,\phi$$

$$-\frac{2B\,\mathrm{Sin}\,\theta\,\mathrm{Cos}\,\phi}{\pi}\left[\mathrm{Cos}\,\psi\,\mathrm{Cos}\,\alpha + \mathrm{Sin}\,\psi\,\mathrm{Sin}\,\alpha\right] + \hbar_1 \qquad \text{…….(14)}$$

Where, $\hbar_1 = \hbar + A - 1 = \mathrm{Constant}$

Equilibrium position and its stability:

For equilibrium position of the system, we take modified potential energy using (14) as

$$V(\phi\,,\psi) = \frac{-1}{2}\mathrm{Cos}^2\,\phi\left[3\mathrm{Cos}^2\,\psi + 1\right] + \frac{5}{2}A\,\mathrm{Cos}^2\,\phi\,\mathrm{Cos}^2\,\psi$$

$$+\ c\ \mathrm{Cos}\phi\mathrm{Cos}\,\psi + f\ \mathrm{Cos}\phi\mathrm{Sin}\,\psi$$

$$+\ \frac{B\,\mathrm{Sin}\,\theta\,\mathrm{Cos}\,\phi}{\pi}\left[\mathrm{Cos}\,\psi\,\mathrm{Cos}\,\alpha + \mathrm{Sin}\,\psi\,\mathrm{Sin}\,\alpha\right] \qquad \text{…… (15)}$$

The equilibrium position is obtained by stationary values $V(\phi,\psi)$ **and hence we have**

$$\frac{\partial v}{\partial \phi} = 0 \ \text{ and } \ \frac{\partial v}{\partial \psi} = 0 \qquad \text{……(16)}$$

Differentiating (15) partially with respect to □ and putting $\frac{\partial v}{\partial \phi} = 0$ **we get**

$$\mathrm{Sin}\,\phi = 0 \Rightarrow \phi = 0 \qquad \text{……(17)}$$

Again on putting =0 in (15) and then differentiating it partially with respect to and using $\frac{\partial v}{\partial \psi} = 0$ **we get**

$$\left[\frac{\partial v}{\partial \phi}\right]_{\psi=\psi_0}^{\phi=0} = (3-5A)\operatorname{Cos}\psi_0 \operatorname{Sin}\psi_0 + f\operatorname{Cos}\psi_0$$

$$-\frac{B\operatorname{Sin}\theta}{\pi}\left(\operatorname{Sin}\psi_0 \operatorname{Cos}\alpha - \operatorname{Cos}\psi_0 \operatorname{Sin}\alpha\right) = 0$$

…(18)

For smallest value of ψ_0**, we have from(18) on putting**

$$\operatorname{Cos}\psi_0 = 1 \text{ and } \operatorname{Sin}\psi_0 = \psi_0$$

$$\psi_0 = \frac{-f\dfrac{B\operatorname{Sin}\theta\operatorname{Sin}\alpha}{\pi}}{3-5A-C-\dfrac{B\operatorname{Sin}\theta\operatorname{Cos}\alpha}{\pi}}$$

……(19)

Thus, equilibrium point is given by

$$\phi = \phi_0 = 0 \; and \;\; \psi = \psi_0 = \frac{-f\dfrac{B\operatorname{Sin}\theta\operatorname{Sin}\alpha}{\pi}}{3-5A-C-\dfrac{B\operatorname{Sin}\theta\operatorname{Cos}\alpha}{\pi}}$$

…(20)

For stability of equilibrium point (ϕ_0, ψ_0) **given in (20), we have**

$$D = \begin{vmatrix} \left[\dfrac{\partial^2 v}{\partial \psi}\right]_{\psi=\psi_0}^{\phi=0} & \left[\dfrac{\partial^2 v}{\partial \phi \partial \psi}\right]_{\psi=\psi_0}^{\phi=0} \\ \left[\dfrac{\partial^2 v}{\partial \psi \partial \phi}\right]_{\psi=\psi_0}^{\phi=0} & \left[\dfrac{\partial^2 v}{\partial \phi^2}\right]_{\psi=\psi_0}^{\phi=0} \end{vmatrix} > 0$$

$$\Rightarrow\begin{bmatrix}(3-5A)\text{Cos}^2\psi_0+1-f\,\text{Sin}\,\psi_0-c\,\text{Cos}\,\psi_0\\ -\dfrac{B\,\text{Sin}\,\theta}{\pi}\left(\text{Cos}\,\psi_0\,\text{Cos}\,\alpha+\text{Sin}\,\psi_0\,\text{Sin}\,\alpha\right)\end{bmatrix}X$$

$$\begin{bmatrix}(3-5A)\text{Cos}^2\psi_0-f\,\text{Sin}\,\psi_0-c\,\text{Cos}\,\psi_0\\ -\dfrac{B\,\text{Sin}\,\theta}{\pi}\left(\text{Cos}\,\psi_0\,\text{Cos}\,\alpha+\text{Sin}\,\psi_0\,\text{Sin}\,\alpha\right)\end{bmatrix}>0$$

.....(21)

On putting to the 2nd factor on L.H.S. of (21), we get

$$\text{Cos}^2\psi_0=1\ and\ \text{Sin}\psi_0=\psi_0=\frac{-f\dfrac{B\,\text{Sin}\,\theta\,\text{Sin}\,\alpha}{\pi}}{3-5A-C-\dfrac{B\,\text{Sin}\,\theta\,\text{Cos}\,\alpha}{\pi}}$$

$$\frac{\left(3-5A-C-\dfrac{B\,\text{Sin}\,\theta\,\text{Cos}\,\alpha}{\pi}\right)^2+\left(f+\dfrac{B\,\text{Sin}\,\theta\,\text{Cos}\,\alpha}{\pi}\right)^2}{3-5A-C-\dfrac{B\,\text{Sin}\,\theta\,\text{Cos}\,\alpha}{\pi}}\geq 0$$

$$i.e.\ \ 3-5A-C-\frac{B\,\text{Sin}\,\theta\,\text{Cos}\,\alpha}{\pi}>0$$

Similarly, we can show that first bracket on the L.H. side of (21) is positive if

$$3-5A-C-\frac{B\,\text{Sin}\,\theta\,\text{Cos}\,\alpha}{\pi}>0$$

Thus, condition (21) holds if , $3\text{-}5A\text{-}C\text{-}\dfrac{B\,\text{Sin}\,\theta\,\text{Cos}\,\alpha}{\pi}>0$

Thus, we conclude that the equilibrium point (ϕ_0,ψ_0) **given in (20) is stable in the sense of Liapuov if** $3-5A-C-\dfrac{B\,\text{Sin}\,\theta\,\text{Cos}\,\alpha}{\pi}$ **is positive.**

3. CONCLUSION

The present research work deals with the effects of small external dissipative and disturbing forces of general nature on the non-linear oscillation of the system of the cable connected satellites in the central gravitation field of the Earth. The satellites are considered as materials particles. The cable connecting the two satellites in supposed to be light, flexible and inextensible. The parametric excitations are found at born the overtones. But to evaluate the amplitude discontinuity effect

accurately, we must know the order of magnitude of parameters, the dissipation co-efficient γ and the amplitude of the external periodic force E. It is clear that these parameters are always concerned with certain model assumptions.

ACKNOWLEDGMENT

One of the authors Dr. Uday Narayan Jha takes this opportunity to thank Prof. (Dr.) Byas Sharma (M.Sc., Ph.D), Retired Professor of Mathematics, B.R.A. Bihar University, Muzaffarpur, Bihar (INDIA), for numerous discussions, valuable suggestion and help in preparing the present research paper for publication. I also feel indebted to some scientific spiritualists who provided us the enthusiasm to undertake scientific studies with a wholesome attitude.

REFERENCES

1. **Singh; R.B. : The three dimensional motion of two connected bodies in elliptical orbit; Bulletin of Moscow, State University, Moscow Mathematics Mechanics No. pp. 59-64.**
2. **Sharma; B. The motion of a system of two Cable-Connected satellites in the atmosphere: Ph. D. thesis, submitted to B. U., Muzaffarpur, 1974.**
3. **Singh. C. P. Motion and stability of inter-connected satellites System in the gravitational field of oblate earth. Ph. D. thesis submitted to B. U., Muzaffarpur, 1983.**
4. **Das; S. K. : Effect of Magnetic force on motion of a System of two cable connected satellites in orbit, (1976). Proc. National Acade. Science; India.**
5. **Sinha; S. K.; Effect of Solar Pressure on the motion and stability of two inter-connected satellites in elliptical orbit. Astrophysics and Space Science 140 pp. 49-54 in 1988.**