PARABOLIC EQUATION SOLUTION OF TROPOSPHERIC WAVE PROPAGATION USING FINITE ELEMENT METEHOD (FEM)

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Abstract : In this work, the parabolic equation applied on radio wave and microwave tropospheric propagation, properly manipulated, and resulting in a one-dimensional form, is solved using the Finite Element Method (FEM). The necessary vertical tropospheric profile characteristics are assigned to each mesh element, while the solution advances in small and constant range segments, each excited by the solution of the previous step. This is leading to a marching algorithm, similar to the widely used Split Step formulation. The surface boundary conditions including the wave polarization and surface conductivity properties are directly applied to the FEM system of equations. Since the FEM system returns the total solution, a technique for the separation of the transmitted and reflected waves is also presented. This method is based on the application of the Discrete Fourier Transform (DFT) in the space domain, which allows for the separation of the existing wave components. Finally, abnormal tropopheric condition propagation is being employed to assess the method, while the results are compared to those obtained using the Advance Refractive Prediction System (AREPS v.3.03) software package.

Keywords : Parabolic equation, Troposheric Profile, Algorithm, FEM, DFT.

I. INTRODUCTION

Since the troposphere refractive index is frequency independent, the lower parts of the atmosphere affect the radiowave propagation in a wide frequency range, from VHF to optical frequencies, whereas, abnormal environmental conditions can end up to ducting phenomena. These result to the trapping of the UHF radio waves and contribute to the over-the –horizon propagation. The modeling of radio wave propagation through the troposphere has been extensively studied, and nowadays a great number of reliable models are in use.

In the past, emphasis was given to geometrical optics techniques. These methods provide a general geometrical description of ray families, propagation through the troposphere. They are based on the discrimination of the medium into sufficiently small segments, with a linearly varying modified refractivity index. In each segment, the radiowave propagating angle is calculated using either the Snell's law or its generalized form, if the Earth's curvature is considered. These methods have main advantages, since their implementation is very simple and the necessary CPU time is very small. On the other hand, ray tracing methods present many disadvantages; for example the radiowave frequency is not accounted for and it is not always clear whether the ray is trapped by the specific duct structure. An alternative approach for tropospheric propagation modeling was developed by Baumgartner and was extended and improved by Baumartner and Shellman. This method, usually known as Waveguide Model or Coupled Mode Technique, is based on a root finding algorithm by tracing the curve defined by

$$G = |R(\theta)\, Rg(\theta)| = 1 \qquad \text{................(1)}$$

Where

R is a complex reflection coefficient over the height h_0,

Rg is the corresponding coefficient below the height h_0,

h_0 is a reference height.

in this work, the parabolic equation is properly manipulated to a one-dimensional form, the solution of which is achieved using the finite Element Method (FEM). The solution advances in space using small range steps. The total field is determined in a two-dimensional tropospheric medium, since azimuth symmetry is assumed. Finally, radar application examples are presented, demonstrating the radiowave propagation under surface ducting conditions, while the method is evaluated through a comparison to the corresponding results from the AREPS package, under the same propagation conditions.

II. Tropospheric Ducts

The index of refraction is defined as

$$n = \sqrt{\varepsilon_r} = c/v \qquad (2)$$

Where

$\mathcal{E}r$ is the dielectric constant of the troposphere,

c is the sprd of light and

υ is the phase velocity of the electromagnetic wave in the medium.

Since n near the earth's surface is slightly greater than unity (1.00025-1.00040), it seems more practical to use the scaled index if refraction N, which is called refractivity and is defined as:

$$N = (n-1)\cdot 10^6 = \frac{77.6p}{T} - \frac{5.6e}{T} + \frac{3.75\cdot 10^5 e}{T^2} \qquad (3)$$

Where

P is the total pressure in mbar,

e is the water vapor pressure and

T is the temperature in °Kelvin.

in order to examine the n gradients, the modified refractivity index is used. it is defined as:

$$M = \left(n - 1 + \frac{h}{a}\right)\cdot 10^6 = N + 0.157h \qquad (4)$$

The computation of the refractive conditions, characterized as sub refraction, standard, surer refraction and Trapping is achieved by its gradient *dM/dh*. tropospheric ducting phenomena occur when either:

$$\frac{dM}{dh} < 0 \qquad (5a)$$

or

$$dN/dh < -157 \qquad (5b)$$

is met.

The tropospheric ducting effects to radio wave propagation are similar to that of the metal waveguides; therefore, only modes with a wavelength shorter than the cut-off wavelength can propagate (the cut=off frequency being a function of the duct's width).

III. Parabolic Equation

Various methods for the solution of the PE were developed and presented to the day. The most efficient algorithm seems to be the split Step Solution which employs the Fast Fourier Transform (FFT) to advance the solution over small range steps. The algorithm has been widely used in many applications. More specifically, Barrios treated horizontally inhomogeneous environments and a terrain model respectively. Craig and Levy applied the Split step solution to assess radar performance under multipath and ducting conditions. Finally, Sevgi and Paker discussed the path loss in HF propagation channels. Alternative algorithms for the solution of the PE wer also proposed. For example, Levy presented a Finite Difference formulation, whereas she has also applied, the horizontal PE solution above a specific height level. finally, Akleman and Sevgi, applied a Finite-Difference Time-Domain(FDTD), and extended its algorithm in order to deal with varying terrain models.

The analysis to follow starts from a Parabolic Equation form taking into account the Earth flattening transformation:

$$\frac{\partial^2 u(x,z)}{\partial z^2} + 2jk\frac{\partial u(x,z)}{\partial x} + k^2\left(n^2 - 1 + \frac{2z}{R}\right)u(x,z) = 0 \qquad (6)$$

Where K = 2π/λ is the free space wave-number,

x is the horizontal propagation distance (range),

z is the heightand

R is the Earth's radius (6378165m).

Assuming that the field slowly varies in the x direction, the partial derivative *ϑu(x,z)/ϑx*, can be analyzed to its partial variations. Thus, for a sufficiently small range step *ϑx*, equation (6) can be written as:

$$\frac{\partial^2 u(x,z)}{\partial z^2} + 2jk\frac{u(x,z) - u(x-\delta x,z)}{\delta x} + k^2\left(n^2 - 1 + \frac{2z}{R}\right)u(x,z) = 0 \qquad (7)$$

or

$$\frac{\partial^2 u(x,z)}{\partial z^2} + \left[\frac{2jk}{\delta x} + k^2\left(n^2 - 1 + \frac{2z}{R}\right)\right]u(x,z) = \frac{2jk}{\delta x}u(x-\delta x,z) \quad (8)$$

Assuming x = δx, the quantity u(x – δx, z) = u(0, z) corresponds to the initial field. Equation (8) is a recursive, One-dimensional form of the parabolic equation. In the applications presented here, a Gaussian shaped field is used; representing the main lobe of a radar system and it is expressed by the Gaussian relation:

$$W(z) = A\exp\left(-\frac{(z-H_0)^2}{k_f}\right) \qquad (9)$$

Where

W(z) is the magnitude of the field in respect to height,

H_0 is the altitude of the radar antenna,

A is the maximum intensity of W at height H0 and

kf is a coefficient which determines the beam width.

IV. Boundary Conditions

The solution of equation (8) using FEM, requires theapplication of boundary conditions at the starting height, z = z_{min}, which in fact is the Earth's surface, and at the maximum altitude considered, z =z_{max}. At the upper artificial boundary, an absorbing condition has to be applied, allowing for the propagation of the waves and at the same time, reducing any possible reflections introduced by the method. Therefore, a first or second order absorbing boundary condition is applied, combined with the z_{max} extension. Alternatively, the non-desirable upper boundary reflections can be eliminated by applying a fictitious absorber or a perfectly Matched Layers (PML) scheme.

The entrance boundary conditions are expressed by the equation:

$$\left[\frac{\partial u}{\partial z} + jkqu\right]_{z=z_{\max}} = 0 \qquad (10)$$

Where q is given by:

$$q = q_v = \sqrt{\frac{\mu_r}{\left(\varepsilon_r + \frac{j\sigma}{\omega\varepsilon_0}\right)}} \qquad (11)$$

or:

$$q = q_H = \sqrt{\frac{\left(\varepsilon_r + \frac{j\sigma}{\omega\varepsilon_0}\right)}{\mu_0}} \qquad (12)$$

For vertical and horizontal polarization respectively. in the equations (11) and (12), ν_r, μ_r are the relative permittivity and permeability of the medium (surface) respectively, ν_0, μ_0, the permittivity and permeability of the free space, and σ is the absolute ground conductivity. For a perfectly conducting surface, equations (11) and (12) are reduced to $\vartheta u(x, 0) / \vartheta z = 0$ for vertical polarization and $\vartheta u(x, 0) = 0$ for horizontal polarization, whereas for an imperfectly conduction ground surface, the above equations can be applied using for example ν_r,= 15 and σ = 0.01S/m.

V. Wave Separation

The solution of equation (8) using the marching algorithm discussed in the previous sections, gives the total field u, which is the combination of the upwards and downwards propagating coefficients. the wave separation methodology, initially introduced for the discrimination of radio waves after their reflection from the ionosphere is based in the application of the Discrete Fourier Transform (DFT) in the space domain.

VI. Results and Discussion

In order to demonstrate the results obtained using the FEM solution of the PE, the method was applied to various frequencies and medium profiles. a radar antenna located at a height of 150m above the see level was assumed in all the examples of this section. the main lobe beam was modeled according to equation(9), with H_0 = 150 and A = 1. Moreover, vertical polarization and a perfectly conduction ground were assumed. The solution of the final system of the FEM equations was obtained using the Bi-Conjugate Gradients stabilized method for a faster convergence. Other solution methods can be implemented as well, as for example the Bi-Conjugate Gradients and the Conjugate Gradient Squared methods.

In order to validate the FEM solution of th PE, the results were compared to those obtained by the AREPS. the AREPS program computes and displays a number of tactical decision aids to assess the influence of the atmosphere and terrain on the performances of electromagnetic (EM) systems. the internal propagation model used by AREPS is the Advance Propagation Model(APM). This is a hybrid model that consists of four sub-models: flat earth, ray optics, extended optics, and split-step parabolic equation (PE). APM effectively merges the Radio Physical optics (RPO) model with the Terrain Parabolic Equation Model(TPEM).

VII. Conclusion

In Practice, the FEM formulation can easily process complex refractivity profiles of any kind, either numerical or analytical. Moreover, the refractive index being independent between consequent range steps, gives the ability to include horizontally inhomogeneous tropospheric profiles. In these cases, the method’s response can be directly adjusted to the refractivity variations, by properly modifying the size of the FEM elements and the range step.

The proposed technique for the separation of the incident and reflected wave components provides a useful tool for many applications. For example, this methodology can be used for the determination of the reflection coefficients and their space distribution. Moreover, it can be used for the of the upper absorbing boundary conditions efficiency evaluation, since the unwanted scattered field components can be calculated.

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