OPTIMALITY AND DUALITY IN NONDIFFERENTIABLE MULTIOBJECTIVE PROGRAMMING WITH GENERALIZED (F, ρ) –UNIVEXITY

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ABSTRCT

In this paper, we have considered nondifferentiable multiobjective optimization problem. A number of duality theorems for Mond-Weir type dual are also established. Duality results have been established assuming the functions to be generalized (F, ρ)-univexity.

Key Words: (F, ρ)-univexity; Sufficiency; Nondifferentiable multiobjective programming; Duality

1. INTRODUCTION:

Wolfe [1] considered dual for nonlinear programming problems. While studying duality under generalized convexity, Mond and Weir [2] proposed a number of different duals for nonlinear programming problems with nonnegative variables and derived various duality theorems under appropriate pseudo-convexity/quasi-convexity assumptions.

Optimality and duality results for several mathematical programs were defined by Rueda et al. [3] by combining the concept of type I functions and univex functions. Optimality and duality results for a multiple–objective program was obtained Mishra [8] by combining the concept of pseudoquasi, type I, quasi-pseudo type I, strictly pseudoquasi type I and univex functions. A new class of generalized type I univex functions was introduced by Mishra et al. [11] by extending weak strictly pseudoquasi type I, strong pseudoquasi type I functions etc.

A nondifferentiable multiple objective programming problem was considered by Mond et al.[10]. Mond-Weir type and Wolfe type duals were formulated. Gulati and Talaat [12] considered a nondifferentiable multiobjective programming problem and Fritz-John and Kuhn-Tucker type sufficient conditions were derived for efficient and properly efficient solutions respectively. Zhang and Mond [9] introduced duality results for nondifferentiable programs under generalized invexity assumptions. Second order Mangasarian type and general Mond-Weir type duals for a class of nondifferentiable multiobjective programming problems was considered by Ahmad and Sharma [10]. Patel [11] has considered Mangasarian type and general Mond-Weir type duals and some duality theorems are established for nondifferentiable multiobjective programming problems under second order (b,F,ρ)-convexity assumptions..

In this paper, we consider a nondifferentiable multiobjective optimization problem. A number of duality theorems for Mond-Weir type dual are also established.

2. NOTATIONS AND PRELIMINARIES:

We consider the following nondifferentiable multiobjective programming problem:

(NMP) Minimize $$f(x) = \left[\left(f_1(x)+(x^tA_1x)^{\frac{1}{2}}\right),\ldots,\left(f_k(x)+(x^tA_kx)^{\frac{1}{2}}\right)\right],$$

subject to $h_j(x) \le 0,$ (2.1)

$x \in X.$ (2.2)

where, f_i, i=1,2,…,k and h_j, j=1,2,…,m, are assumed to be continuously differentiable functions, X be an open convex subset of R^n and A_i are n x n positive semidefinite symmetric matrix.

Let U denote the set of feasible solutions of (NMP).

Definition 2.1: A feasible solution x^0 is efficient if there is no other feasible solution x for (NMP) such that

$$f_i(x) + (x^tA_ix)^{\frac{1}{2}} \le f_i(x^0) + (x^{0t}A_ix^0)^{\frac{1}{2}}, \quad \text{for } i=1,2,\ldots,k,$$

and

$$f_s(x) + (x^tA_sx)^{\frac{1}{2}} < f_s(x^0) + (x^{0t}A_sx^0)^{\frac{1}{2}}, \quad \text{for some } s.$$

Definition 2.2: An efficient solution x^0 of (NMP) is said to be properly efficient solution for (NMP) if there exists a scalar M > 0 such that for every feasible x

$$f_i(x) + (x^tA_ix)^{\frac{1}{2}} < f_i(x^0) + (x^{0t}A_ix^0)^{\frac{1}{2}}$$

$$\Rightarrow \left[\left(f_i(x^0) + (x^{0t}A_ix^0)^{\frac{1}{2}}\right) - \left(f_i(x) + (x^tA_ix)^{\frac{1}{2}}\right)\right]$$

$$\le M\left[\left(f_s(x) + (x^tA_sx)^{\frac{1}{2}}\right) - \left(f_s(x^0) + (x^{0t}A_sx^0)^{\frac{1}{2}}\right)\right]$$

for some s such that

$$f_s(x) + (x^tA_sx)^{\frac{1}{2}} > f_s(x^0) + (x^{0t}A_sx^0)^{\frac{1}{2}}.$$

Let X be an open convex subset of R^n.

Now, we introduce a class of type I (F,ρ)-univex functions and their generalizations for nondifferentiable multiobjective programming problem which will be used to derive some important properties of (NMP) and other results.

Let X be an open convex subset of R^n and let R_+ be the set of positive real numbers. Let the functions f_i i=1,2,…,k; h_j, j=1,2,…,m; η and ρ be as follows: $f_i, h_j : X \to R$, $w_i \in R^n$, A_i are nxn positive semidefinite symmetric matrix,

θ_0,θ_1:X x $R^n \rightarrow R$, η : XxX $\rightarrow R^n$, $w_i \in R^n$, b_0,b_1:XxX→R_+, $\rho = (\rho_i^1,\rho_j^2)$, $\rho_i^1=(\rho_1^1,\rho_2^1,\ldots,\rho_k^1) \in R^k$,

$\rho_j^2 = (\rho_1^2,\rho_2^2,\ldots,\rho_m^2) \in R^m$.

Definition 2.3: The problem (NMP) is said to be strong pseudoquasi type I (F,ρ)-univex at x, $x^0 \in X$, if there exist real valued functions b_0, b_1, θ_0, θ_1, ρ and $w_i \in R^n$ such that

$$b_0(x,x^0)\theta_0\left[\{f_i(x) + x^tA_iw_i\} - \{f_i(x^0) + x^{0t}A_iw_i\}\right] \le 0$$

$$\Rightarrow F[x,x^0;(\eta(x,x^0))^t\nabla f_i(x^0) + A_iw_i^0)] + \rho_i^1 d^2(x,x^0) \le 0,$$

$$- b_1(x, x^0)\theta_1[h_j(x^0)] \leqq 0 \Rightarrow F[x,x^0;(\eta(x,x^0))^t\nabla h_j(x^0)] + \rho_j^2 d^2(x,x^0) \leqq 0.$$

for all i={1,2,…,k} and j={1,2,…,m}. If (NMP) is strong pseudoquasi type I (F,ρ)-univex at x,$x^0 \in X$, (NMP) is said to strong pseudoquasi type I (F,ρ)-univex on X.

Definition 2.4: The problem (NMP) is weak quasi strictly pseudo type I (F,ρ)-univex at x,$x^0 \in X$, if there exist real-valued functions b_0, b_1, θ_0, θ_1, ρ and $w_i \in R^n$ such that

$$b_0(x,x^0)\theta_0\left[\{f_i(x) + x^tA_iw_i\} - \{f_i(x^0) + x^{0t}A_iw_i\}\right] \le 0$$

$$\Rightarrow F[x,x^0;(\eta(x,x^0))^t\nabla f_i(x^0) + A_iw_i^0)] + \rho_i^1 d^2(x,x^0) < 0,$$

$$- b_1(x, x^0)\theta_1[h_j(x^0)] \leqq 0 \Rightarrow F[x,x^0;(\eta(x,x^0))^t\nabla h_j(x^0)] + \rho_j^2 d^2(x,x^0) < 0.$$

for all i={1,2,…,k} and j={1,2,…,m}. If (MP) is weak quasi strictly pseudo type I (F,ρ)-univex at x,$x^0 \in X$, (MP) is said to be weak quasi strictly pseudo type I (F,ρ)-univex on X.

Definition 2.5: The problem (NMP) is weak strictly pseudo type I (F,ρ)-univex at x,$x^0 \in X$, if there exist real-valued functions b_0, b_1, θ_0, θ_1, ρ and $w_i \in R^n$ such that

$$b_0(x,x^0)\theta_0\left[\{f_i(x) + x^tA_iw_i\} - \{f_i(x^0) + x^{0t}A_iw_i\}\right] \le 0$$

$$\Rightarrow F[x,x^0;(\eta(x,x^0))^t\nabla f_i(x^0) + A_iw_i^0)] + \rho_i^1 d^2(x,x^0) < 0,$$

$$- b_1(x, x^0)\theta_1[h_j(x^0)] \leqq 0 \Rightarrow F[x,x^0;(\eta(x,x^0))^t\nabla h_j(x^0)] + \rho_j^2 d^2(x,x^0) < 0.$$

for all i={1,2,…,k} and j={1,2,…,m}. If (NMP) is weak strictly pseudo type I (F,ρ)-univex at x,$x^0 \in X$, (NMP) is said to be weak strictly pseudo type I (F,ρ)-univex on X.

3. OPTIMALITY CONDITIONS:

We establish some sufficient optimality condition for x^0 to be an efficient solution of problem (NMP) under various generalized type I (F,ρ)-univex functions defined in the previous section.

Theorem 3.1: (Sufficiency): Suppose that

(i) x,$x^0 \in U$, (ii) There exist $\mu^0 \in R^k$, $\mu^0 > 0, \lambda \in R^m$, $\lambda^0 \geq 0$ and $w_i^0 \in R^n$, such that

(a) $\mu^0[\nabla f_i(x^0) + A_iw_i^0] + \lambda^0\nabla h_j(x^0) = 0$,

(b) $\lambda^0\nabla h_j(x^0) = 0$,

(c) $\mu^0 e=1$, where $e = (1,\ldots\ldots,1)^T \in R^k$.

(iii) The problem (NMP) is strong pseudoquasi type I (F,ρ)-univex at $x, x^0 \in U$ with respect to some b_0, b_1, θ_0, θ_1, ρ and $w_i \in R^n$ for all feasible x. Then x^0 is an efficient solution to (NMP).

Proof: Suppose contrary to the result that x^0 is not an efficient solution to (NMP). Then there exists a feasible solution x to (NMP) such that

$$[f_i(x)+(x^t A_i w_i)] \leq [f_i(x^0)+(x^{0t} A_i w_i)]$$

By the properties of b_0 and θ_0 the above inequality, we have

$$b_0(x,x^0)\theta_0[\{f_i(x) +(x^t A_i w_i)\}-\{f_i(x^0)+(x^{0t} A_i w_i)\}] \leq 0. \quad (3.1)$$

By the feasibility of x^0, we have $-\lambda^0 \nabla h_j(x^0) \leqq 0$.

By the properties of b_1 and θ_1 from above inequality, we have

$$-b_1(x,x^0)\theta_1[\lambda^0 \nabla h_j(x^0)] \leqq 0. \quad (3.2)$$

By inequalities (3.1) and (3.2) and condition (iii), we have

$$F(x,x^0;(\mu^0 \nabla f_i(x^0) + A_i w_i^0)+\rho_i^1 d^2(x,x^0)) \leq 0$$

and

$$F(x,x^0;(\lambda^0 \nabla h_j(x^0)+\rho_j^2 d^2(x,x^0))) \leqq 0, \text{ since } \mu^0 > 0,$$

The above inequalities give $F(x,x^0;[\mu^0 \nabla f_i(x^0) + A_i w_i^0 + \lambda^0 \nabla h_j(x^0)] + (\rho_i^1+\rho_j^2)d^2(x,x^0)) < 0$,

(3.3)

which contradict condition (iii). This completes the proof.

Theorem 4.3.2: (Sufficiency): Suppose that

(i) $x,x^0 \in U$, (ii) There exist $\mu^0 \in R^k$, $\mu^0 > 0, \lambda \in R^m$, $\lambda^0 \geq 0$ and $w_i^0 \in R^n$, such that

(a) $\mu^0[\nabla f_i(x^0)+ A_i w_i^0]+\lambda^0 \nabla h_j(x^0) = 0$,

(b) $\lambda^0 \nabla h_j(x^0) = 0$,

(c) $\mu^0 e = 1$, where $e=(1,\ldots\ldots,1)^T \in R^k$.

(iii) The problem (NMP) is weak strictly pseudoquasi type I (F,ρ)-univex at $x, x^0 \in U$ with respect to some $b_0,b_1,\theta_0,\theta_1$, ρ and $w_i \in R^n$ for all feasible x, then x^0 is an efficient solution to (NMP).

Proof: Suppose contrary to the result that x^0 is not an efficient solution to (NMP). Then there exists a feasible solution x to (NMP) such that

$$[f_i(x)+(x^t A_i w_i)] \leq [f_i(x^0)+(x^{0t} A_i w_i)].$$

By the properties of b_0 and θ_0 and the above inequality, we get (3.1). By the feasibility of x^0 the properties of b_1 and θ_1 and the condition (iii), we have

$$F(x,x^0;(\nabla f_i(x^0) + A_i w_i^0)+ \rho_i^1 d^2(x,x^0)) < 0$$

and

$$F(x,x^0;(\lambda^0 \nabla h_j(x^0)+\rho_j^2 d^2(x,x^0))) \leq 0, \text{since } \mu^0 \geq 0,$$

The above inequalities give $F(x,x^0;[\mu^0 \nabla f_i(x^0) + A_i w_i^0 + \lambda^0 \nabla h_j(x^0)] + (\rho_i^1+\rho_j^2)d^2(x,x^0)) < 0$, which contradict condition (iii). This completes the proof.

Theorem 4.3.3: (Sufficiency): Suppose that

(i) $x, x^0 \in U$, (ii) There exist $\mu^0 \in R^k$, $\mu^0 > 0, \lambda \in R^m$, $\lambda^0 \geq 0$ and $w_i^0 \in R^n$, such that

(a) $\mu^0[\nabla f_i(x^0) + A_i w_i^0] + \lambda^0 \nabla h_j(x^0) = 0,$

(b) $\lambda^0 \nabla h_j(x^0) = 0,$

(c) $\mu^0 e = 1$, where $e = (1, \ldots\ldots, 1)^T \in R^k$.

(iii) The problem (NMP) is weak strictly pseudo type I (F,ρ)-univex at x, $x^0 \in U$ with respect to some $b_0, b_1, \theta_0, \theta_1$, ρ and $w_i \in R^n$ for all feasible x, then x^0 is an efficient solution to (NMP).

Proof: Suppose contrary to the result that x^0 is not an efficient solution to (NMP). Then there exists a feasible solution x to (NMP) such that

$$[f_i(x) + (x^t A_i w_i)] \leq [f_i(x^0) + (x^{0t} A_i w_i)].$$

By the properties of b_0 and θ_0 and the above inequality, we get (3.1).

By the feasibility of x^0 the properties of b_1 and θ_1 we get (3.2). By inequalities (3.1) and (3.2) and condition (iii), we have $F(x, x^0; (\nabla f_i(x^0) + A_i w_i^0) + \rho_i^1 d^2(x, x^0)) < 0$

and

$F(x, x^0; (\lambda^0 \nabla h_j(x^0) + \rho_j^2 d^2(x, x^0))) < 0$, since $\mu^0 \geq 0$,

The above inequalities give $F(x, x^0; [\mu^0 \nabla f_i(x^0) + A_i w_i^0 + \lambda^0 \nabla h_j(x^0)] + (\rho_i^1 + \rho_j^2) d^2(x, x^0)) < 0$,

which contradict condition (iii). This completes the proof.

4. MOND-WEIR TYPE DUALITY:

We present some weak and strong duality theorems for (NMP) and the following Mond-Weir dual problem:

(NMWMD) Maximize $[f_i(u) + u^t A_i w_i]$,

subject to $\mu[\nabla f_i(u) + A_i w_i] + \lambda \nabla h_j(u) = 0,$

$\lambda \nabla h_j(u) \geqq 0,$

$\lambda \geq 0$, $\mu \geq 0$, and $\mu e = 1$, where $e = (1, \ldots\ldots, 1)^T \in R^k$. Denote by U^0 the set of all the feasible solutions of problem (NMWMD).

Theorem 4.1: (Weak Duality): Suppose that

(i) $x \in U$, (ii) $(u, \mu, \lambda) \in U^0$ and $\mu > 0$, (iii) The problem (NMP) is strong pseudoquasi type I (F,ρ)-univex at u with respect to some $b_0, b_1, \theta_0, \theta_1$, ρ and $w_i \in R^n$ then

$$[f_i(x) + (x^t A_i w_i)] \nleq [f_i(u) + (u^t A_i w_i)]$$

Proof: Suppose contrary to the result the above inequality holds,

i.e, $[f_i(x) + (x^t A_i w_i)] \leq [f_i(u) + (u^t A_i w_i)]$.

By the property of b_0 and θ_0 and the above inequality, we have

$$b_0(x,u)\theta_0[\{f_i(x)+(x^tA_iw_i)\}-\{f_i(u)+(u^tA_iw_i)\}] \leq 0 \qquad (4.1)$$

By the feasibility of (u,μ,λ), we have $-\lambda^0\nabla h_j(u) \leqq 0$.

By the properties of b_1 and θ_1 we get

$$-b_1(x,u)\theta_1[\lambda\nabla h_j(u)] \leqq 0 \qquad (4.2)$$

By the inequalities (4.1) and (4.2) and condition (iii), we have

$$F(x,u;(\nabla f_i(u)+A_iw_i)+\rho_i^1 d^2(x,u)) \leq 0$$

and

$$F(x,u;(\lambda\nabla h_j(u)+\rho_j^2 d^2(x,u))) \leq 0, \text{ since } \mu > 0,$$

The above inequalities give $F(x,u;([\mu\nabla f_i(u)+A_iw_i+\lambda\nabla h_j(u)]+(\rho_i^1+\rho_j^2)d^2(x,u)) < 0$,

which contradicts (iii). This completes the proof.

Theorem 4.2 : (Weak Duality): Suppose that

(i) $x\in U$ (ii) $(u,\mu,\lambda)\in U^0$ and $\mu > 0$, (iii) Problem (NMP) is weak strictly pseudoquasi type I (F,ρ)-univex at u with respect to some $b_0, b_1, \theta_0, \theta_1, \rho$ and $w_i \in R^n$ then

$$[f_i(x)+(x^tA_iw_i)] \not\leq [f_i(u)+(u^tA_iw_i)].$$

Proof: Suppose contrary to the result the above inequality holds,

i.e, $[f_i(x)+(x^tA_iw_i)] \leq [f_i(u)+(u^tA_iw_i)]$.

By the properties of b_0 and θ_0 and the above inequality, we get (4.1). By the feasibility of (u,μ,λ) and properties of b_1 and θ_1 we get (4.2).

By the inequalities (4.1) and (4.2) and condition (iii),

we have

$$F(x,u;(\nabla f_i(u)+A_iw_i)+\rho_i^1 d^2(x,u)) \leq 0$$

and

$$F(x,u;(\lambda\nabla h_j(u)+\rho_j^2 d^2(x,u))) \leq 0, \text{since } \mu^0 \geq 0,$$

The above inequalities give $F(x,u;([\mu\nabla f_i(u)+A_iw_i+\lambda\nabla h_j(u)]+(\rho_i^1+\rho_j^2)d^2(x,u)) < 0$,

which contradicts (iii). This completes the proof.

Theorem 4.3: (Weak Duality): Suppose that

(i) $x\in U$ (ii) $(u,\mu,\lambda)\in U^0$ and $\mu^0 \geq 0$, (iii) Problem (NMP) is weak strictly pseudo type I (F,ρ)-univex at u with respect to some b_0, b_1, θ_0, θ_1, ρ and $w_i \in R^n$ then

$$[f_i(x)+(x^tA_iw_i)] \not\leq [f_i(u)+(u^tA_iw_i)].$$

Proof: Suppose contrary to the result the above inequality holds,

i.e, $[f_i(x)+(x^tA_iw_i)] \leq [f_i(u)+(u^tA_iw_i)]$.

By the properties of b_0 and θ_0 and the above inequality, we get (4.1) and the feasibility of (u,μ,λ) and properties of b_1 and θ_1 we get (4.2).

By the inequalities (4.1) and (4.2) and condition (iii), we have

$F(x,u;(\nabla f_i(u) + A_i w_i) + \rho_i^1 d^2(x,u)) < 0$

and

$F(x,u;(\lambda \nabla h_j(u) + \rho_j^2 d^2(x,u))) < 0.$

which contradicts condition (iii). This completes the proof.

Theorem 4.4: (Strong Duality): Let u^0 be an efficient solution for (NMP) and u^0 satisfies a constraint qualification for (NMP) (in Maruscíac [17]). Then there exist $\mu^0 \in R^k$ and $\lambda^0 \in R^m$ such that (u^0, μ^0, λ^0) is feasible for (NMWMD). If any of the weak duality in theorems (4.1- 4.3) also holds. Then (u^0, μ^0, λ^0) is efficient solution (NMWMD).

Proof: Since u^0 is efficient for (NMP) and satisfies the constraint qualification for (NMP), then from the Kuhn-Tucker necessary optimality condition, we obtain $\mu^0 > 0$ and $\lambda^0 \geq 0$, such that $(\mu^0 \nabla f_i(u^0) + A_i w_i^0) + \lambda^0 \nabla h_j(u^0) = 0,\ \lambda^0 h_j(u^0) = 0,$

the vector μ^0 may be normalized according to $\mu^0 e = 1$. $\mu^0 > 0$, which gives that the triple (u^0, μ^0, λ^0) is feasible for (NMWMD). The efficiency of (u^0, μ^0, λ^0) for (NMWMD) follows from weak duality theorem. Thus completes the proof.

5. GENERAL MOND-WEIR TYPE DUALITY:

We consider a general Mond-Weir type of dual problem to (NMP) and establish weak and strong duality theorems under some mild assumption. We consider the following general Mond-Weir type dual problem:

(GNMWMD) Maximize $[f_i(u) + u^t A_i w_i] + \lambda_{J_0} h_{J_0}(u)e$ (5.1)

subject to $\mu[\nabla f_i(u) + A_i w_i] + \lambda \nabla h_j(u) = 0,$ (5.2)

$\lambda_{J_q} h_{J_q} \geqq 0, 1 \leqq q \leqq r$ (5.3)

$\lambda \geq 0$, $\mu \geq 0$ and $\mu e = 1$, where $e = (1,\ldots..,1)^T \in R^k$, $J_q, 1 \leqq q \leqq r$, are partitions of the set N.

Theorem 5.1: (Weak Duality): Suppose that for all feasible x for (NMP) and for all feasible for $(u, \mu\ \mu, \lambda)$ (GNMWMD):

(a) $\mu > 0$ and $(f + \lambda_{J_0} h_{J_0}(.)e, \lambda_{J_q} h_{J_q}(.))$ is pseudoquasi type I (F, ρ)-univex at u for each q, $1 \leq q \leq r$ with respect to some b_0, b_1, θ_0, θ_1 and ρ ;

(b) $(f + \lambda_{J_0} h_{J_0}(.)e, \lambda_{J_q} h_{J_q}(.))$ is weak strictly pseudoquasi type I (F, ρ)-univex at u for each q, $1 \leq q \leq r$ with respect to some b_0, b_1, θ_0, θ_1 and ρ ;

(c) $(f+\lambda_{J_0}h_{J_0}(.)e,\ \lambda_{J_q}h_{J_q}(.))$ is weak strictly pseudo type I (F, ρ)-univex at u for each q, $1 \leq q \leq r$ with respect to some b_0, b_1, θ_0, θ_1, ρ and $w_i \in R^n$;

then

$$[f_i(x)+(x^tA_iw_i)] \nleq [f_i(u)+(u^tA_iw_i)] + \lambda_{J_0}h_{J_0}(u)e.$$

Proof: Suppose contrary to the result the above inequality holds. Thus, we have

$$[f_i(x)+(x^tA_iw_i)] \leq [f_i(u)+(u^tA_iw_i)] + \lambda_{J_0}h_{J_0}(u)e.$$

Since x is feasible for (NMP) and $\lambda \geq 0$, the above inequality implies that

$$[f_i(x)+(x^tA_iw_i)]+\lambda_{J_0}h_{J_0}(x)e \leq [f_i(u)+(u^tA_iw_i)]+\lambda_{J_0}h_{J_0}(u)e. \qquad (5.4)$$

By the feasibility of (u, μ, λ) inequality (5.3) gives

$$-\lambda_{J_q}h_{J_q}(u) \geqq 0, \quad 1 \leqq q \leqq r, \qquad (5.5)$$

Since θ_0 and θ_1 are increasing, from (5.4) and (5.5) , we have

$$b_0(x,u)\theta_0[\{f_i(x)+(x^tA_iw_i)+\lambda_{J_0}h_{J_0}(x)e\}-\{f_i(u)+(u^tA_iw_i)+\lambda_{J_0}h_{J_0}(u)e\}] \leq 0 \qquad (5.6)$$

$$-b_1(x,u)\theta_1\{\lambda_{J_q}h_{J_q}(u)\} \leqq 0, \ 1 \leqq q \leqq r. \qquad (5.7)$$

By condition (a), from (5.6) and (5.7), we have

$$F(x,u,([\mu\nabla f_i(u) + A_iw_i + \lambda_{J_0}h_{J_0}(u)e] + (\rho_i^1+\rho_j^2)d^2(x,u)) < 0,$$

$$F(x,u,(\lambda_{J_q}\nabla h_{J_q}(u))+\rho_j^2d^2(x,u)) \leqq 0, \ 1 \leqq q \leqq r.$$

Since $\mu > 0$, the above inequalities give

$$F(x,u;(\mu\nabla f_i(u) + A_iw_i + \sum_{q=0}^{r}\lambda\nabla_{J_q}h_{J_q}(u)) + (\rho_i^1+\rho_j^2)d^2(x,u)) < 0. \qquad (5.8)$$

Since, $J_q, 0 \leqq q \leqq r$, are partitions of the set N, (5.8) is equivalent to

$$F(x,u;([\mu\nabla f_i(u)+A_iw_i+\lambda\nabla h_j(u)] + (\rho_i^1+\rho_j^2)d^2(x,u))) < 0,$$

which contradicts (5.1), By condition (b), from (5.6) and (5.7), we have

$$F(x,u;(\mu\nabla f_i(u) + A_iw_i + \lambda_{J_0}\nabla h_{J_0}(u)e) + (\rho_i^1+\rho_j^2)d^2(x,u)) < 0,$$

$$F(x,u;[\lambda_{J_q}\nabla h_{J_q}(u)]+\rho_j^2d^2(x,u)) \leq 0, \ 1 \leqq q \leqq r.$$

Since, $\mu \geq 0$, the above inequalities give (5.8), which again contradicts (5.1).

By condition (c), (5.6) and (5.7), we have,

$$F(x,u;(\nabla f_i(u) + A_iw_i + \lambda_{J_0}h_{J_0}(u)e) + (\rho_i^1+\rho_j^2)d^2(x,u)) < 0,$$

$$F(x,u;[\lambda_{J_q}\nabla h_{J_q}(u)]+\rho_j^2d^2(x,u)) \leq 0, \ 1 \leqq q \leqq r.$$

Since, $\mu \geq 0$, the above inequalities give (5.8), which again contradicts (5.1).

This completes the proof.

Theorem 5.2: (Strong Duality): Let u^0 be an efficient solution for (NMP) and u^0 satisfies a constraint qualification for (NMP). Then there exist $\mu^0 \in R^k$ and $\lambda^0 \in R^m$ such that (u^0, μ^0, λ^0) is feasible for (GNMWMD). If any of the weak duality in theorem 5.1 holds, then (u^0, μ^0, λ^0) is an efficient solution for (GNMWMD).

Proof: Since u^0 is efficient for (NMP) and satisfies a generalized constraint qualification, by the Kuhn-Tucker necessary optimality condition (see Maeda[20]), there exist $\mu^0 > 0$ and $\lambda^0 \geqq 0$, such that

$$(\mu^0 \nabla f_i(u^0) + A_i w_i^0) + \lambda^0 \nabla h_j(u^0) = 0,$$

$$\lambda^0 h_j(u^0) = 0,\ 1 \leqq i \leqq k,$$

The vector μ^0 may be normalized according to $\mu^0 e = 1$. $\mu^0 > 0$, which gives that the triple (u^0, μ^0, λ^0) is feasible for (GNMWMD). The efficiency of follows from weak duality theorem 5.1 this completes the proof.

6. CONCLUSION

We have used generalized type - I vector valued functions to generalized univex type- I vector-valued functions. We consider a nondifferentiable multiobjective optimization problem involving generalized type-I function with (F,ρ)-univexity. Kuhn-Tucker type sufficient optimality conditions are obtained for a feasible solution to be an efficient solution. Mond-Weir and general Mond-Weir type duality results are also presented. Duality results have been established assuming the functions to be generalized (F, ρ)-univexity.

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