A Methodology on a class of Trilevel Programming Problems

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Abstract. This paper deals with theory and methodology on a special class of Trilevel Programming Problems with non-linear objectives. A quadratic (indefinite) programming problem is a mathematical programming problem which is a product of two linear factors. In this article, the piece–wise indefinite quadratic programming problem (PIQP) is considered. Here, the objective function is a product of two continuous piecewise linear functions defined on a non-empty and compact feasible region. Here, the optimality criterion is explained and established to solve PIQP. While optimizing PIQP, we come across certain variables which will not satisfy the optimality condition. For these variable, cases have been discussed so as to move from one basic feasible solution to another basic feasible solution till we reach the optimality. A numerical example is discussed to decipher the methodology.

Keywords: indefinite programs; quadratic programming problem; optimal solution, piecewise linear function; continuity. quasi–concave function

1. Introduction

Quadratic programming is an important class of non-linear programming where the objective function is quadratic and the constraints linear. Aneja et al. [1] considered a special class of quadratic programs. They dealt with the maximization and minimization cases along with the assumption that two linear factors being non-negative. Cabot[3] focused on a special class of problem of maximizing the sum of certain quasi-concave functions over a convex region. Chen and Huang [5], Kough [10] gave methods to obtain the global optimum solution by using Benders cut. Pardalos et al. [14] proposed a new and efficient branch and bound algorithm to find global optimum of large scale problems. Shi et al. [15] in 2005 proposed a multiple criteria quadratic program.

2. Trilevel Indefinite Quadratic Programming Problem

Indefinite quadratic programming problem has extensive applications in realistic as well as material world. It can be seen in corporate planning; production planning problems , health care, financial planning etc. The indefinite quadratic programming deals with product of two piecewise linear functions and each of which is continuous in nature. The decision maker at first level optimizes his objective function first, then for a given value of the variables controlled by the decision maker at first level, second decision maker optimizes his objective and then again the third one optimizes his function for given values of the variables under the control of first & second decision makers.The objective function at each of the three levels is product of two positive affine functions .

2. Mathematical Model of Trilevel Programming Problem

Mathematically, the Trilevel indefinite quadratic programming problem is defined as:

(PIQP) : $\text{Max } Z_1 (X) = P_1 (X).P_2 (X)$

$\text{Max } Z_2 (X) = Q_1(X).Q_2(X)$

$\text{Max } Z_3 (X) = R_1(X)R_2(X)$

Where X= (x_1, x_1, x_3) ; x_1 is the variable controlled by first decision maker; x_2 is the variable controlled by second decision maker; x_3 is the variable controlled by third decision maker respectively over the convex polyhedron set S.

The optimal solution for this problem exists Since the objective functions at each of the three levels is product of piece–wise continuous linear functions as also functions being indefinite quadratic problem. Solving this problem by its conventional method increases the size of the problem. Thus, arises the need of developing an algorithm to obtain the solution of (PIQP).Hence, an algorithm for piece–wise indefinite quadratic programming problem is employed to optimize (PIQP). The result which establishes the condition that sum of quasi-concave functions is quasi-concave plays a very important role in finding optimal solution to the quadratic trilevel programming problems

3. Methodology to solve (PIQP)

The piecewise indefinite quadratic programming problem (PIQP) is defined as the transformation for piecewise linear functions ,let the partition of the variables

The piecewise indefinite quadratic programming problem (PIQP) is defined as:

$\text{Max } Z(X) = \Sigma\, P(x)Q(x)$

subject to

$AX = b$

$X \in S$

Here,

$AX = b$

which is rewritten in the form

$$B{*}X_B{*} + N{*}X_N = b$$

Here, B^* is the n × n basis matrix and X_B is the basic feasible solution corresponding to the basis matrix B

Let $X^* = (X_{B*}, X_{N*})$ be a basic feasible solution corresponding to this basis structure.

4. Optimality condition

A non-basic variable can change its value from its current breakpoint value either in the left side direction or in the right side direction. The left hand side reduced cost is denoted by $\mathbf{a}^-$ and the right hand side reduced cost is denoted by. A non degenerate basic feasible solution is optimal solution of the problem under consideration only if $\mathbf{a}^- \geq 0$ and $\mathbf{a}^+ \leq 0$

5. Conclusion

In this research article, an efficient and innovative idea is proposed so as to optimize Trilevel Programming Problem piece–wise indefinite quadratic programming problem. The method incorporates those objective functions which are quasiconcave. A result has been proved exhibiting the conditions where sum of quasiconcave functions will be quasiconcave.

6. References

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