A Review of Nuclear Shell Model

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Abstract:

Shell model provides us better understanding about the various properties of the nucleus and this model is highly successful to explain many properties of the nucleus. With in the nucleus it is very difficult to define the potential because many factors decide the nature of this potential therefore various potentials was proposed by different physicists. Here we have applied finite deep square well potential, harmonic oscillator potential, wood sexon potential to predict the magic numbers but these model predict the magic number 2,8,20 only which match with the experimental results then spin orbit interaction provide a way to explain other magic numbers too. Latest development, recent theories, success and failure of this model are also discuss here.

Keywords: shell model, magic numbers, potential etc

Introduction:

The electrons in atom are filled in various shells and subshells. Atoms consist 2,10,18,36,54 and 86 electrons have all their shells completely filled. Such atoms represent the inert gas configuration for gases like helium, neon, argon, krypton, xenon, radon etc. such atoms are more stable in comparison of other atoms. Similarly in the nucleus such type of situation also exist and nuclei having 2,8,20,28,50,82,126 number of protons or number of neutrons or total number of nucleons are more stable in comparison of other nuclei. Such numbers are said to be magic numbers and nucleus having these numbers are said to be magic nuclei. Along with these number nuclei having 14,28,40 number of protons or number of neutrons or total number of nucleons are intermediate stable in comparison of other nuclei. Such numbers are said to be semi magic numbers and nucleus having these numbers are said to be semi magic nuclei.

Evidence for existence of magic number:

1. Mayer in 1948 suggested that nuclei with a magic number of nucleons are especially abundant in nature.
2. If we plot a curve between binding energy per nucleon vs total number of nucleons A, then curve shows that binding energy suddenly increases corresponding to magic number because binding energy curve shows peak corresponding to magic numbers.
3. ${}^{4}_{2}He$ and ${}^{16}_{8}O$ nuclei are more stable in comparison of near by nucleus because such nuclei are doubly magic nuclei. In lighter nuclei binding energy curve shows peaks corresponding to such nuclei.
4. Number of stable isotopes per element

Elements	Number of stable isotopes
K_{19}	3
In_{49}	2
Ti_{22}	2
Ca_{20}	6
Sn_{50}	10
Pb_{82}	4
Sc_{21}	1
Sb_{51}	2
Bi_{83}	1
Ni_{28}	5
Be_{4}	4
S_{16}	6
Mo_{42}	7

From this table it may be concluded that number of stable isotopes for Z=20,28,50 and 82 are much larger then corresponding to Neighboring isotopes. Sulphur and Molybdenum have number of neutrons N= 20 and N=50 respectively.

5. Number of stable isotones

Number of neutrons (N)	Stable isotones
19	0
20	5
21	1
49	1
50	6
51	1
81	1
82	7
83	1

From this table it may be concluded that number of stable isotones exist for N=20,50 and 82and are much larger then corresponding to Neighboring isotones.

6. Alpha decay energies are very smooth function of A for a given value of Z but shows sharp discontinuity at N =126. It shows that magic character of N=126.
7. Decay product of radioactive series

Mass no.	Series	Parents	Half life	Stable end product
4n	Thorium	$^{232}_{90}Th$	1.39×10^{10}	$^{208}_{82}Pb$
4n+1	Neptunium	$^{237}_{93}Np$	2.25×10^{6}	$^{209}_{83}Bi$
4n+2	Uranium	$^{238}_{92}U$	4.51×10^{9}	$^{206}_{82}Pb$
4n+3	Actinium	$^{235}_{92}U$	7.07×10^{8}	$^{207}_{82}Pb$

From the table it may be concluded that Z=82 and N=126 are particularly stable nuclei.

8. Weak binding of the first nucleon outside a closed shell is shown by the low probabilities for the capture of neutrons by nuclei .

Number of Neutrons	Cross-section in Milli Barn (mb) approx..
19	2
20	0.41
21	12
28	2
49	19
50	0.65
51	6.40
82	10
126	8

The cross-section for these nuclei are very much lower than those of neighboring nuclei, the low cross-section is an indication of large level spacing in the compound nucleus formed by the neutron capture. Calcium, tin and lead have very much smaller cross-section then there neighbors.

9. The doubly magic nuclei when Z and N are both magic numbers like as ${}^{4}_{2}He$, ${}^{16}_{8}O$, ${}^{40}_{20}Ca$ and ${}^{208}_{82}Pb$ are particular stable and such nuclei shows peaks in binding energy curve.

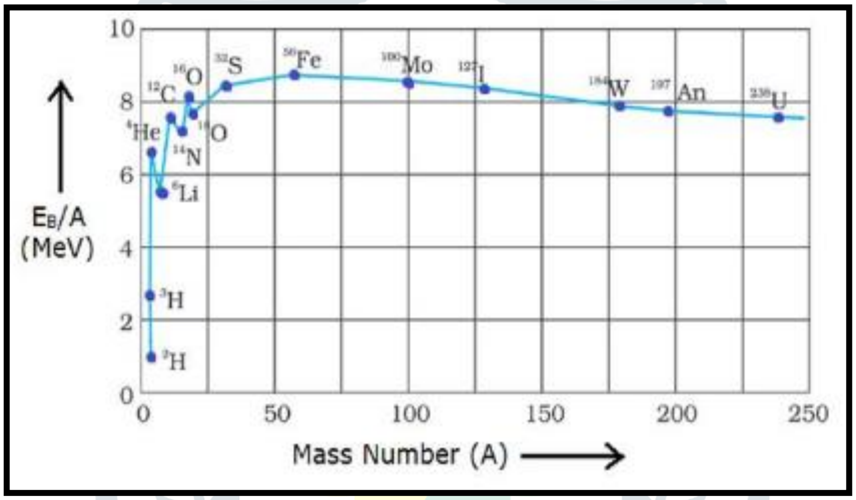

Fig.1.1 Binding Energy Curve

10. When the parent nucleus has 126 neutrons, such nucleon have relatively long half lives and low alpha energies.
11. The asymmetry of the fission of uranium could involve the substructure of nuclei having expressed the existence of magic numbers.
12. When the electric quadrupole moment of magic nuclei is measured then value of EQM tends to be zero or nearly so, basically lesser value of EQM shows the more stability corresponding to magic nuclei.
13. The binding energy of next neutron and proton after a magic number is comparatively very small.

Nucleus	No. of Neutrons	Separation energy in MeV
${}^{15}_{8}O$	7	13.2
${}^{16}_{8}O$	8	15.7
${}^{17}_{8}O$	9	4.14
${}^{39}_{20}Ca$	19	13.3
${}^{40}_{20}Ca$	20	15.7
${}^{41}_{20}Ca$	21	8.4
${}^{47}_{20}Ca$	27	7.3
${}^{48}_{20}Ca$	28	9.9
${}^{49}_{20}Ca$	29	5.1

From above table it may be concluded that separation energy of the last neutrons suddenly decreases when neutrons number are 8,20 and 28.

14. Energies of the emitted beta particles are specially large when the number of neutrons and protons in the product nucleus (daughter nucleus) corresponds to a magic number. The result of investigations on alpha decay and beta decay shows that there are energy discontinuous of above 2 MeV in the neighborhood of magic numbers.
15. Elements with Z= magic number have a large abundance then those of nearby elements and this is shown by the composition of earth, sun, stars etc.

Results and Discussion:

Square well potential:

As we know that nuclear force is a short range, strongest known force in nature and such forces are attractive force and when such force act they act in full strength (saturation force) otherwise such force do not act. Japanese Scientist Yukawa explained the theory of nuclear force on the basis of π meson exchange. When separation between two nucleons is small enough to exchange the π meson then such force produce. This separation decide the range of the nuclear force so potential can be defined as

$$\begin{aligned} V(r) &= -V_0 \qquad & r < r_0 \\ &= 0 \qquad & r > r_0 \end{aligned}$$

If we solve schrodinger equation for applying above square well potential then energy eigen value of nucleons obtained in the sequence 1s,1p,1d,2s,1f and so on

State	Orbital Quantum number
3p……	1……..
1i	6
2f	3
3s	0
1h	5
2d	2
1g	4
2p	1
1f	3
2s	0
1d	2
1p	1
1s	0

For a given value of l , magnetic orbital quantum number have the value $-l$ to $+l$ i.e $(2l+1)$ values. Since nucleon have two different orientation so number of nucleons in given l values should be $2(2l+1)$.

Level	Value of l	Number of nucleons 2(2l+1)	Magic numbers
1s	0	2	2
1p	1	6	8
1d	2	10	18
2s	0	2	20
1f	3	14	34
2p	1	6	40
1g	4	18	58
2d	2	10	68
1h	5	22	90
3s	0	2	92
2f	3	14	106
1i	6	26	132
3p	1	6	138

This model predicts the value of magic number 2,8,18,20,34,40,58,68,90,92,106,132 and 138 but these values partially resemble with observed magic numbers. Only in these value 2,8,20 are magic numbers other numbers are not magic numbers.

Harmonic oscillator potential:

In this model predicted potential $V(r) = -V_0 + \frac{1}{2}m\omega^2r^2$

If we solve schrodinger equation for applying above harmonic oscillator potential then energy eigen value of nucleons obtained in the sequence 1s,1p,2s,1d,2p,1f and so on.

Nuclear levels and magic numbers predicted by harmonic oscillator potential

Energy levels	Value of l	Number of nucleons $\sum_l 2(2l+1)$	Magic number
1s	0	2	2
1p	1	6	8
2s,1d	0,2	12	20
2p,1f	1,3	20	40
3s,2d,1g	0,2,4	30	70
3p,2f,1h	1,3,5	42	112
4s,3d,2g,1i	0,2,4,6	56	168

So this model predict the following magic numbers 2,8,20,40,70,112 and 168 etc and only three 2,8,20 in this list resembles with observed magic number.

Woods -Saxon potential:

This potential expressed in the form of $V(r) = -\frac{V_0}{1+e^{\frac{R-r}{a}}}$

Here value of a= 0.52 fermi and R is the mean nuclear radius.
If we solve schrodinger equation for applying above potential then predicted magic numbers would be 2,8,20,40,58,92,112.

Spin orbit coupling:

This concept was proposed independently in 1949 by M.G Mayer in USA and J.H.D Jensen in Germany. This model was on the basis of spin orbit interaction. According to shell theory, L-S coupling holds good for very lighter nuclei in which value of l are very small. The intrinsic spin angular momentum S of the system are coupled together in to a total spin angular momentum and orbital angular momentum of the system are coupled together into a total orbital angular momentum thereafter total orbital angular momentum and total spin angular momentum are coupled together. For heavy nuclei J-J coupling holds good. In this coupling spin angular momentum and orbital angular momentum of each particle are first coupled together to form total angular momentum for that particle. This coupling holds for the great majority of the nuclei.

In mathematical form we have

$$\vec{S}=\sqrt{s(s+1)}\hbar$$
$$\vec{L}=\sqrt{l(l+1)}\hbar$$
$$\vec{J}=\sqrt{j(j+1)}\hbar$$

Total angular momentum is a vector resulting from the coupling of the orbital and spin angular momentum

$$\vec{J}=\vec{L}+\vec{S}$$
$$j=l+\tfrac{1}{2} \text{ if } \vec{L} \text{ is parallel to } \vec{S}$$
$$j=l-\tfrac{1}{2} \text{ if } \vec{L} \text{ is antiparallel to } \vec{S}$$

In the absence of the spin orbit interaction the energy of state does not depends upon total angular momentum $\vec{J}$.

Mathematically we have

$$\vec{J}.\vec{J}=(\vec{L}+\vec{S}).(\vec{L}+\vec{S})$$
$$J^2=L^2+S^2+2\,\vec{L}.\vec{S}$$

And expression of energy splitting

$$\Delta E_{l,s}=\frac{(2l+1)}{2}<V_{l,s}(r)>$$

$V_{l,s}$ is negative and so energy corresponding to $j=l+\frac{1}{2}$ is always lesser than the level corresponding to $j=l-\frac{1}{2}$

For the s state $l=0$, s= $\frac{1}{2}$ so j= $\frac{1}{2}$ this energy level ${}^{2}_{\frac{1}{2}}s$ remains unsplitted due to absence of spin orbit interaction.

For the p state $l=1$, s= $\frac{1}{2}$ so j= $\frac{1}{2}$, $\frac{3}{2}$ this energy level ${}^{2}_{\frac{1}{2}}p, {}^{2}_{\frac{3}{2}}p$ remains splitted into two energy levels due to presence of spin orbit interaction.

For the d state $l=2$, s= $\frac{1}{2}$ so j= $\frac{3}{2}$, $\frac{5}{2}$ this energy level ${}^{2}_{\frac{3}{2}}d, {}^{2}_{\frac{5}{2}}d$ remains splitted into two energy levels due to presence of spin orbit interaction. and so on ……..

According to this model

Energy Levels	Magic Number
$\overset{1}{\frac{1}{2}}s$	2
$\overset{1}{\frac{3}{2}}p, \overset{1}{\frac{1}{2}}p$	8
$\overset{1}{\frac{5}{2}}d, \overset{2}{\frac{1}{2}}s, \overset{1}{\frac{3}{2}}d$	20
$\overset{1}{\frac{7}{2}}f$	28
$\overset{2}{\frac{3}{2}}p, \overset{1}{\frac{5}{2}}f, \overset{2}{\frac{1}{2}}p, \overset{1}{\frac{9}{2}}g$	50
$\overset{1}{\frac{7}{2}}g, \overset{2}{\frac{5}{2}}d, \overset{2}{\frac{3}{2}}d, \overset{3}{\frac{1}{2}}s, \overset{1}{\frac{11}{2}}h$	82
$\overset{1}{\frac{9}{2}}h, \overset{2}{\frac{7}{2}}f, \overset{2}{\frac{5}{2}}f, \overset{3}{\frac{3}{2}}p, \overset{3}{\frac{1}{2}}p, \overset{1}{\frac{13}{2}}i$	126
$\overset{2}{\frac{9}{2}}g, \overset{3}{\frac{5}{2}}d, \overset{1}{\frac{11}{2}}i, \overset{2}{\frac{7}{2}}g, \overset{4}{\frac{1}{2}}s, \overset{3}{\frac{3}{2}}d, \overset{1}{\frac{15}{2}}j$	184

Magic numbers 2,8,20,28,50,82,126 and 184 are predicted.

Shell Configurations-Populations of Sub levels

N	*l*	n*l*	Energy Levels	Magic Number
1	0	1s	$\overset{1}{\frac{1}{2}}s$	2
2	1	1p	$\overset{1}{\frac{3}{2}}p, \overset{1}{\frac{1}{2}}p$	8
3	2,0	1d,2s	$\overset{1}{\frac{5}{2}}d, \overset{2}{\frac{1}{2}}s, \overset{1}{\frac{3}{2}}d$	20
4	3	1f	$\overset{1}{\frac{7}{2}}f$	28
5	3,1	2p,1f,1g	$\overset{2}{\frac{3}{2}}p, \overset{1}{\frac{5}{2}}f, \overset{2}{\frac{1}{2}}p, \overset{1}{\frac{9}{2}}g$	50
6	4,2,0	1g,2d,3s,1h	$\overset{1}{\frac{7}{2}}g, \overset{2}{\frac{5}{2}}d, \overset{2}{\frac{3}{2}}d, \overset{3}{\frac{1}{2}}s, \overset{1}{\frac{11}{2}}h$	82
7	5,3,1	1h,2f,3p,1i	$\overset{1}{\frac{9}{2}}h, \overset{2}{\frac{7}{2}}f, \overset{2}{\frac{5}{2}}f, \overset{3}{\frac{3}{2}}p, \overset{3}{\frac{1}{2}}p, \overset{1}{\frac{13}{2}}i$	126
8	6,4,2,0	2g,3d,1i,4s1j	$\overset{2}{\frac{9}{2}}g, \overset{3}{\frac{5}{2}}d, \overset{1}{\frac{11}{2}}i, \overset{2}{\frac{7}{2}}g, \overset{4}{\frac{1}{2}}s, \overset{3}{\frac{3}{2}}d, \overset{1}{\frac{15}{2}}j$	184

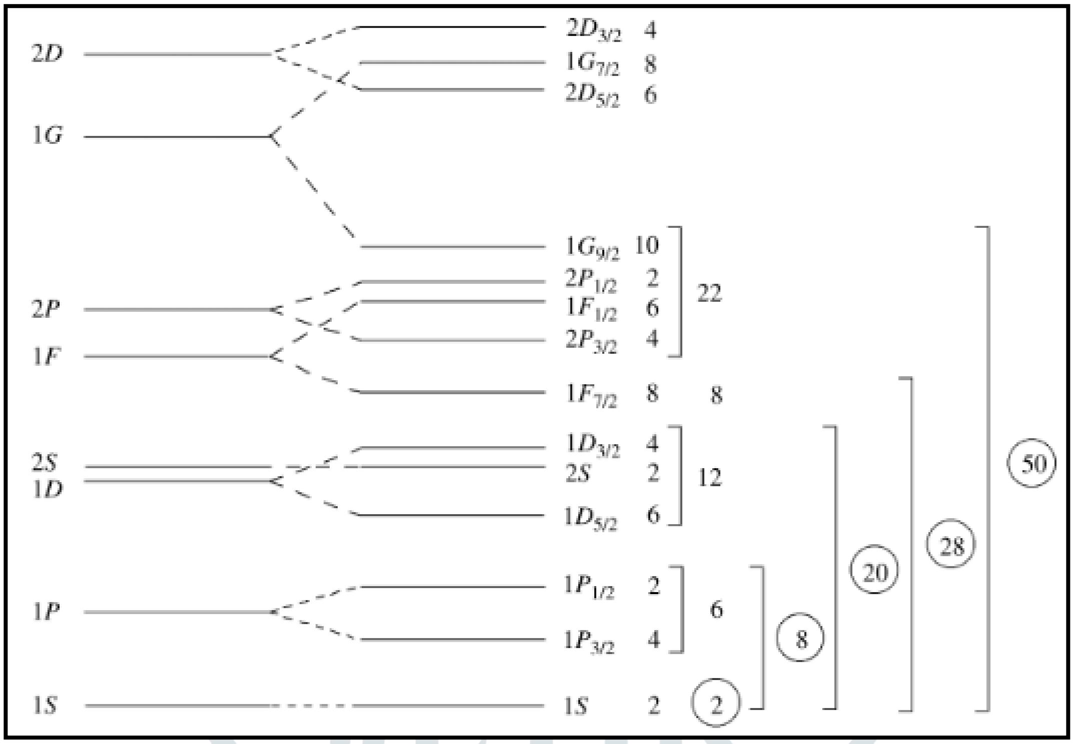

Fig.1.2 Spin Orbit Interaction Ensure Magic Numbers in Shell Model

Prediction of the shell model:

1. Even even nuclei have total ground state angular momentum =0 and such nuclei have even parity. There is no exception of these rule.
2. With an odd number of nucleons, nucleons pair off as far as possible.
3. In odd number of nucleons resultant angular momentum and spin direction are just that of the single odd nucleons.
4. For odd odd nucleus total angular momentum is equal to the vector sum of the odd neutron and odd proton J values.
5. For odd odd nucleus parity= $(-1)^{l_n+l_p}$
6. L.W Nordheim Rule-
   If For two odd odd nucleons $j_1 + l_1 + j_2 + l_2 =$ an even number
   Then resultant angular momentum J= $|J_1 - J_2|$
   If For two odd odd nucleons $j_1 + l_1 + j_2 + l_2 =$ an odd number
   Then resultant angular momentum J= $J_1 + J_2$
7. In an odd nucleus the total angular momentum of the nucleus is equal to the angular momentum of the last unpaired nucleon and so the magnetic moment of the nucleus is produce due to odd nucleon only.
8. EQM for odd A, odd Z nuclide is
   $Q = \frac{2J-1}{2J+1}(r_{r.m.s})^2$
   For a uniform charge distribution $(r_{r.m.s})^2 = \frac{3}{5}R^2$ where R= 1.25 $A^{\frac{1}{3}}$ fermi is said to be coulomb radius.
9. Theory of beta decay shows that life time can be understood in terms of relative parity and angular momenta of the state involved.
10. Shell model can explain magnetic dipole and electric quadrupole moment, isomerism of different nuclei, extra stability of magic nuclei, spin and parity of odd -odd nuclei etc.

Drawbacks of shell model:

1. Shell model fails to explain Spin value of certain nuclei for example $^{23}_{11}Na$ has angular momentum $\frac{3}{2}$ instead of $\frac{5}{2}$. $^{55}_{25}Mn$ has $\frac{5}{2}$. Instead of $\frac{7}{2}$.
2. Shell model is unable to explain the energy of first excited states I even-even nuclei.
3. It is unable to explain magnetic moment of some nuclei and electric quadrupole moment of many nuclei.

Duflo Zuker shell model:

Recently Duflo Zuker shell model is very popular and providing very good result in wide range. In this model there are two different models DZ_{10} formula represented with 10 terms and DZ_{33} formula represents with 33 terms. In brief according to these formula B.E expression can be described as

$$\text{B.E} = < H_m > -E_c - E_{sym} + E_p$$

Where monopole Hamiltonian $< H_m >$ represents an average mean field extracted from the interacting shell model. E_c represents the coulomb energy, E_{sym} represents the symmetry energy and E_p represents the pairing energy term.

H.Nakada and K.Sugiura Model:

In 2013 this model predict the magic numbers of nuclei with semi realistic nucleon-nucleon interactions by self consistant mean fields and calculations was done with the help of M3Y-P6 and P7 semi realistic nucleon nucleon interaction. Here Hamiltonian of the system was taken as

$$H = H_N + V_C - H_{CM}$$

Where H_N= nuclear Hamiltonian
V_C= coulomb interaction
H_{CM}= centre of mass Hamiltonian

The magic number Z= 2,8,20,28,50,82 and N= 2,8,20,28,50,82,126 have been established around the beta stability line but it has been clarify by experiments that N= 8,20,28 magicity is eroded far off the beta stability line.

Shell model Algorithm:

Valance shell nucleons interact through an effective interaction, now a days various shell model code like as DUSM, and FDUO code are used to predict better results.

Different shell model codes:

Sr. No.	Group	Code	Scientist
1	Permutation group	DUSM	Novoselsky,Vallieres 1989
2	Pseudo LS pairs	FDUO	Wu, Vallieres 1989
3	Mscheme	MG	Whitehead, Watt 1977
4	J-J Coupling	OAK.RIDGE	Halbert, MC Grogry, Wang 1969

Ref: The nuclear shell model Michel Vallieres and Huawu (1991)

FDUO shell model code: this model can be used for even-even nucleus.

L S Coupling: (sd shell), SU(4) Multiplex, Applied by Wigner in 1937. Here potential is isospin independent applied by Elliott, 1958.

J J Coupling: in this coupling seniority scheme was applied by Racah in 1949. Here pair formation take place due to short range of interaction.

Shell model Monte Carlo: this method are based on Monte Carlo evolution of path integral obtained by H.S Transformation.

Note- For this research paper, work is done during the session 2010-12 and onwards. This research paper is written by M.Sc students of Meerut College, Meerut. **Neetika,** M.Sc Student (Session 2010-12), **Ankit Chauhan**, M.Sc Student (Session 2010-12), **Dharmendra Singh Raghav,** M.Sc Student (Session 2010-12) under the research guidance of **Dr. Lalit Kumar** Associate Professor, Department of Physics, Meerut College Meerut for PG students.

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