Glimpse of Gravitational Wave, Detector Sensitivity and its Application

Dr Shailendra Kumar Srivastava
Associate Professor
Department of Physics
MGPG College ,Gorakhpur

Abstract

Gravitational waves are propagating fluctuations of gravitational fields, that is, ”ripples” in space time, generated mainly by moving massive bodies. These distortions of space time travel with the speed of light. Every body in the path of such a wave feels a tidal gravitational force that acts perpendicular to the waves direction of propagation; these forces change the distance between points, and the size of the changes is proportional to the distance between the points. Gravitational waves can be detected by devices which measure the induced length changes. The frequencies and the amplitudes of the waves are related to the motion of the masses involved. Thus, the analysis of gravitational waveforms allows us to learn about their source and, if there are more than two detectors involved in observation, to estimate the distance and position of their source on the sky.

Keywords: Gravitational Waves, Gravitational Fields, Tidal Gravitational force.

1. INTRODUCTION

Einstein first postulated the existence of gravitational waves in 1916 as a consequence of his theory of General Relativity, but no direct detection of such waves has been made yet. The best evidence thus far for their existence is due to the work of 1993 Nobel laureates Joseph Taylor and Russell Hulse. They observed, in 1974, two neutron stars orbiting faster and faster around each other, exactly what would be expected if the binary neutron star was losing energy in the form of emitted gravitational waves. The predicted rate of orbital acceleration caused by gravitational – 1 – radiation emission according to general relativity was verified observationally, with high precision. Cosmic gravitational waves, upon arriving on earth, are much weaker than the corresponding electromagnetic waves. The reason is that strong gravitational waves are emitted by very massive compact sources undergoing very violent dynamics. These kinds of sources are not very common and so the corresponding gravitational waves come from large astronomical distances. On the other hand, the waves thus produced propagate essentially unscathed through space, without being scattered or absorbed from intervening matter.

1.1 Why are Gravitational Waves Interesting?

Detection of gravitational waves is important for two reasons: First, their detection is expected to open up a new window for observational astronomy since the information carried by gravitational waves is very different from that carried by electromagnetic waves. This new window onto universe will complement our view of the cosmos and will help us unveil the fabric of space time around black-holes, observe directly the formation of black holes or the merging of binary systems consisting of black holes or neutron stars, search for rapidly spinning neutron stars, dig deep into the very early moments of the origin of the universe, and look at the very center of the galaxies where supermassive black holes weighting millions of solar masses are hidden. These are only a few of the great scientific discoveries that scientists will witness during the first decade of the 21st century. Second, detecting gravitational waves is important for our understanding of the fundamental laws of physics; the proof that gravitational waves exist will verify a fundamental 85-year-old prediction of general relativity. Also, by comparing the arrival times of light and gravitational waves, from, e.g., supernovae, Einstein's prediction that light and gravitational waves travel at the same speed could be checked. Finally, we could verify that they have the polarization predicted by general relativity.

1.2 How We Will Detect Them?

Up to now, the only indication of the existence of gravitational waves is the indirect evidence that the orbital energy in the Hulse-Taylor binary pulsar is drained away at a rate consistent with the prediction of general relativity. The gravitational wave is a signal, the shape of which depends upon the changes in the gravitational field of its source. As it has been mentioned earlier, any body in the path of the wave will feel an oscillating tidal gravitational force that acts in a plane perpendicular to the waves direction of propagation. This means that a group of freely moving masses, placed on a plane perpendicular to the direction of propagation of the wave, will oscillate as long as the wave passes through them, and the distance between them will vary as a function of time as in Figure 1. Thus, the detection of gravitational waves can be accomplished by monitoring the tiny changes in the distance between – 2 – freely moving test masses. These changes are extremely small; for example, when the Hulse-Taylor binary system finally merges, the strong gravitational wave signal that will be emitted will induce changes in the distance of two particles on earth, that are 1 km apart much smaller than the diameter of the atomic nucleus! To measure such motions of macroscopic objects is a tremendous challenge for experimentalists. As early as the mid-1960s, Joseph Weber designed and constructed heavy metal bars, seismically isolated, to which a set of piezoelectric strain transducers were bonded in such a way that they could detect vibrations of the bar if it had been excited by a gravitational wave. Today, there are a number of such apparatuses operating around the world which have achieved unprecedented sensitivities, but they still are not sensitive enough to detect gravitational waves. Another form of gravitational wave detector that is more promising uses laser beams to measure the distance between two well-separated masses. Such devices are basically kilometer sized laser interferometers consisting of three masses placed in an L-shaped configuration. The laser beams are reflected back and forth between the mirrors attached to the three masses, the mirrors lying several kilometers away from each other. A gravitational wave passing by will cause the lengths of the two arms to oscillate with time. When one arm contracts, the other expands, and this pattern alternates. The result is that the interference pattern of the two laser beams changes with time. With this technique, higher sensitivities could be achieved than are possible with the bar detectors. It is expected that laser interferometric detectors are the ones that will provide us with the first direct detection of gravitational wave.

2. DETECTION OF GRAVITATIONAL WAVES

The first attempt to detect gravitational waves was undertaken by the pioneer Joseph Weber during the early 1960s. He developed the first resonant mass detector and inspired many other physicists to build new detectors and to explore from a theoretical viewpoint possible cosmic sources of gravitational radiation. A pair of masses joined by a spring can be viewed as the simplest conceivable detector; In practice, a cylindrical massive metal bar or even a massive sphere is used instead of this simple system. When a gravitational wave hits such a device, it causes the bar to vibrate. By monitoring this vibration, we can reconstruct the true waveform. The next step, following the idea of resonant mass detectors, was the replacement of the spring by pendulums. In this new detector the motions induced by a passing-by gravitational wave would be detected by monitoring, via laser interferometry, the relative change in the distance of two freely suspended bodies. The use of interferometry is probably the most decisive step in our attempt to detect gravitational wave signals. In what follows, we will discuss both resonant bars and laser interferometric detectors. Although the basic principle of such detectors is very simple, the sensitivity of detectors is limited by various sources of noise. The internal noise of the detectors can be Gaussian or non-Gaussian. The non-Gaussian noise may occur several times per day such as strain releases in the suspension systems which isolate the detector from any environmental mechanical source of noise, and the only way to remove this type of noise is via comparisons of the data streams from various detectors. The so-called Gaussian noise obeys the probability distribution of Gaussian statistics and can be characterized by a spectral density $S_n(f)$. The observed signal at the output of a detector consists of the true gravitational wave strain h and Gaussian noise. The optimal method to detect a gravitational wave signal leads to the following signal-to-noise ratio:

$$\left(\frac{S}{N}\right)^2_{\rm opt} = 2\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)}df,$$

where h˜(f) is the Fourier transform of the signal waveform. It is clear from this expression that the sensitivity of gravitational wave detectors is limited by noise.

2.1 Cosmological Gravitational Waves

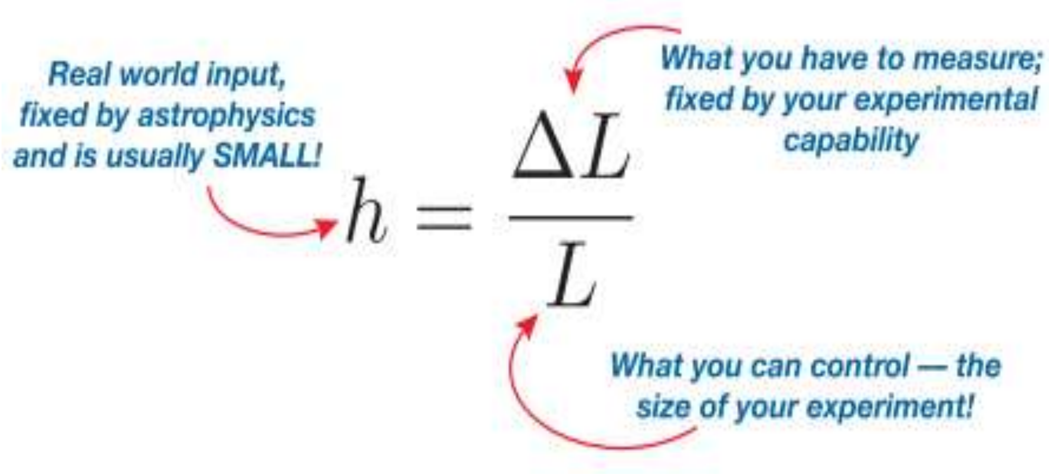
One of the strongest pieces of evidence in favour of the Big Bang scenario is the 2.7 K cosmic microwave background radiation. This thermal radiation first bathed the universe around 380,000 years after the Big Bang. By contrast, the gravitational radiation background anticipated by theorists was produced at Planck times, i.e., at 10^{-32} sec or earlier after the Big Bang. Such gravitational waves have travelled almost unimpeded through the universe since they were generated. The observation of cosmological gravitational waves will be one of the most important contributions of gravitational wave astronomy. These primordial gravitational waves will be, in a sense, another source of noise for our detectors and so they will have to be much stronger than any other internal detector noise in order to be detected. Otherwise, confidence in detecting such primordial gravitational waves could be gained by using a system of two detectors and cross-correlating their outputs. The two LIGO detectors are well placed for such a correlation.

3. Detector Sensitivity

•When you decide to build a detector, you think about the physical effect you have to measure. We have seen that gravitational waves change the proper distance between particles.
• When you decide to build a detector, you think about the physical effect you have to measure. We have seen that gravitational waves change the proper distance between particles.
We characterize this distance by the strain $h = \Delta L/L$. This fundamental definition guides our basic thinking about detector design. If ΔL is what we have to measure, over the distance L, then the kind of astrophysical strain from typical astrophysical objects is roughly

$$h = \frac{\Delta L}{L} \sim 10^{-21} \sim \frac{\text{Diameter of H atom}}{1\ \text{AU}}$$

•The way these quantities enter in the process of experiment design is shown schematically below:



•There are two ways to go about this. You could decide what astrophysical sources you are interested in, and determine what detector is needed, or you can decide what detector you can build (L is determined by size and pocketbook, whereas ΔL is fixed by the ingenuity of your experimentalists). But often the design problem is an optimization of both astrophysics and capability.

•In the modern era, gravitational wave detection technology is dominated by laser interferometers, which we will focus on here. In general, an interferometric observatory has its best response at the transfer frequency f', where gravitational wavelengths are roughly the distance probed by the time of flight of the lasers:

$$f_\star = \frac{c}{2\pi L}$$

•If you build a detector, the principle goal is to determine what gravitational waves the instrument will be sensitive to. We characterize the noise in the instrument and the instrument's response to gravitational waves using a sensitivity curve.

• Sensitivity curves plot the strength a source must have, as a function of gravitational wave frequency, to be detectable. There are two standard curves used by the community:

! Strain Sensitivity. This plots the gravitational wave strain amplitude h versus gravitational wave frequency f.

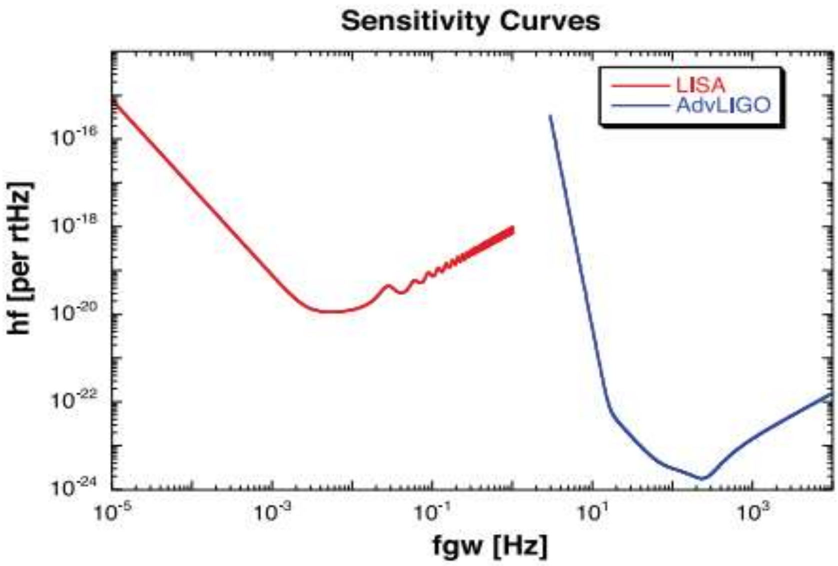
! Strain Spectral Amplitude. This plots the square root of the power spectral density,

$h_f = \sqrt{S_h}$ versus gravitational wave frequency f. The power spectral density is the power per unit frequency and is often a more desirable quantity to work with because gravitational wave sources often evolve dramatically in frequency during observations.

• The strain sensitivity of a detector, h^D, builds up over time. If you know the observation time T_{obs} and the spectral amplitude curve (like those plotted above) you can convert between the two via

$$h_f^D = h^D \sqrt{T_{obs}}$$

• The sensitivity for LIGO and LISA are shown below. Your own LISA curves can be created using the online tool at www.srl.caltech.edu/~shane/sensitivity/MakeCurve.html.



• LISA has armlengths of $L = 5 \times 10^9$ m, which if you consider its transfer frequency f' makes it more sensitive at lower frequencies. LIGO has armlengths of L = 4 km, but the arms are Fabry-Perot cavities, and the laser light bounces back and forth ~ 100 times; this puts its prime sensitivity at a much higher frequency.

4. Sources And Sensitivity Curves

•Sensitivity curves are used to determine whether or not a source is detectable. Rudimentarily, if the strength of the source places it above the sensitivity curve, it can be detected! How do I plot sources on these curves? First, it depends on what kind of curve you are looking at; second, it depends on what kind of source you are working with!

• If you are talking about observing sources that are evolving slowly (the are approximately monochromatic) then the spectral amplitude and strain are related by

$$h_f = h\sqrt{T_{obs}}$$

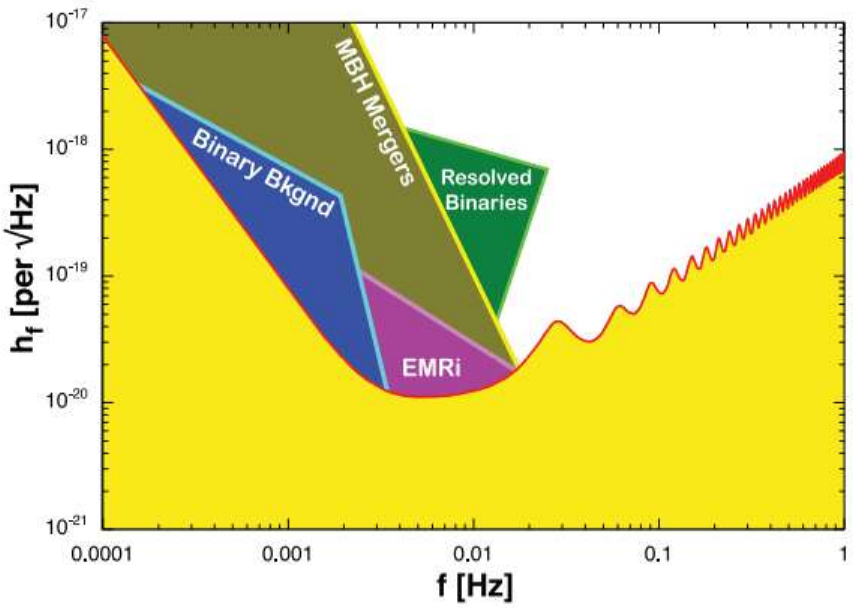
• If you are talking about a short-lived ("bursting") source with a characteristic width τ , then to a good approximation the bandwidth of the source in frequency space is $\Delta f \sim \tau^{-1}$ and the spectral amplitude and strain are related by

$$h_f = \frac{h}{\sqrt{\Delta f}} = h\sqrt{\tau}$$

• The fundamental metric for detection is the SNR ρ (signal to noise ratio) defined as

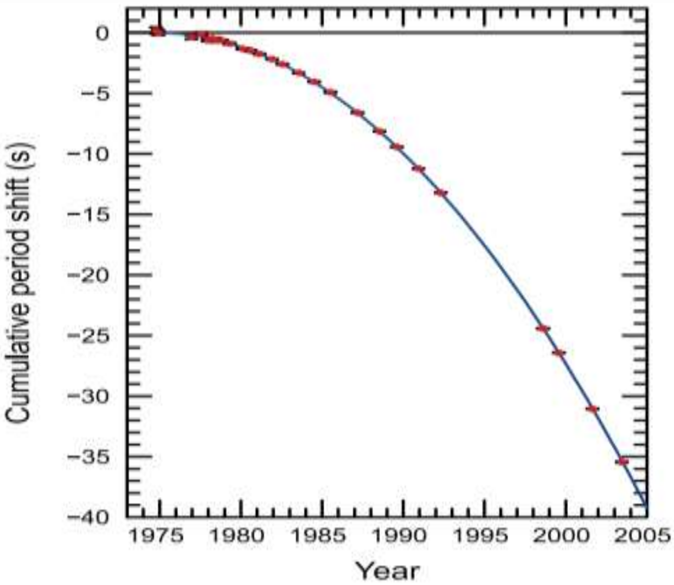
$$\rho \sim \frac{h_f^{src}}{h_f^D}$$

• To use this you need to know how to compute $h^{src}{}_f$. A good starting point is the pocket formulae from the last section.



Application: Binary Pulsar

•Early on we became confident in the existence of gravitational waves because we could observe their astrophysical influence. The first case of this was the pulsar, PSR B1913 + 16, my colloquially known as "The Binary Pulsar," or the "Hulse-Taylor Binary Pulsar," after the two radio astronomers who discovered it in 1974. bullet The Binary Pulsar is famous because it is slowly spiraling together. As shown in the figure below, the rate at which the binary is losing energy from its orbit is precisely what is expected from general relativity! This is the strongest, indirect observational evidence for the existence of gravitational waves. Joe Taylor and Russell Hulse received the Nobel Prize for this discovery in 1993.



• Let's use our formulae for inspiralling binaries to examine the binary pulsar in detail. The physical parameters of this system are given in the table below.

Symbol	*Name*	*Value*
m_1	primary mass	$1.441 M_\odot$
m_2	secondary mass	$1.387 M_\odot$
P_{orb}	orbital period	7.751939106 hr
a	semi-major axis	1.9501×10^9 m
e	eccentricity	0.617131
D	distance	21,000 lyr

• If one computes the yearly change in semi-major axis, one finds

$$\left\langle \frac{da}{dt} \right\rangle = 3.5259 \frac{m}{yr}$$

which is precisely the measured value from radio astronomy observations!

•Because gravitational waves are slowly bleeding energy and angular momentum out of the system, the two neutron stars will one day come into contact, and coalesce into a single, compact remnant. The time for that to happen is

$$\tau_{merge} = 3.02 \times 10^8 \ yr$$

• This is well outside the lifetime of the average astronomer, and longer than the entire history of observational astronomy on the planet Earth! It is, however, much shorter than a Hubble time! This suggests the since (a) there are many binary systems in the galaxy, and (b) neutron stars are not an uncommon end state for massive stars to evolve to, then there should be many binary neutron stars coalescing in the Universe as a function of time.

•This is the first inkling we have that there could be many such sources in the sky, and that perhaps observing them in gravitational waves could be a useful observational exercise.

•If we are going to contemplate observing then, we should have some inkling of their strength. What is the scaling amplitude, h_o of the Hulse-Taylor binary pulsar?

$$h_o = 4.5 \times 10^{-23}$$

This number is extremely small, but we haven't talked about whether it is detectable or not.

Conclusion

In this paper gravitational wave as Detector Sensitivity with its application has been focussed. These primordial gravitational waves will be, in a sense, another source of noise for our detectors and so they will have to be much stronger than any other internal detector noise in order to be detected. Otherwise, confidence in detecting such primordial gravitational waves could be gained by using a system of two detectors and cross-correlating their outputs. The two LIGO detectors are well placed for such a correlation.

References

[1] Fundamentals of Interferometric Gravitational Wave Detectors, P.R. Saulson, World Scientific (1994).
[2] The Detection of Gravitational Waves, D.G. Blair, Cambridge University Press (1991). [3] Relativistic Gravitation and Gravitational Radiation, Cambridge University Press, Editors J-A. Marck and J-P. Lasota (1997).

[4] Gravitational Radiation, K.S. Thorne, in S.W. Hawking and W. Israel (eds), 300 Years of Gravitation, (Cambridge University Press, Cambridge).
[5] Gravitational Wave Physics, K.D. Kokkotas, Encyclopedia of Physical Science and Technology, 3rd Edition, Volume 7 Academic Press, (2002)
[6] An Overview of Gravitational-Wave Sources C. Cutler and K.P. Thorne gr-qc/0204090 April 30th 2002.
[7] mini-GRAIL http://www.minigrail.nl
[8] LIGO http://www.ligo.caltech.edu
[9] VIRGO http://www.pi.infn.it/virgo/virgoHome.html
[10] GEO http://www.geo600.uni-hannover.de
[11] B.C. Barish, "LIGO: Status and Prospects" Proc. of Conference on Gravitational Wave Detection, Tokyo, Eds. K. Tsubono, M.K. Fujimoto, K. Kuroda, Universal Academy Press Inc, Tokyo (1997) 163-173
[12] J.-Y. Vinet et al, Proc. of Gravitation and Cosmology, ICGC-95 Conference, Pune, India 13- 19 Dec 1995 Astrophysics and Space Science Library, Kluwer Academic Publishers 211 (1997) 89-93
[13] J. Hough et al "GEO 600: Current status and some aspects of the design" Proc. of Conference on Gravitational Wave Detection, Tokyo, Eds. K. Tsubono, M.K. Fujimoto, K. Kuroda, Universal Academy Press Inc, Tokyo (1997) 175-182
[14] K. Tsubono and the TAMA collaboration, "TAMA project" Proc. of Conference on Gravitational Wave Detection, Tokyo, Eds. K. Tsubono, M.K. Fujimoto, K. Kuroda, Universal Academy Press Inc, Tokyo (1997) 183-191
[15] C.M. Will, "Theory and experiment in gravitational physics" Revised edition, Cambridge University Press, Cambridge (1983)
[16] R.A. Hulse Rev. Mod. Phys. 66 (1994) 699