Norm Equilibrium and its use in Game theory

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Abstract:- In this paper we have studied the Norm equilibrium and its use inGame theory. Grauer, L.V. and Petrosian, L.A. strong Nash equilibrium in multistage Games - International Game theory Raview[1].

The theoretical foundations of the solution concept include the assumption that the game to be played is common knowledge. The concept of Norm equilibrium for random matchinggames is introduced in this paper. Also some examples about the concept of Norm equilibrium is illustrate and to point out its interesting aspects and some weaknesses.

Key word:- Norm Equilibrium, Nash equilibrium, social norm, social standard of behavior, transition probability, status levels, status distribution,and Game theory.

1. Introduction :-

Norm Equilibrium and its use in a Game:

The pure strategy for a player of type i is a mapping $s_i : X \longrightarrow A_i$ specifying a choice of action $s_i (x, z) \in A_i$ in each stage game when players withstatus levels $x \in X_i$ and $z \in X_j (i \neq j)$ are matched .

Set of all pure strategies for a player of type i is denoted by S_i. Wecall a pair of strategies (s_1, s_2) prescribed to all payers of types 1 and 2 respectively a social standard of behavior (SSB) and denoted it by $\sigma = (\sigma_1 ,$

σ_2).

The social standard of behavior prescribes that all players in the same situation do the same thing. A pair $\beta = (r, \sigma)$ will be referred to as a socialnorm.

Easily we can extend the definition of a social standard of behaviourto allow for the prescription of a random action for some matchings. To avoid the additional notational complexity , we allow the SSB to assign random actions for some matchings of status pairs that can arise only out of equilibrium.

In each period and in each matching, only the following be common knowledge:

(i) The status of the players,

(ii) The game Γ,

(iii) The transition function, and

(iv) The social standard of behaviour of the players. Thus, the history of the plays a player has chosen in that past and the current status distribution may not be known precisely. They become known to eachplayer only to the extent they are reflected in the status levels of the matched players.

To derive expressions for the future value of each status level we define the characteristic function (for given r and σ) $\xi_i : X_i \times X \longrightarrow \{0, 1\}$, as follows:

$$\xi_i(y, x, z) = \begin{cases} 1 & \text{if } y = r_i(x, z, \sigma_i(x, z)) \\ 0 & \text{otherwise.} \end{cases}$$

If a player i chooses σ_i , the transition probability from the status level x to the status level y is defined as a function of the opposite status distribution, p_j as:

$$q^i_{xy}(r, \sigma_i, p_j) = \sum_{z \in X_j} p_j(z)\, \xi_i(y, x, z) \quad (i \neq j),$$

Let $Q_i(r, \sigma_i, p_j)$ be the $K_i \times K_i$ matrix, an element of which is denoted by

$q^i_{xy}(r, \sigma_i, p_j)$, and

we assume that

$$Q(r, \sigma, p) = (Q_1(r, \sigma_1, p_2), Q_2(r, \sigma_2, p_1)).$$

It is given that $\beta = (r, \sigma)$ and if all the players follow a SSB σ, the transition probability $Q_i(r, \sigma, p) = Q_i(\beta, p)$ unambiguously characterizes the future statusdistribution for each of the two player sets.

Also if the current status distribution is p, then the distribution of status levels kperiods, $p^{(k)}(\beta, p)$, can be defined as:

2. Definition:-

$$p_i^{(1)}(\beta, p) = p_i Q_i(r, \sigma_i, p_j)$$

And $$p_i^{(k)}(\beta, p) = p_i^{(k-1)} Q_i(r, \sigma_i, p_j^{(k-1)})$$

Let us denote by $p^{(k)}(x, s_i; \beta, p)$ the status distribution in the k – tv period if theSSB is σ, the current status distribution is p, but a player i with status level $x \in X_i$ changes his strategy to $s_i \in S_i$.

Each player set is a continuum and each player is of measure zero, the distribution function will not depend on the actions of any individual playercan be written as $p^{(k)}(\beta, p)$.

The player sets were finite, however, an individual player's deviation from the social standard of behaviour would alter the probabilitydistribution of status levels in future periods.

A social norm β is stationary at a status distribution pIf $p^{(k)}(\beta, p) = p$ for all k = 1, 2, ….

Clearly, if this holds for k = 1, it must also hold for higher k. we generally restrict attention to stationary social norms since this limits substantially theinformation required to follow the social norm.

Given (β, p), if a player of type i with status level x chooses a strategy $s_i \in S_i$,his expected payoff in each period is defined by,

$$\Pi_i(x, s_i\,; \beta, p) = \sum_{z\in X_j} p_j(z)\pi_i(s_i(x, z), \sigma_j(z, x)).$$

If a player of type i chooses s_i that is the SSB itself, i.e., $s_i = \sigma_i$, then his immediate expected payoff is denoted by $\Pi_i(x, \sigma_i\,; \beta, p)$.

Let us suppose that the social norm $\beta = (r, \sigma)$ is stationary at the status distribution p. Then for each i = 1, 2, there is a well-defined associated presentdiscounted payoff for each status x and for each Markov strategy s_i.

These payoffs are defined by simultaneously solving for all $x\in X_i$:

$$v_i^\infty(x, s_i; \beta, p) = \Pi_i(x, s_i\,; \beta, p) + \delta \sum_{z\in X_j} p_j(z)v_i^\infty(r_i(x, z, s_i(x, z)), s_i; \beta, p)$$

If s_i is the SSB itself, i.e. , when $s_i = \sigma_i$, then his present discounted payoff isdenoted by:

$$v_i^\infty(x, \sigma_i; \beta, p) = \Pi_i(x, \sigma_i\,; \beta, p) + \delta \sum_{z\in X_j} p_j(z)v_i^\infty(r_i(x, z, \sigma_i(x, z)),$$

2.1. Definition:-

A triplet $(\beta^*, p^*) = (r^*, \sigma^*, p^*)$ is called a norm equilibrium of $\Gamma^\infty(\delta)$ if the following holds:

(a) β^* is stationary at p^*, and
(b) For all i = 1, 2, $x \in X_i$ and $s_i \in S_i$,

$$v_i^\infty(x, \sigma_i^*\,; \beta^*, p^*) \geq v_i^\infty(x, s_i\,; \beta^*, p^*)$$

Some examples about the concept of norm equilibrium will be illustrated now and to point out its interesting aspects and some weaknesses. With the help of such examples we will show that any individually rational utility level in a two person stage game can be achieved as norm equilibrium for a social norm utilizing only two status levels.

σ_i;, p)

$$= \Pi_i(x, \sigma_i ; \beta, p) + \delta \sum_{y \in X_j} Q_{xy}^i(r, \sigma, p)\, v_i^{\infty}(y, \sigma_I; \beta, p)$$

Let us suppose that there be only two status levels for each type. i.e.let $K_1 = K_2 = 2$. Let the set of statuses by X = {G, B}. Also we assume the following stage game Γ_1 with M > 0:

	C	D	P
C	4,4	0,5	-1, -100
D	5,0	1,1	0, -M
P	-100, -1	-M, 0	-100, -100

Where C = cooperate actions.

D = defect actions.

P = Punishment action.

2.2. Examples:-

2.2.1. We consider the social norm (β, p) defined as follows.

$$r_i(x, z, \alpha) = \begin{cases} G & \text{if } (x, z, \alpha) = (G, G, C) \text{ or } (G, B, D), \\ B & \text{otherwise.} \end{cases}$$

$$\sigma_i(x, z) = \begin{cases} C & \text{if } x = z = G \\ D & \text{otherwise} \end{cases}$$

$P_i(G) = 1$ and $p_i(B) = 0$

The social norm prescribes that a player should choose C if both he and his opponent are good, i.e., status G, and should defect (choose D) if either is bad(status B).

A player's status is revised according to r. A player with status G remains a G remains a G so long as he follows the prescribed social standard ofbehaviour but changes to bad, B, if he deviates from it. The status level B is "absorbing" in the sense that a B remains a B regardless of his action.

The present discounted payoff for a player of type i with status x along the equilibrium path, v_i^∞ (x), is :

For social norm, $v_i^\infty(G) = 4/(1 - \delta)$and

$v_i^\infty(B) = 1/(1 - \delta)$

In view of previous result, the triplet (β, p) is a norm equilibrium if $\delta \geq ¼$. Thisis exactly the condition necessary to make the prescribed behaviour a perfect equilibrium for the fixed player repeated game with this stage game.

The probability distribution over status levels is degenerate is not important. If for i = 1, 2, $p_i(B) \leq r$ and $p_i(G) \geq 1 - r$, $r > 0$ then similar calculations reveal that (β, p) would be a norm equilibrium for $\delta \geq 1/ [4 - 3r]$.

Hence, the presence of a small proportion of status B people in thesociety increases the threshold discount factor which is consistent with this norm being equilibrium, but does so continuously.

We observation first that while the outcome is the same as at a conventional Nash equilibrium, Playing the equilibrium strategy in a conventional Nash equilibrium may required a vast amount of information that is summarized by the status level in the norm equilibrium. To see this suppose itis possible to obtain records of the past plays of a given player, say player 1.
Suppose further player 1 has played D sometime in the past, say in period t. It does not necessarily imply that he has deviated from the equilibrium path sincethis is the prescribed behaviour against some opponents.

Now, we must check whether or not the player matched against player 1 in period t, player 2 had played D before. If player 2 had, we must check the history of the player who was matched with player 2 when he playedD, and so on.

The second observation is that equilibrium associated with the socialnorm in example 1 is not optimal, in the sense that for some parameter values itwill not support cooperation, while for other social norms cooperation might result.

2.2.II. We consider the following social norm and status distribution (β, p) = (r, σ, p) with the same stage game Γ_1.

r_I (x, z, α) =

σ_I (x, z) =

G if (x, z, α) = (G, G, C) or (G, B, P)

B otherwise.

C if (x, z) = (G, G),

P if (x, z) = (G, B),

D otherwise.

P_i (G) = 1 and p_i(B) = 0

This social norm differs from that in the previous example. It prescribes that agood player should "punish" his opponent if the opponent is bad (status B) byplaying action P.

A player with status G retains that status so long as he follows thesocial standard of behaviour and reverts to B if he deviates from it. The statuslevel B is absorbing, as before.

The corresponding discounted payoffs are:

v_i^∞ (G) = 4/(1 – δ) and v_i^∞ (B) = 0

In light of previous result three inequalities must be satisfied for this to be a norm equilibrium, associated with matchings of a G with another G, a G with aB, and a B with a G.

The inequality associated with the last match, a B meeting a G, is vacuously satisfied the SSB prescribes that the B play a one shot best responsein this case. The other two inequalities are respectively the following:

$1 \leq \delta(4/(1 - \delta) - 0)$ and

$M + 1 \leq \delta\ (4/(1 - \delta) - 0)$

Easily it can be checked that these constraints are satisfied if

$\delta \geq \max \{(1 + M)/)(5 + M), 1/5\}$.

Thus, there are pairs of M and δ for which this social norm can support cooperation, (C, C), while the social norm of example 1 cannot.

In previous example, we first note that in the social norm we have

$r_i(G, B, P) = G$,

But $r_i(G, G, P) = r_i(B, B, P) = B$

Also we note in this example that, as in the previous example, while the probability distribution over status levels is degenerate the social norm would still have been part of an equilibrium with a positive proportion of people having status B. As in that example, the minimum δ that was consistent with equilibrium would increase as the proportion of people with status B increased.

Thirdly we wish to make in this example is that it is easier to supportcooperation with random matching than if there were a fixed match.
Cooperation is sustained through the threat of punishments to a player howdeviates from cooperative behaviour.

Random matching allows severe punishments to be shared by manypeople – the different partners that will be matched with the player to be punished which lowers the cost per opponent of carrying out the punishment, which allows more severe punishments than might otherwise be possible.

In previous example we also observe that when a player deviates from the SSB, he will become status B. The SSB prescribes that in this case, heis to be punished forever.

If M > 1, this is not a best response to D, hence there is a cost to the

G. For a fixed matching, this cost would be borne by the same status G player each period, while in the random matching case, a given G player will not bematched each period with given status B player, and hence will not incur thecost of punishing every period.

As a result, the constraint associated with a G matched with a B will be more easily satisfied in the random matching case than in the fixed matchingcase.

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