The Coreflective Hull of Fuzzy Sierpinski Space in C-FTS

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Abstract

In this paper, we determine the coreflective hull of the fuzzy Sierpinski space in the category of constant-generated fuzzy topological spaces.

1. Introduction

It is well-known that the subcategory **FTS$_0$** of all **T$_0$** fuzzy topological spaces of **FTS** of all fuzzy topological spaces is the epireflective hull of the fuzzy Sierpinski space I_s (cf.[9]) in **FTS.** Then the question arises – what is the coreflective hull of the fuzzy Sierpinski space the category **FTS.** We know that the subcategory of Meet-complete topological spaces is the coreflective hull of the two-point Sierpinski space in the category **TOP** of all topological spaces (cf.[7]). It has been shown in [11] that the analogous result in **FTS** is not true, i.e., the subcategory of meet-complete fuzzy topological spaces (spaces in which topologies are closed under arbitrary meets) is not the coreflective hull of I_s in the category **FTS.** In this paper, it is shown that like **FTS**, in the category **C-FTS** of constant-generated fuzzy topological spaces, the coreflective hull of the fuzzy Sierpinski space is not the category of meet-complete fuzzy topological spaces.

Keywords: *Coreflective subcategory; Coreflective hull, Constant-generated fuzzy topological spaces, fuzzy Sierpinski space.*

2. Preliminaries

For fuzzy topological concepts, we refer [2] but recall a few here, for convenience. Throughout, let I denote the interval [0, 1].

Let X be a non-empty set. A fuzzy set in X is a function from X to I=[0,1]. If $t \in I$, then $\underline{t}$ denotes the *constant fuzzy set* in X, which takes value t everywhere. In particular, $\underline{0}$ and $\underline{1}$ denote the constant fuzzy sets taking values 0 and 1 respectively.

Definition: *Given a set X, $x \in X$ and $t \in (0,1]$, x_t denotes the fuzzy set in X which takes value t at x and value 0 elsewhere. x_t is often called a* fuzzy point (*with* value t *and* support x).

The *complement* of μ is the fuzzy set $1 - \mu$, defined as $(1 - \mu)(x) = 1 - \mu(x), \forall x \in X$.

Definition(Chang [2]): A collection δ of fuzzy sets in X with $\underline{0}$ and $\underline{1}$, which is closed under finite meets and arbitrary joins is called a **fuzzy topology** on X and the pair (X,δ) a **fuzzy topological space**.

Definition (Lowen [8]): *Let X be a non-empty set. A subset* δ *of* I^X *which is closed under arbitrary joins and finite meets and which contains all constant fuzzy sets is called a* fuzzy topology *on X.*

The members of δ are called open (or δ -open) fuzzy sets in X and their complements are called closed fuzzy sets in X. The smallest (resp. the largest) fuzzy topology on X is called the *indiscrete* (resp. *discrete*) fuzzy topology on X.

Definition: A mapping $f : (X, \delta) \to (X', \delta')$ between fuzzy topological spaces is called ***fuzzy continuous*** if $f^{\leftarrow}(\mu) \in \delta, \forall \mu \in \delta'$(where $f^{\leftarrow}(\mu) = \mu \circ f$).

Let (X, δ) be a fuzzy topological space, Y *a* set and f: $X \longrightarrow Y$ a surjective mapping. Then

$$\delta/f = \{\alpha \in I^Y\colon\ f^{\leftarrow}(\alpha) \in \delta\ \}.$$

is clearly a fuzzy topology on Y, called the **quotient fuzzy topology** on Y with respect to f, while $(Y, \delta/f)$ is then called the *quotient space* of (X, δ) with respect to f. The resulting continuous mapping $f\colon (X, \delta) \to (Y, \delta/f)$ is called a **quotient space**.

All category-theoretic notions and results used here, but not defined or explained, are fairly standard by now (and can be found in [1]). However, for convenience, we recall some of the categorical notions used in the sequel (subcategories are always assumed to be full and isomorphism-closed).

FTS shall denote the category of fuzzy topological spaces in Chang's sense and continuous functions and $\mathbf{FTS_0}$ denote the category of T_0-fuzzy topological spaces. **C-FTS** will denote the category of fuzzy topological spaces in Lowen's sense and continuous functions and $\mathbf{C\text{-}FTS_0}$ denotes the full subcategory of **C-FTS** containing all T_0-fuzzy topological spaces. Of course, **TOP** is just the category of usual topological spaces and continuous maps.

Definition([9]): *A fuzzy topological space* (X, δ) *is said to be* T_0 *if for all distinct* $x, y \in X$, $\exists \mu \in \delta$ *such that* $\mu(x) \neq \mu(y)$.

Definition: *The fuzzy topology on I, generated by {id}, where id is the identity function, both in the sense of Chang and in the sense ofLowen, will be called the* fuzzy Sierpinski topology. *The resulting fuzzy topologies will be denoted by* δ_S *and* C-δ_S *respectively.* (I, δ_S) *and* $(I$, C-$\delta_S)$ *will be denoted by* I_S (cf. [10]) *and* C-I_S (cf. [9]) *respectively* (*hence it is clear that an arbitrary member of* C-I_S *is going to be of the form* $(\underline{t} \wedge id) \vee \underline{r}$ *with* $t, r \in I$; cf. [9]).

Definition: *A subcategory* **U** *of a category* **C** *is said to be* coreflective *in* **C** *if for each object X in* **C**, $\exists$ *an object* X_U *in* **U** *and an* **X**-*morphism.*

The notions of reflective and coreflective subcategories are very important in category theory and several such interesting and useful subcategories have been studied by many authors. In particular, Herrlich and Strecker (([4], [5] and [6]) have extensively investigated such subcategories in the categories which occur in topology (e.g., the categories of topological spaces, uniform spaces, etc.).

We begin with a preliminary examination of the coreflective subcategories of **C-FTS**. We find that the characterization of coreflective subcategories of **C-FTS** is similar to that of the coreflective subcategories of **TOP** ([6]).

We recall some of the definitions used in the sequel.

Definition: *An epimorphism* $e: A \to B$ *in a category* **C** *is called an* extremal epimorphism *if whenever* $e = m \circ f$ *in* **C**, *where* m *is a* **C**- *monomorphism, then* m *is an* **C**-*isomorphism.* B *is called an* extremal quotient *of* A *if there exists an extremal epimorphism from* A *to* B.

We know that in **TOP**, extremal epimorphisms are precisely the quotient maps; hence the extremal quotients in **TOP** are the quotients in **TOP**. As both **C-FTS** and **FTS** are topological over **SET**, extremal epimorphisms are precisely the quotient maps in both **C-FTS** and **FTS** (cf. [5]) and so extremal quotients are the quotients in both **C-FTS** and **FTS**.

Definition: *A category* **C** *is said to have the* unique extremal epi- mono factorization property *if* (i) *each* **C**-*morphism* f *admits an extremal epi-mono factorization in* **C**, *say* $f = g \circ h$, *where* h *is an extremal epimor- phism and* g *is a monomorphism and* (ii) *if* $f = g' \circ h'$ *is another extremal epi-mono factorization of* f *in* **C**, *then there exists a* **C**-*isomorphism* u *such that* $u \circ h' = h$ *and* $g \circ u = g'$

If in addition, **C** *is also closed under the composition of extremal epimor- phisms, then* **C** *said to admit the* strong unique extremal epi-mono factor- ization property.

Theorem 2.1. ([5]) *If* **C** *is a well-powered category, which has products and the epi-mono factorization property, then* **C** *has the unique extremal epi-mono factorization property.*

Definition: *A morphism* $f : X \to Y$ *in a category* **C** *is called* constant *if for each* **C**-*object* Z *and each pair of* **C**-*morphisms* $g, h : Z \to X$, $f \circ g = f \circ h$.

It is known that the constant morphisms in **TOP** are precisely the constant maps (cf. [6]). We note that in **C-FTS**, like **TOP**, there is exactly one fuzzy topology on a single-point set. As a consequence of this, in **C- FTS** also, the constant maps are continuous. But, in contrast to **TOP** and **C-FTS**, in **FTS**, there can be many fuzzy topologies on a single-point set and hence constant maps need not be continuous in **FTS**.

Definition: *A category* **C** *is said to be* constant-generated *if for each pair* (X, Y) *of* **C**-*objects:* (i) $\mathbf{C}(X, Y) \neq \emptyset$ *and* (ii) *for every distinct pair* $f, g : X \to Y$ *of* **C**-*morphisms, there exists a* **C**-*object* Z *and a constant* **C**-*morphism* $k : Z \to X$ *such that* $g \circ k \neq f \circ k$.

TOP is well-known to be constant-generated (cf. [6]). Like **TOP**, **C- FTS** is also constant-generated; the main reason being the continuity of constant maps in both the categories. We observe that for some pair (X, Y) of **FTS**-objects, we may have $\mathbf{FTS}(X, Y) = \emptyset$; in particular, if (X, δ) is an indiscrete fuzzy space and (Y, Δ) is a discrete fuzzy space in **FTS**, then there does not exist any continuous map from X to Y. So **FTS** is not constant- generated.

We now state the following results from [5] which will be used in the sequel.

Theorem 2.2 ([5]) *Let* **C** *be a well-powered category, which has coproducts and the extremal epi-mono factorization property. Then a subcategory* **U** *of* **C** *is monocoreflective in* **C** *if and only if* **U** *is closed under the formation of coproducts and extremal quotients.*

Theorem 2.3. ([5]) *If* **U** *is a coreflective subcategory of a constant- generated category* **C**, *then* **U** *is both monocoreflective and epicoreflective in* **C**.

Theorems 2.2 and 2.3 can be combined to yield:

Theorem 2.4. *Let* **C** *be a well-powered category, which has coproducts and the extremal epi-mono factorization property. If* **C** *is also constant- generated, then a subcategory* **U** *of* **C** *is coreflective in* **C** *if and only if* **U** *is closed under the formation of coproducts and extremal quotients.*

Theorem 2.5. ([5]) *If* **C** *is a well-powered category, which has coproducts and the extremal epi-mono factorization property, then the monocoreflective hull in* **C** *of a class* **A** *of* **C**-*objects exists. Furthermore, if* **C** *has the strong unique extremal epi-mono factorization property, then the objects of this monocoreflective hull of* **A** *in* **C** *are exactly all the extremal quotients of coproducts of objects in* **A**.

In view of Theorems 2.3 and 2.5, we have the following:

Theorem 2.6. *If* **C** *is a well-powered and constant-generated category, which has coproducts and strong unique extremal epi-mono factorization property, then the coreflective hull in* **C** *of a class* **A** *of* **C**-*objects exists and the objects of this coreflective hull of* **A** *in* **C** *are exactly all the extremal quotients of coproducts of objects in* **A**.

3. The coreflective subcategories of C-FTS

As **SET** is well-powered and has epi-mono factorization property and as both **C-FTS** and **FTS** are topological over **SET** (cf. [5], [6] resp.), so both **C-FTS** and **FTS** are well-powered and have epi-mono factorization property. Both **C-FTS** and **FTS** have products, so by Theorem 2.1, both the categories have unique extremal epi-mono factorization property. As the composition of quotient maps (extremal epimorphisms) in both **C-FTS** and **FTS** is a quotient map, so both have strong unique epi-mono factorization property. Also, **C-FTS** is constant-generated, while **FTS** is not constant-generated. So Theorem 2.5 is applicable to **C-FTS**, as a consequence of which, we are in a position to characterize coreflective subcategories and the objects of the coreflective hull of any class of **C-FTS**-objects. Thus, we are led to the following:

Theorem 3.1([12])

(*i*) *A subcategory* **U** *of* **C-FTS** *is coreflective iff it is closed under the formation of coproducts and quotients.*

(ii) *In* **C-FTS**, *the coreflective hull of any* $A \in ob$**C-FTS** *always exists. Moreover, its objects are precisely the quotients of the coproducts of copies of* A**.**

3.1.The coproducts of copies of a fuzzy topological space in C-FTS

We proceed to give an internal description of the coproducts in **C-FTS** of copies of any fuzzy topological space, which we shall then use for our main results. Let $(X,\delta) \in ob$**C-FTS** and J be some index set. Put $X_j = X \times \{j\}, j \in J$, and denote $\bigcup_{j\in J} X_j$ by $\boldsymbol{X_j}$. For each $\mu \in \delta$, define $\mu_j : \boldsymbol{X_j} \longrightarrow \boldsymbol{I}$ as $\mu_j(x, j) = \mu(x)$ and put $\delta_j = \{\mu_j \mid \mu \in \delta\}, j \in J\}$. Then δ_j is a fuzzy topology on X_j (and (X_j, δ_j) is homeomorphic to (X, δ)). Let $\delta^+ = \{\nu \in I^{X_j} \mid \nu|_{X_j} \in \delta_j, \forall \boldsymbol{j} \in J\}$. It can be verified that ($\boldsymbol{X_j}, \delta$) is the coproduct of $|J|$ copies of (X, δ) in **C-FTS**.

Let $[X]_{\mathbf{C\text{-}FTS}}$ denote the coreflective hull in **C-FTS** of a **C-FTS**-object X.

Proposition 3.1. *Let* $X = (X, \delta)$ *be* **C-FTS**-*object. Then* $Y = (Y, \Delta)$ *is an object of* $[X]_{\mathbf{C\text{-}FTS}}$ *iff* $\exists$ *a family* $\{(Y_j, \Delta_j) \mid j \in J\}$ *of fuzzy subspaces of* Y *such that* $Y = \bigcup_{j\in J} Y_j$, *each* Y_j *is a quotient of* (X, δ), $\boldsymbol{j} \in J$, *and for each* $\mu \in I^Y$, μ *is open in* Y *iff each* $\mu|_Y$ *is open in* $Y_j, j \in J$.

4. The coreflective hull of the fuzzy Sierpinski space C-I_S

Consider the two-point Sierpinski topological space 2_s. The following result gives the descriptions of the epireflective and the coreflective hulls of 2_s in **TOP**.

Proposition 4.1. ([9]) *The category* **C-FTS**$_0$ *of all* T_0*-fuzzy spaces is the epireflective hull of* C-I_S *in the category* **C-FTS**.

We show that the subcategory of those fuzzy topological spaces (X,δ) for which δ is closed under arbitrary meets, does not form the coreflective hull of C-I_S in the category **C-FTS**. We then also describe what this coreflective hull is.

Let the fuzzy topological counterpart of meet-complete topological spaces be called meet-complete fuzzy topological spaces, i.e., fuzzy spaces whose fuzzy topologies are closed under arbitrary meets. Let **MC-C-FTS** denote the category of all such spaces in **C-FTS**. We now show that **MC-C-FTS** is coreflective in **C-FTS**.

Theorem 4.1.([11]). MC-C-FTS *is a coreflective subcategory of* **C-FTS**.

Although **MC-C-FTS** is a coreflective subcategory of **C-FTS**, contrary to what one might expect, we now show by means of the following example that it is *not* the coreflective hull of C-I_S in **C-FTS**.

Example: *Consider the fuzzy topological space* C-I_D = (I,C-δ_D), *where* C-δ_D =< {id, $1 - id$} >. *Then* (I,C-δ_D) $\in$ *ob***MC-C-FTS**. *However,* (I,C-δ_D) *cannot be a quotient of a coproduct of copies of* C-I_S*'s. To see this, consider the possibility of the existence of a quotient map* q : ((C-I_S), δ^+) $\rightarrow$ (I,C-δ_D). *Then as* $id \in$ C-δ_D, $q^{\leftarrow}(id) = id \circ q = q$ *must be in* δ^+*,i.e.,* $q|_{\text{C}-I_{S_{j'}}} = id_{j'}$ *for some* $j' \in J$ *(since* q *is surjective). As* $1 - id \in$C-δ_D, $q^{\leftarrow}(1 - id)|_{\text{C}-I_{S_{j'}}} = (1 - q)|_{\text{C}-I_{S_{j'}}} = 1 - id_{j'} \in \delta_{S_{j'}}$ *for* $j' \in J$*, which is clearly false.*

Thus (I,C-δ_D) *cannot be a quotient of a coproduct of copies of* C-I_S*'s and so* C-I_D *is not an object of the coreflective hull of* C-I_S.

From the above, it is clear that finding all quotients of C-I_S will help us to determine [C-I_S]$_{\text{C-FTS}}$. This is done through the following result. Before proceeding, we note that any open fuzzy set μ in C-I_S is of the form $(\underline{t} \wedge id) \vee \underline{r}$, $t, s \in I$, i.e., $\mu|_{[s,t]} : [s, t] \rightarrow [s, t]$ is a bijection, $\mu|_{[0,s)} = \underline{s}$ and $\mu|_{(t,1]} = \underline{t}$.

Proposition 4.2. *A fuzzy topological space* (X, δ) *is a quotient of* C-I_S *iff* $|X| \leq |I|$ *and* δ =< α >, *where for some partition* {X_1, X_2, X_3} *of* X *and for some* $s, t \in I$, $\alpha|_{X_1} : X_1 \rightarrow [s, t]$ *is a bijection,* $\alpha|_{X_2} = \underline{s}$, $\alpha|_{X_3} = \underline{t}$.

Proof: Let (X, δ) be a quotient of C-I_S. Then there exists some quotient map $q : (I,\text{C-}\delta_S) \rightarrow (X, \delta)$ in **C-FTS**. As q is surjective, $|X| \leq |I|$.

Let $\mu \in \delta$. We note that:

(i) If $q^{\leftarrow}(\mu) = \underline{t}$, then $\mu = \underline{t}$, $t \in I$.

(ii) If $q^{\leftarrow}(\mu) = id$, then q is a bijection (as $|X| \leq |I|$) and $\mu = q^{-1}$.

(iii) If $q^{\leftarrow}(\mu) = id \wedge \underline{t}$, for some $t \in (0, 1)$, then $\mu \circ q|_{[0,t]} = id|_{[0,t]}$, whereby $q|_{[0,t]}$ is injective and so $\mu|_{q([0,t])} : q([0, t]) \rightarrow [0,t]$ is bijective such that $\mu|_{q((t,1])} = \underline{t}$.

(iv) If $q^{\leftarrow}(\mu) = id \vee \underline{t}$ for some $t \in (0, 1)$, then $\mu \circ q|_{[t,1]} = id|_{[t,1]}$, whereby $q|_{[t,1]}$ is injective and $\mu|_{q([t,1])}$ is injective such that $\mu(q([t, 1])) = [t, 1]$ and $\mu|_{q([0,t))} = \underline{t}$.

(v) If $q^{\leftarrow}(\mu) = (\underline{t} \wedge id) \vee \underline{s}$, for some $s, t \in (0, 1)$, $s < t$, then $\mu \circ q|_{[s,t]} = id|_{[s,t]}$, whereby $q|_{[s,t]}$ is injective and $\mu|_{q([s,t])}$ is injective such that $\mu(q([s, t])) = [s, t]$ and $\mu|_{q([0,s))} = \underline{s}$, $\mu|_{q((t,1])} = \underline{t}$.

Consider the case when q is bijective. As q is a quotient map and $id \in$ C-I_S, $q^{-1} \in \delta$. Clearly $\underline{t} \in \delta$, $\forall t \in I$. Put $\alpha = q^{-1}$. Then α is bijective. We show that $\delta =< \alpha >$. As for every subbasic open fuzzy set μ in C-I_S, $\exists$ some $\nu \in < \alpha >$ such that $q^{\leftarrow}(\nu) = \mu$ and as $q^{\leftarrow}$ is arbitrary join- and arbitrary meet- preserving, for each $\mu \in$C-δ_S, $\exists \nu \in < \alpha >$ such that $q^{\leftarrow}(\nu) = \mu$. Hence $\delta =< \alpha >$, where α is a bijection on X.

If q is not bijective, then $\nexists$ any $\nu \in \delta$ such that $q^{\leftarrow}(\nu) = id$. So, in view of (iii), (iv), (v) (above mentioned cases), we consider the following cases.

Consider the case when q is not bijective, but for some $t \in (0, 1)$, $q|_{[0,t]}$ is injective. Then by (iii), $\exists \mu \in \delta$ such that $\mu \circ q = \underline{t} \wedge id$, whereby there exists a partition $\{X_1, X_2\}$ of X and $t \in (0, 1)$ such that $\mu|_{X_1} : X_1 \rightarrow [0,t]$ is bijective

and $\mu|_{X_2} = \underline{t}$. Since q is not injective, $\nexists \beta \in \delta$ such that $\beta \circ q = id$. Moreover, if $\beta \in \delta$, then $\beta \circ q = (\underline{v} \wedge id) \vee \underline{u}$ implies that $\beta|_{q([u,v])} : q([u, v]) \rightarrow [u, v]$ is bijective for some $v \leq t$ or β is a constant fuzzy set. Thus $\delta =< \mu >$, where for some partition $\{X_1, X_2\}$ of X, for some $t \in (0, 1)$, $\mu|_{X_1}$ is injective, $\mu(X_1) = [0, t]$ and $\mu|_{X_2} = \underline{t}$.

Consider the case when q is not bijective, but for some $t \in (0, 1)$, $q|_{[t,1]}$ is injective. Then by (iv), $\exists \mu \in \delta$ such that $\mu \circ q = id \vee \underline{t}$, implying that $\mu \circ q|_{[t,1]} = id|_{[t,1]}$, whereby $\exists$ a partition $\{X_1, X_2\}$ of X such that $\mu|_{X_1} : X_1 \longrightarrow [t, 1]$ is bijective and $\mu|_{X_2} = \underline{t}$. Since q is not injective, $\nexists \beta \in \delta$ such that $\beta \circ q = id$. Moreover, if $\beta \in \delta$, then $\beta \circ q = (\underline{v} \wedge id) \vee \underline{u}$, $u, v \in I$ implies that $\beta|_{q([u,v])} : q([u, v]) \rightarrow [u, v]$ is bijective for some $u \geq t$ or β is a constant fuzzy set. Thus $\delta =< \mu >$, where for some partition $\{X_1, X_2\}$ of X and for some $t \in (0, 1)$, $\mu|_{X_1}$ is injective, $\mu(X_1) = [t, 1]$ and $\mu|_{X_2} = \underline{t}$.

Consider the case when q is not bijective, but for some $s, t \in (0, 1)$, $q|_{[s,t]}$ is injective. Then by (v), $\exists \mu \in \delta$ such that $\mu \circ q = (\underline{t} \wedge id) \vee \underline{s}$, implying that $\mu \circ q|_{[s,t]} = id|_{[s,t]}$, whereby $\exists$ some partition $\{X_1, X_2, X_3\}$ of X such that $\mu|_{X_1} : X_1 \rightarrow [s, t]$ is bijective, $\mu|_{X_2} = \underline{s}$ and $\mu|_{X_3} = \underline{t}$. Since q is not bijective, so $\nexists \beta \in \delta$ such that $\beta \circ q = id$. Since for any $t \in (0, 1)$, $q|_{[0,t]}$ is not injective, so $\nexists \beta \in \delta$ such that $\beta \circ q = id \wedge \underline{t}$ for any $t \in (0, 1)$. Moreover, if $\beta \in \delta$, then $\beta \circ q = (\underline{v} \wedge id) \vee \underline{u}$, $u, v \in I$ implies that $\beta|_{q([u,v])} : q([u, v]) \rightarrow [u, v]$ is bijective for some $u \geq s$ and $v \leq t$ or β is a constant fuzzy set. Thus $\delta =< \mu >$, where for some partition $\{X_1, X_2, X_3\}$ of X, $\mu|_{X_1} : X_1 \rightarrow [s, t]$ is bijective, $\mu(X_1) = [s, t]$, $\mu|_{X_2} = \underline{s}$ and $\mu|_{X_3} = \underline{t}$.

Consider now the case when q is not bijective such that for distinct $s, t \in I$, $q|_{[s,t]}$ is not injective. Then $\nexists\ \mu \in \delta$ such that $\mu \circ q = (\underline{t} \wedge id) \vee \underline{s}$, unless $s = t$. Thus δ is indiscrete.

So if (X, δ) is a quotient of C-I_S, then $\delta =< \alpha >$, where for some partition $\{X_1, X_2, X_3\}$ of X and some $s, t \in I$, $\mu|_{X_1}$ is bijective, $\mu(X_1) = [s, t]$, $\mu|_{X_2} = \underline{s}$ and $\mu|_{X_3} = \underline{t}$.

Conversely, let $(X, \delta) \in ob$**C-FTS** such that $|X| \le |I|$ and $\delta =< \alpha >$, where for some partition $\{X_1, X_2, X_3\}$ of X, $\mu|_{X_1} : X_1 \to [s, t]$ is injective, $\mu(X_1) = [s, t]$, $\mu|_{X_2} = \underline{s}$ and $\mu|_{X_3} = \underline{t}$. Let $q : (I, \text{C-}\delta_S) \to (X, \delta)$ be a map such that $q|_{[s,t]}$ is injective, $q([s, t]) = X_1$ and $q([0, s)) = X_2$, $q((t, 1]) = X_3$. As $q^{\leftarrow}$ is arbitrary join- and arbitrary meet- preserving, it is sufficient to show that for any subbasic open fuzzy set μ in X, $q^{\leftarrow}(\mu) \in \text{C-}\delta_S$. Then for $\mu = \alpha$, $q^{\leftarrow}(\mu)|_{[s,t]} = id|_{[s,t]}$ and $q^{\leftarrow}(\mu)|_{[0,s)} = \underline{s}$, $q^{\leftarrow}(\mu)|_{(t,1]} = \underline{t}$ and so $q^{\leftarrow}(\mu) = (\underline{t} \wedge id) \vee \underline{s}$. For $\mu = \underline{t}$, $t \in I$, $q^{\leftarrow}(\mu) = \underline{t}$, $t \in I$. Hence $q^{\leftarrow}(\mu) \in \text{C-}\delta_S$ for each $\mu \in \delta$.

Next, let $q^{\leftarrow}(\mu) \in \text{C-}\delta_S$, for some $\mu \in I^X$. We wish to show that $\mu \in \delta$. As $q^{\leftarrow}(\mu) \in \text{C-}\delta_S$, $q^{\leftarrow}(\mu) = (\underline{v} \wedge id) \vee \underline{u}$, $u, v \in I$, implies that $\mu|_{q([u,v])} : q[u, v] \to [u, v]$ is bijective for some $u \ge s$ and $v \le t$, $\mu|_{q([0,u))} = \underline{u}$ and $\mu|_{q((v,1])} = \underline{v}$, i.e., $\mu = (\underline{v} \wedge \alpha) \vee \underline{u}$, whereby $\mu \in \delta$.

We have thus shown that $\mu \in \delta \Leftrightarrow q^{\leftarrow}(\mu) \in \text{C-}\delta_S$, which in turn shows that (X, δ) is a quotient of $(I, \text{C-}\delta_S)$.

We now characterize $[\text{C-}I_S]_{\textbf{C-FTS}}$, the coreflective hull in **C-FTS** of C-I_S.

Theorem 4.2. *A fuzzy topological space (X, δ) is an object of $[\text{C-}I_S]_{\textbf{C-FTS}}$ iff it satisfies the following two conditions:*

(a) *$X = \bigcup_{j \in J} X_j$, for some index set J such that for each $j \in J$, $|X_j| \le |I|$ and the subspace fuzzy topology δ_j on X_j is $< \alpha >$, where for some partition $\{X_1, X_2, X_3\}$ of X_j and for some $s, t \in I$, $\alpha|_{X_1} : X_1 \to [s, t]$ is a bijection and $\alpha|_{X_2} = \underline{s}$, $\alpha|_{X_3} = \underline{t}$,*

(b) *for each $\mu \in I^X$, $\mu \in \delta$ iff $\mu|_{X_j} \in \delta_j$, for each $j \in J$.*

Proof: The proof directly follows from Propositions 3.1 and 4.2.

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