# STUDY ON ISOTROPIC RATE OF EMISSION OF POWER IN TWO AND THREE LEVELS OF THE SYSTEM

**Dr. Motiur Rahman Ansari**
Assistant Professor, Department of Physics
Maulana Azad College of Engineering and Technology, Neora, Patna.

***Abstract:*** The main aim of this study has considered absorption and emission phenomenon where only two and three levels of the system take part and the dispersion phenomenon is dealt with at infrared frequency. The method has been used to treat the problem of co-related emission and photon echoes. It is relatively easy to make input and output calculations for a ruby laser in a hypothetical steady radiating state. Thus the rate equation approach to the problem provides a quantitive analysis of power emission from the system.

**KEY WORDS: Emission, Infrared frequency, Radiation, Transition.**

## I. INTRODUCTION

We limit our consideration to absorption and emission phenomenon up to two and three levels of the system. For two levels system, a net absorption takes place in thermal equilibrium condition. To have a net emission, the bulk matter is to be excited such that the equilibrium population between the levels is inverted. We note that $\omega_{12}$ is the product of Einstein's co-efficient of absorption of radiation $B_{12}$ and photon density p(υ). As the system radiates, the photon density grows and ultimately attains a steady value. It is therefore interesting to solve the rate equation taking as a function of time, to begin with, and independent of time under the steady-state condition.

## II. THEORY

We are all familiar with the process of spontaneous emission, in which an atom in an excited state $\boldsymbol{E_i}$ can emit quantum radiation of frequency $\boldsymbol{\upsilon_{ij}}$, thereby dropping to a lower energy state $\boldsymbol{E_j}$, according to the relation

$$\boldsymbol{E_i} - \boldsymbol{E_j} = \mathrm{h}\,\boldsymbol{\upsilon_{ij}} \tag{1}$$

where h is the Planck's constant. These jump occur at a rate $\boldsymbol{A_{ij}}$ with a resultant spatially isotropic rate of emission of power $\boldsymbol{n_i}$ $\boldsymbol{A_{ij}}$ h$\boldsymbol{\upsilon_{ij}}$, Where $\boldsymbol{n_i}$is the population of atoms in the excited states. Somewhat less familiar is the concept that these same atoms can be stimulated to emit radiation $\boldsymbol{\upsilon_{ij}}$ by being bathed in the radiation of the same frequency. The physical phenomenon which makes the laser possible is that of this stimulated emission of radiation. The rate of these stimulated jumps is proportional to the energy density u ($\boldsymbol{\upsilon_{ij}}$) of the radiation and to the population difference $\boldsymbol{n_i - n_j}$, between the upper and lower energy states. Furthermore, the stimulated radiation exhibits the same directional and polarization characteristics as that of the stimulating radiation. This is the process that gives rise to the amplification and directional properties of lasers. Einstein showed that in the steady-state, an expression of the form,

$$\mathrm{P}\,(\boldsymbol{\upsilon})\,\boldsymbol{B_{12}N_1} = \mathrm{P}\,(\boldsymbol{\upsilon})\,\boldsymbol{B_{21}N_2} + \mathrm{A}\boldsymbol{N_2} \tag{2}$$

must be true to account for the transitions that take place under the influence of a broadened radiation field where P($\boldsymbol{\upsilon}$) is the energy per unit volume per unit frequency interval A and B are the spontaneous and induced transition rate coefficients ( or Einstein's coefficient). Einstein derived them originally, not based on field quantization, but rather by the use of classical arguments and thermodynamic considerations. The expression (e.g. $\boldsymbol{B_{21}}$=$\boldsymbol{B_{12}}$ , $\boldsymbol{A_{12}}/\boldsymbol{B_{12}} = \frac{\boldsymbol{8\pi h\upsilon^3\ \eta^3}}{\boldsymbol{c^3}}$ ) are found to agree with the results obtained based on field quantization.

The present work can define a transition rate from level 1to level 2 in the presence of a single-mode field as,

$$\boldsymbol{\omega_{12}} = \mathrm{K} < \mathrm{n} > \tag{3}$$

whereas the transition rate from level 2 to level 1 is given by

$$\omega_{21} = \mathrm{K} < \mathrm{n} > + \mathrm{A} \tag{4}$$

As we pass from the single-mode radiation case to the broadband radiation case, the equation for the population difference in terms of the transition rates still holds, but now the transition rates $\boldsymbol{\omega_{12}}$ and $\boldsymbol{\omega_{21}}$ given by (2.3) and (2.4) must be generalized by summation or integration over frequency. If the frequency distribution of the radiation field represented by < n($\boldsymbol{\upsilon}$) > and the mode density P($\boldsymbol{\upsilon}$) are each assumed to be very slowly concerning the line shape factor contained in K($\boldsymbol{\upsilon}$) $\mathrm{K} = \frac{\pi\,\omega}{\pi\,\varepsilon}\ \frac{|\mu_{12}|^2}{3}$ $g_L(\boldsymbol{\omega}.\ )\frac{1}{V}$ , all frequency dependence other than the line shape factor may be removed from the integral. With this assumption, we arrive at,

$$\boldsymbol{\omega}_{12} = \mathrm{P}\,(\boldsymbol{\upsilon})B_{12} \tag{5}$$

$$\boldsymbol{\omega_{21}} = \mathrm{P}\,(\boldsymbol{\upsilon})\,B_{21} + \mathrm{A} \tag{6}$$

where P($\boldsymbol{\upsilon}$) is the energy per unit volume per unit frequency interval defined by

$$\mathrm{P}\,(\boldsymbol{\upsilon}) = \mathrm{P}\,(\boldsymbol{\upsilon})\,\mathrm{h} < \mathrm{n}\,(\boldsymbol{\upsilon}) > \tag{7}$$

and $B_{12} = B_{21} = \mathrm{B} = (\frac{1}{h\upsilon})\,\mathrm{V}\int_0^\infty Kd$

ith A given by $\mathrm{A} = \frac{1}{T_{sp}} = \mathrm{V}\int_0^\infty K(\omega)\,P(\omega)\,dw$

(where $T_{sp}$, $K(\omega)$ and $P(\omega)\,d\boldsymbol{\omega}$ represent spontaneous emission time, single-mode spontaneous emission rate and number of modes per unit volume in a frequency range $d\omega$, respectively) We see that the ratio of A to B is as follows:

$$\frac{A}{B} = h(\upsilon)\, P(\upsilon) = \frac{8\pi\, h\, \upsilon^3\, \eta^3}{c^3} \tag{8}$$

(from the results of mode density relationships). From (7) and (8) we find

$$\frac{A}{BP} = \frac{1}{<n>} \tag{9}$$

where <n> is the expectation value of the number of photons in a single mode. From (9) we thus see that the induced transition rate B (emission or absorption) is <n> times that for spontaneous emissions.

The laser consists of a vast number of atomic amplifiers placed between two partially reflecting mirrors which cause radiation to travel back and forth through the amplifying medium. The electromagnetic field, building up within the laser, may be regarded as a field in a cavity that is weakly coupled to the outside. The different types of electromagnetic oscillations of the laser regarded as an isolated cavity are the well-known modes of oscillations or briefly modes.

It is relatively easy to make input and output calculations for a ruby laser in a hypothetical steady radiating state. Such calculations are of little value, however, because of the large intensity fluctuations which seem inherent in the situation.

The power generated at the frequency $\upsilon_{21}$in a uniformly excited ruby laser of volume, V is

$$P_0 = \omega_{21}(N_2 - N_1)\ V\ h\upsilon_{21} \tag{10}$$

Here $\omega_{21}$ is proportional to the radiation density, and $N_2 = N_1$ depends upon the radiation density as well as on the intensity of excitation. Starting with zero radiation density at the frequency $\upsilon_{21}$ when the threshold is first reached, the radiation density starts to fall again and an oscillation of intensity ensues. This oscillation is called pulsation.

## III. Transition probability from T - matrix consideration:

For the interaction problem of radiation and two-level system, it is essential to determine the complex probability amplitudes a (t) and b (t) for the two states │a > and │b > respectively that solve the mechanical part of the interaction problem, because, these directly (and their different real combinations $r_1$ *and* $r_2$ and$r_3$) give the various physical quantities of interest such as power emission from the system, polarization of the system, etc. Direct solutions of the differential equation for a(t) and b(t) which are obtained from the Schroedinger equation of the problem appeared in literature and were first given by Einstein in connection with the derivation of induced transition probabilities.

It is interesting to note that the T-matrices of interaction obtained by earlier workers have the elements which are just the probability amplitudes and their conjugates and are recognizable as Kayley-Klein parameters which are intimately connected with spatial rotation on quantum mechanics.

When the system is initially in its higher state represented by the ket│a >, the state vector │ψ>, later t, is essentially the transformation of │a > by a matrix

$$T(t) = \begin{pmatrix} a(t) & -b^+(t) \\ b(t) & a^+(t) \end{pmatrix} \tag{11}$$

In terms of the three well known real functions $\omega_1$,$\omega_2$ and $\omega_3$($\omega'_1$,$\omega'_2$ *and* $\omega'_3$ in the primed frame; Hamiltonians in the primed frame appear as a constant.) of Feynman equation $\frac{d\vec{r}}{dt} = (\vec{\omega} \times \vec{r})$, the interaction matrices have been expressed as form = $\underline{+}$ 1 transition

$$T(t) = \begin{pmatrix} \cos\frac{\Omega t}{2} - \frac{i\omega'_3}{\Omega}\sin\frac{\Omega t}{2} & \frac{-i}{\Omega}(\omega'_1 - i\omega'_2)\ \sin\frac{\Omega t}{2} \\ \frac{-i}{\Omega}(\omega'_1 - i\omega'_2)\ \sin\frac{\Omega t}{2} & \cos\frac{\Omega t}{2} + \frac{i\omega'_3}{\Omega}\sin\frac{\Omega t}{2} \end{pmatrix} \tag{12}$$

with $= (\ \omega'^2_1 + \omega'^2_2 + \omega'^2_3\ )$ ; and for Δm = 0 transitions

$$T(t) = \begin{pmatrix} \cos\frac{\Omega t}{2} - \frac{i\omega'_2}{\Omega}\sin\frac{\Omega t}{2} & \frac{-i\omega'_1}{\Omega}\sin\frac{\Omega t}{2} \\ \frac{i\omega'_1}{\Omega}\sin\frac{\Omega t}{2} & \cos\frac{\Omega t}{2} + \frac{i\omega'_3}{\Omega}\sin\frac{\Omega t}{2} \end{pmatrix} \tag{13}$$

with $\Omega = (\ \omega'^2_1 + \omega'^2_2\ )^{1/2}$ Since the above matrices refer to the rotating coordinate system (1', 2', 3') we obtain, on comparison of the elements of (11) and (13), the probability amplitudes, to be given by

$$a'(t) = \cos\frac{\Omega t}{2} - \frac{i\omega'_2}{\Omega}\sin\frac{\Omega t}{2},\quad b'(t) = \frac{-i\omega'_1}{\Omega}\sin\frac{\Omega t}{2} \tag{14}$$

for Δm = 0 case of transition.

The transition probability │b (t) │$^2$ (=│b' (t) │$^2$) for the system (initially in the higher state) to be in the lower state after a time t due to the interaction is therefore given buy

$$|b(t)|^2 = \frac{\omega'^2}{\Omega^2}\ \sin^2\frac{\Omega t}{2}$$

$$= \frac{(\frac{\mu_{ab}\, E_0}{\hbar})^2}{(\frac{\mu_{ab}\, E_0}{\hbar})^2 + (\omega_0 - \omega)}\ \sin^2 \tfrac{1}{2}\, t\sqrt{(\frac{\mu_{ab}\, E_0}{\hbar})^2 + (\omega_0 - \omega)} \tag{15}$$

(from the explicit expression for $\omega_1$'s given by Feynman et al for a coherent applied field E = $E_0 \cos\omega t$ appear to be constants in a rotating frame obtained by rotation of the (1-2) plane about the 3rd axis with an angular velocity **ω**, $\omega_3 = \omega_0$ is the transition frequency of the two states). Under the resonance condition $\omega_0 = \omega$ the transition probability (15) changes to

$$|b(t)|^2 = \sin^2 \tfrac{1}{2}\, t\,(\frac{\mu_{ab}\, E_0}{\hbar}) \text{ where, } \alpha = \tfrac{1}{2}(\frac{\mu_{ab}\, E_0}{\hbar}) \tag{16}$$

The above results are identical to the results obtained for occupation probability for a two-level spin system in a time-harmonic field through solving the Schrodinger equation.

## IV. Solution for the Inverted Population Difference between the LASING LEVELS of a 3-levels system:

Literature pointcut that the most widely used quantum mechanical analysis, for the complete and rigorous treatment of a collection of multilevel atoms is the density matrix approach. However, they are not very different from the approximate equations obtained with simple heuristic arguments.

The approximate approach we will develop in this section amounts in essence to treating each separate transition in a 3-level (or a multilevel) system as a separate two-level transition and then adding up the various rate equation terms to find the total rate equation for each level in the system. Siegman points out that if applied signals are present at or near various transition frequencies $\omega_{ij}$ between various levels $E_i$ and $E_j$, then if none of the applied signals is too strong, the following general principles can be applied:

(a) As far as the stimulated response on any particular transition is concerned, that transition may be treated as if it were simply an elementary two-level transition between the two energy levels $E_i$ and $E_j$."

The induced response on the transition, in each case, will be proportional to the population difference $\Delta N$ on the transition and will be independent of the populations of all the other energy levels, as well as independent of the presence of any allied signals on other transitions (provided they keep $\Delta N$ unchanged).

(b) Only the populations $n_i$(t) of the two levels will be directly changed by the presence of an applied signal on the transition, and these population changes (or more precisely rates of change) can be described in the same rate equation terms as in the equivalent two-level case.

(c) A signal applied at or near a given transition frequency will excite a significant response on that transition only. Multiple signals applied simultaneously to several different transitions in the same atomic system will not directly interact with each other.

In other words, however, leaving the indirect effects, it is not the presence or absence of the other signals that counts, but simply the population difference that is present, regardless of how it is brought about.

## V. POWER EMITTED AND TIME FOR MAXIMUM POWER OUTPUT:

The power P(t), emitted from a three-level system at any time t is

P(t)= $\Delta n(t)$The power P(t), emitted from a three-level system at any time t is

$$\text{P(t)}= \Delta n(t)\ \omega_{12} h\upsilon_{12} \tag{17}$$

Where $\Delta n(t)$ is the population difference at any time t between the lasing levels and is given by

$$\Delta n(t) = n_2(\text{t}) - n_2(\text{t})=(A_2 - A_1) +(B_2 - B_1)e^{\alpha t}+(C_2 - C_1)e^{\beta t} \tag{18}$$

On substitution of the values of the constants (i.e. A, B, etc.) and using the approximation that the spontaneous transition probabilities are much smaller than the induced ones, for the output power at any time t, we have,

$$\begin{aligned}\text{P}(t) = {} & \frac{N}{3}\left[P_{21}(e_3-1) - P_{22}(e_1-1)\right] h\upsilon_{12}+\left\{\left[\frac{n_2^e - n_1^e}{2}\right]\cdot\left[1-\frac{\omega_{12}-\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right] - \frac{N}{6\omega_{12}}\left[P_{21}(e_3-1) - P_{21}(e_1-1)\right]\right. \\ & -\left[1+\frac{\omega_{12}+\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right]\left[\omega_{12}^2+\omega_{13}^2\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}\right]\cdot t\,\frac{[N-3n_2^e]}{2\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\Big\}\ \omega_{12}\ h\upsilon_{12} + \left\{\left[\frac{n_2^e-n_1^e}{2}\right]\cdot\right. \\ & \left[1+\frac{\omega_{12}+\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right] - \frac{N}{6\omega_{12}}\left[P_{21}(e_3-1) - P_{32}(e_1-1)\right]\cdot\left[1-\frac{\omega_{12}+\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right] \\ & +\frac{[N-3n_2^e]}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\Big\}\ \omega_{12}\ h\upsilon_{12} - \left[\frac{\omega_{12}+\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right]\cdot \text{t}\end{aligned} \tag{19}$$

At time=0, the power output is P(t=0) = $[n_2^e - n_1^e]\ \omega_{12} h\upsilon_{12}$ and is a negative quantity, since $n_2^e < n_1^e$ .

For large value of t, the power output is

$$P(t)_{large} = \frac{N}{3}\left[P_{21}(e_3-1) - P_{22}(e_1-1)\right] h\upsilon_{12} \tag{20}$$

To find the time $t_m$, at which the output power will be maximum, we have

$$\frac{dP(t)}{dt} = [\alpha(B_2 - B_1)\ e^{\alpha t_m} + \beta\ (C_2 - C_1)\ e^{\beta t_m}]\ \omega_{12}\ h\upsilon_{12}\ = 0$$

$$\text{or, } \frac{e^{\alpha t_m}}{e^{\beta t_m}} = -\frac{(C_2 - C_1)}{(B_2 - B_1)}\frac{\beta}{\alpha}$$

$$\text{or, } e^{t_m(\alpha-\beta)} = -\frac{(C_2 - C_1)}{(B_2 - B_1)}\frac{h+D}{h-D}$$

$$\text{or, } t_m = \frac{1}{0.4343\ D}\log\left[-\frac{(C_2 - C_1)}{(B_2 - B_1)}\frac{h+D}{h-D}\right] \tag{21}$$

Again from the expression (2.31) for example,

$$P(t)_{large} = \frac{N}{3}\left[P_{21}(e_3-1) - P_{32}(e_1-1)\right] h\upsilon_{12}$$

we have, for microwave frequency range in steady condition,

$$\text{P (t)} = \frac{N}{3}\left[\ P_{21}\left(e^{-(E_2-E_1)/KT} - 1\right)\ - P_{32}\left(e^{-(E_3-E_2)/KT} - 1\right)\right] h\upsilon_{12}$$

$$= \frac{N}{3}\left[\ P_{21}\left(-\frac{h\upsilon_{12}}{KT}\right) - P_{32}\left(-\frac{h\upsilon_{32}}{KT}\right)\right] h\upsilon_{12}$$

For $h\upsilon_{ij} \ll$ KT

$$= \frac{N}{3}\frac{h}{KT}\{P_{32}\upsilon_{32} - P_{21}\upsilon_{21}\} \tag{22}$$

This is exactly the same result as was obtained by Bloembergen.

## VI. CONCLUSION :

In this topic we have solved the rate equations for the power output from the emissive system at any instant of time considering $\boldsymbol{\omega_{12}}$ as time-independent. the power emitted is found to vary with time which ultimately attains a constant value.

The induced response on the transition in each case will be proportional to the population difference on the transition and will be independent of the population of all of the other energy levels as well as independent of the presence of any applied signals on other transition. Only the population of two levels will be directly changed by the presence of an applied signal on that particular transition and these population changes can be described in the same rate equation terms as in the equivalent two levels case.

Thus the rate equation approach to the problem provides a quantitive analysis of power emission from the system but it does neither give other physical quantities of interest namely as the polarization induced in the system nor is suitable for any qualitative analysis of the problem.

## REFERENCES

**[1]** Singer J.R. 1959 Quantum Electronics and Coherent Light, John Wiley & Sons Inc., NewYork
**[2]** Bloembergen. 1984 Enrico Fermi Proceedings of the international school of Physics, Vol. XXXI,
**[3]** Yari A.1997 Quantum Electronics John Wiley and Sons
**[4]** Vuylateke A.A. 1960 Elements of Maser Theory D. Van Nostrant Company, INC.N.J. Newyork.
**[5]** Venkatesh H.G.1991 Ind.J.Physics 45, 97–106.
**[6]** Lamb W.E.1964 Physics Rev. 134, 1429.
**[7]** Anderson P.W. 1987 J. Appl. Phys. 23, 1049.
**[8]** Feynman R.P, Vernon F.L. and Hellwarth R.W. J. 1997 Appl. Phys. 28, 49.
**[9]**. Dicke R.H. 1954 Physics Rev. 93, 99.