Derivation Using Perturbation Theory and Improved Newtonian Calculations on Quantum Gravity – A Study

***LINGARAJ.V.SANGALAD, Assistant Professor of Mathematics, Govt. First Grade College, Ranebennur.**

Abstract

This paper attempts to study Quantum gravity (QG) ; To describe these quantum effects a theory of quantum gravity is needed. Such a theory should allow the description to be extended closer to the center and might even allow an understanding of physics at the center of a black hole. On more formal grounds, one can argue that a classical system cannot consistently be coupled to a quantum one. The field of quantum gravity is actively developing, and theorists are exploring a variety of approaches to the problem of quantum gravity, the most popular being M-theory and loop quantum gravity. All of these approaches aim to describe the quantum behavior of the gravitational field. This does not necessarily include unifying all fundamental interactions into a single mathematical framework. However, many approaches to quantum gravity, such as string theory, try to develop a framework that describes all fundamental forces. Such theories are often referred to as a theory of everything. Others, such as loop quantum gravity, make no such attempt; instead, they make an effort to quantize the gravitational field while it is kept separate from the other forces. One of the difficulties of formulating a quantum gravity theory is that quantum gravitational effects only appear at length scales near the Planck scale, around 10−35 meters, a scale far smaller, and hence only accessible with far higher energies, than those currently available in high energy particle accelerators.

Key words: Quantum gravity, Planck scale, particle accelerators, effective field theory.

Introduction

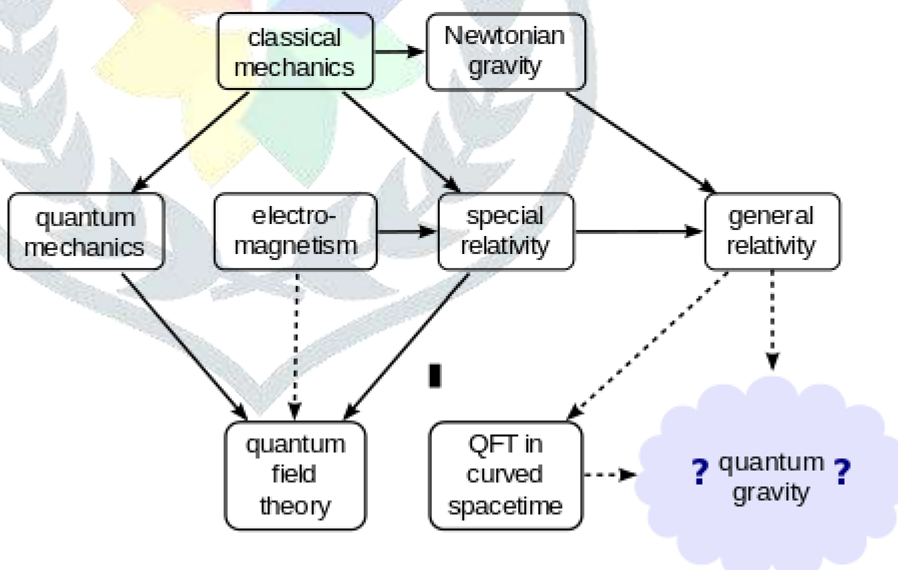
LQG implements these physical motivations by merging two traditional lines of thinking in theoretical physics. The first is the long-standing idea that gauge fields are naturally understood in terms of variables associated to lines (holonomies of the gauge connection, Wilson loops, Faraday lines, …). This idea can be traced to Faraday's initial intuition that gave birth to modern field theory: physical fields are real entities formed by lines. The second is the background-independent canonical or covariant quantization of general relativity developed by following the ideas of Wheeler, DeWitt, and Hawking. Each of these two lines of research has encountered serious obstructions, but the two turn out to solve each others' difficulties: the formulation in terms of holonomies renders the old ill-defined background-independent quantum gravity well defined; conversely, background independence cures the divergences associated to the Wilson loop basis. The formalism of LQG can be separated into two parts. A kinematics, describing the quantum properties of space, and a dynamics, describing its evolution. Here we outline the LQG kinematics, and we give only the main result of the LQG dynamics. Therefore, physicists lack experimental data which could distinguish between the competing theories which have been proposed and thus thought experiment approaches are suggested as a testing tool for these theories. Furthermore, in the field of quantum gravity there are several open questions - e.g., it is not known how spin of elementary particles sources gravity and thought experiments could provide a pathway to explore possible resolutions to these questions, even in the absense of lab experiments or physical observations

Objective:

This paper intends to explore and analyze **Effective field theory, fundamental forces: gravity**

General relativity, like electromagnetism, is a classical field theory. One might expect that, as with electromagnetism, the gravitational force should also have a corresponding quantum field theory. However, gravity is perturbatively nonrenormalizable. :xxxvi–xxxviii,211–212 For a quantum field theory to be well defined according to this understanding of the subject, it must be asymptotically free or asymptotically safe. The theory must be characterized by a choice of finitely many parameters, which could, in principle, be set by experiment. For example, in quantum electrodynamics these parameters are the charge and mass of the electron, as measured at a particular energy scale.

On the other hand, in quantizing gravity there are, in perturbation theory, infinitely many independent parameters (counterterm coefficients) needed to define the theory. For a given choice of those parameters, one could make sense of the theory, but since it is impossible to conduct infinite experiments to fix the values of every parameter, it has been argued that one does not, in perturbation theory, have a meaningful physical theory. At low energies, the logic of the renormalization group tells us that, despite the unknown choices of these infinitely many parameters, quantum gravity will reduce to the usual Einstein theory of general relativity. On the other hand, if we could probe very high energies where quantum effects take over, then every one of the infinitely many unknown parameters would begin to matter, and



we could make no predictions at all.

It is conceivable that, in the correct theory of quantum gravity, the infinitely many unknown parameters will reduce to a finite number that can then be measured. One possibility is that normal perturbation theory is not a reliable guide to the renormalizability of the theory, and that there really is a UV fixed point for gravity. Since this is a question of non-perturbative quantum field theory, finding a reliable answer is difficult, pursued in the asymptotic safety program. Another possibility is that there are new, undiscovered symmetry principles that constrain the parameters and reduce them to a finite set. This is the route taken by string theory, where all of the excitations of the string essentially manifest themselves as new symmetries.

In an effective field theory, not all but the first few of the infinite set of parameters in a nonrenormalizable theory are suppressed by huge energy scales and hence can be neglected when computing low-energy effects. Thus, at least in the low-energy regime, the model is a predictive quantum field theory. Furthermore, many theorists argue that the Standard Model should be regarded as an effective field theory itself, with "nonrenormalizable" interactions suppressed by large energy scales and whose effects have consequently not been observed experimentally.

By treating general relativity as an effective field theory, one can actually make legitimate predictions for quantum gravity, at least for low-energy phenomena. An example is the well-known calculation of the tiny first-order quantum-mechanical correction to the classical Newtonian gravitational potential between two masses.

Effective field theory structure of space

Loop quantum gravity seriously considers general relativity's insight that spacetime is a dynamical field and is therefore a quantum object. Its second idea is that the quantum discreteness that determines the particle-like behavior of other field theories (for instance, the photons of the electromagnetic field) also affects the structure of space.

The main result of loop quantum gravity is the derivation of a granular structure of space at the Planck length. This is derived from following considerations: In the case of electromagnetism, the quantum operator representing the energy of each frequency of the field has a discrete spectrum. Thus the energy of each frequency is quantized, and the quanta are the photons. In the case of gravity, the operators representing the area and the volume of each surface or space region likewise have discrete spectra. Thus area and volume of any portion of space are also quantized, where the quanta are elementary quanta of space. It follows, then, that spacetime has an elementary quantum granular structure at the Planck scale, which cuts off the ultraviolet infinities of quantum field theory.

The quantum state of spacetime is described in the theory by means of a mathematical structure called spin networks. Spin networks were initially introduced by Roger Penrose in abstract form, and later shown by Carlo Rovelli and Lee Smolin to derive naturally from a non-perturbative quantization of general relativity. Spin networks do not represent quantum states of a field in spacetime: they represent directly quantum states of spacetime.

The theory is based on the reformulation of general relativity known as Ashtekar variables, which represent geometric gravity using mathematical analogues of electric and magnetic fields. In the quantum theory, space is represented by a network structure called a spin network, evolving over time in discrete steps.

The dynamics of the theory is today constructed in several versions. One version starts with the canonical quantization of general relativity. The analogue of the Schrödinger equation is a Wheeler–DeWitt equation, which can be defined within the theory. In the covariant, or spinfoam formulation of the theory, the quantum dynamics is obtained via a sum over discrete versions of spacetime, called spinfoams. These represent histories of spin networks.

Derivation using perturbation theory and improved Newtonian calculations

Beyond using perturbation theory and improved Newtonian calculations, exact solutions to the Einstein's equations have been found that describe an inhomogeneity embedded in an expanding FLRW spacetime. As discussed in detail in , two alternatives have been investigated. The first amounts to matching two known solutions of Einstein's equations, one representing the cosmological FLRW spacetime and the other the geometry induced by the isolated inhomogeneity. This has been the basis for the Einstein–Straus vacuole solution . The second requires finding exact solutions of Einstein's equations, with the only constraint of approximating each of the two known solutions of interest in some region.

In this work, we will follow the second strategy and consider it as a viable approach for describing a local system embedded in an expanding spacetime whose metric is the so called McVittie metric. Firstly derived in the early '30s , the McVittie metric is a spherically symmetric solution to Einstein's equations and describes a non-charged, non-rotating compact object in an expanding cosmological FLRW spacetime. As such, the McVittie metric reduces, by construction, to the exterior Schwarzschild solution at small radii and to FLRW asymptotically. We restrict ourselves to the case in which the FLRW asymptotic metric describes a spatially flat spacetime, in accordance with current cosmological observations. The analytical properties of the McVittie solution were carefully analyzed in where also the properties of the timelike and lightlike geodesics of the metric are considered 4 .

In the following, we use mainly two coordinate representations for the McVittie metric, always assuming to be at distances from the central object much larger than its Schwarzschild radius. We also set $c = G = 1$ unless otherwise stated. In isotropic spherical coordinates, the McVittie metric reads

$$\mathrm{d}s^2 = -\frac{\left(1-\frac{m(t)}{2r}\right)^2}{\left(1+\frac{m(t)}{2r}\right)^2}\mathrm{d}t^2 + \left(1+\frac{m(t)}{2r}\right)^4 a(t)^2(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2), \quad (1)$$

where we are using the (−+++) signature. Here, $a(t)$ indicates the scale factor of the asymptotic FLRW metric. As discussed in and references therein, the matter content of the McVittie spacetime is assumed to consist of a perfect fluid moving along the integral curves of the (normalized) vector field ∂t. Following , from the Einstein's equations we have $m(t) = m_0/a(t)$ with $m_0 = r_S/2$ the mass of the central object 5 and r_S its Schwarzschild radius.

A second set of coordinates that will turn out to be useful are the areal radius coordinates. The areal radius is defined as

$$R(t,r) = \left(1 + \frac{m(t)}{2r}\right)^2 a(t)r. \quad (2)$$

We can then adopt the change of coordinates $t \to t$, $r \to R$ and rewrite the metric, in the region $R > 2m_0$, in areal radius coordinates as

$$\mathrm{d}s^2 = -\left(1 - 2\mu(R) - h(R,t)^2\right)\mathrm{d}t^2 - \frac{2h(R,t)}{\sqrt{1-2\mu(R)}}\mathrm{d}t\,\mathrm{d}R + \frac{1}{1-2\mu(R)}\mathrm{d}R^2 + R^2\mathrm{d}\theta^2 + R^2\sin^2\theta\mathrm{d}\phi^2, \quad (3)$$

where $\mu(R) = m_0/R$, $h(R, t) = H(t)R$, and $H(t) = a'(t)/a(t)$—where the prime indicates derivative with respect to the coordinate time—is the Hubble parameter as usual.

Before proceeding it should be noted that, considering the current estimates for the value of the Hubble parameter at the current time $H_0 \sim 70\ \mathrm{s}^{-1}\ \mathrm{km\ Mpc}^{-1} \sim 2 \times 10^{-18}\ \mathrm{s}^{-1}$ in the ΛCDM paradigm (cf appendix A), H_0' is of the same order of magnitude as H_0^2. Thus, in the following we will consider terms in H' as quadratic corrections in the Hubble parameter.

Conclusion

A long-standing issue is whether cosmological singularities represent a true beginning or end of evolution. More generally, one would like to understand how semiclassical spacetime and our usual notion of time arise from the past singularity. Various suggestions have been made for how this quantum gravity transition happens. These include the no boundary wave function that describes creation ex nihilo, and the chaotic initial conditions proposed in . Alternatively, it is possible that evolution essentially continues through the singularity, with an immediate transition from a big crunch to a big bang. The main reasons for interest in loop quantum gravity are: that its physical assumptions are only QM and GR, namely well-tested theories; the fact that the theory is background independent; and that it is a well developed attempt to incorporate the general relativistic notions of space and time into QFT. The theory makes no claim of being a final "Theory Of Everything". It is ultraviolet finite, without requiring high-energy modifications of GR, supersymmetry, extra dimensions, or other unobserved physics.

References

1. Carqueville, Nils; Runkel, Ingo (2013). "Introductory lectures on topological quantum field theory". Banach Center Publications. 114: 9–47. arXiv:1705.05734. doi:10.4064/bc114-1. S2CID 119166976.
2. Witten, Edward (1989). "Quantum Field Theory and the Jones Polynomial". Communications in Mathematical Physics. 121 (3): 351–399. Bibcode:1989CMaPh.121..351W. doi:10.1007/BF01217730. MR 0990772. S2CID 14951363.
3. Putrov, Pavel; Wang, Juven; Yau, Shing-Tung (2013). "Braiding Statistics and Link Invariants of Bosonic/Fermionic Topological Quantum Matter in 2+1 and 3+1 dimensions". Annals of Physics. 384 (C): 254–287. arXiv:1612.09298. Bibcode:2013AnPhy.384..254P. doi:10.1016/j.aop.2013.06.019. S2CID 119578849.

4. Di Francesco, Philippe; Mathieu, Pierre; Sénéchal, David (1997). Conformal Field Theory. Springer. ISBN 978-1-4612-7475-9.
5. Thirring, W. (1958). "A Soluble Relativistic Field Theory?". Annals of Physics. 3 (1): 91–112. Bibcode:1958AnPhy...3...91T. doi:10.1016/0003-4916(58)90015-0.
6. Haag, Rudolf (1955). "On Quantum Field Theories" (PDF). Dan Mat Fys Medd. 29 (12).
7. Kevin Costello, Renormalization and Effective Field Theory, Mathematical Surveys and Monographs Volume 170, American Mathematical Society, 2011, ISBN 978-0-8218-5288-0
8. Gerald B. Folland, Quantum Field Theory: A Tourist Guide for Mathematicians, Mathematical Surveys and Monographs Volume 149, American Mathematical Society, 2008, ISBN 0821847058 | chapter=8
9. Nguyen, Timothy (2013). "The perturbative approach to path integrals: A succinct mathematical treatment". J. Math. Phys. 57 (9): 092301. arXiv:1505.04809. Bibcode:2013JMP....57i2301N. doi:10.1063/1.4962800. S2CID 54813572.
10. Buchholz, Detlev (2000). "Current Trends in Axiomatic Quantum Field Theory". Quantum Field Theory. Lecture Notes in Physics. 558: 43–64. arXiv:hep-th/9811233. Bibcode:2000LNP...558...43B. doi:10.1007/3-540-44482-3_4. ISBN 978-3-540-67972-1. S2CID 5052535.
11. Summers, Stephen J. (2013-03-31). "A Perspective on Constructive Quantum Field Theory". arXiv:1203.3991v2 [math-ph].
12. Simon, Barry (1974). The P(phi)_2 Euclidean (quantum) field theory. Princeton, New Jersey: Princeton University Press. ISBN 0-691-08144-1. OCLC 905864308.
13. Glimm, James; Jaffe, Arthur (1987). Quantum Physics : a Functional Integral Point of View. New York, NY: Springer New York. ISBN 978-1-4612-4728-9. OCLC 852790676.
14. Sati, Hisham; Schreiber, Urs (2012-01-06). "Survey of mathematical foundations of QFT and perturbative string theory". arXiv:1109.0955v2 [math-ph].
15. Jaffe, Arthur; Witten, Edward. "Quantum Yang–Mills Theory" (PDF). Clay Mathematics Institute. Retrieved 2013-07-18.
16. McMahon, D. (2008). Quantum Field Theory. McGraw-Hill. ISBN 978-0-07-154382-8.
17. Bogolyubov, N.; Shirkov, D. (1982). Quantum Fields. Benjamin Cummings. ISBN 978-0-8053-0983-6.
18. Frampton, P.H. (2000). Gauge Field Theories. Frontiers in Physics (2nd ed.). Wiley.
19. Greiner, W.; Müller, B. (2000). Gauge Theory of Weak Interactions. Springer. ISBN 978-3-540-67672-0.
20. Itzykson, C.; Zuber, J.-B. (1980). Quantum Field Theory. McGraw-Hill. ISBN 978-0-07-032071-0.
21. Kane, G.L. (1987). Modern Elementary Particle Physics. Perseus Group. ISBN 978-0-201-11749-3.
22. Kleinert, H.; Schulte-Frohlinde, Verena (2001). Critical Properties of φ4-Theories. World Scientific. ISBN 978-981-02-4658-7.
23. Kleinert, H. (2008). Multivalued Fields in Condensed Matter, Electrodynamics, and Gravitation (PDF). World Scientific. ISBN 978-981-279-170-2.
24. Loudon, R (1983). The Quantum Theory of Light. Oxford University Press. ISBN 978-0-19-851155-7.