# CERTAIN PARTITION THEOERMS

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***Abstract:*** **In this paper, we have taken certain partition theorem due to Slater'5; and making use of known identities to establish some new partition theorems in original research work.**

***Key words:*** **Generalized hyper geometric function and Gauss hyper geometric function and Ordinary hyper-geometric series; identities, known transformation formulae.**

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## 1. *Introduction:* 

**In this paper , an attempt has been made to established certain partition theorems similar to Roger's – Ramanujan theorems by adapting the pattern of Hirechorn [1], Subbraao and Agrawal [1] and Singh, S.N. [3],. We consider the following identities due to Slate [5];**

$$\sum_{n=0}^{\infty}\frac{q^{2n^2}}{(q;q)_{2n}}=\frac{1}{(q^2,q^3,q^4,q^5,q^{11},q^{12},q^{13},q^{14};q^{16})_\infty} \tag{1}$$

$[Slater^5;(83)P.160]$

$$\sum_{n=0}^{\infty}\frac{q^{2n^2}}{(q;q)_{2n}}=\frac{1}{(q,q^3,q^4,q^5q^7,q^9,q^{11},q^{12},q^{13},q^{15},q^{16},q^{17},q^{19};q^{20})_\infty} \tag{2}$$

$[Slater^5;(79)P.160]$

$$\sum_{n=0}^{\infty}\frac{q^{2(2n+1)}}{(q;q)_{2n+1}}=\frac{1}{(q,q^3,q^5,q^7,q^9,q^{11},q^{13},q^{15};q^{16})_\infty} \tag{3}$$

$[Slater^5;(84)P.161]$

$$\sum_{n=0}^{\infty}\frac{q^{2(2n-1)}}{(q;q)_{2n}}=\frac{1}{(q,q^3,q^5,q^7,q^9,q^{11},q^{13},q^{15};q^{16})_\infty} \tag{4}$$

$[Slater^5;(85)P.161]$

$$\sum_{n=0}^{\infty}\frac{q^{2(2n+1)}}{(q;q)_{2n+1}}=\frac{1}{(q,q^4,q^6,q^7,q^9,q^{10},q^{12},q^{15};q^{16})_\infty} \tag{5}$$

$[Slater^5;(86)P.161]$

## 2. *Main Results:*

**We shall prove and establish the following interesting partition theorems:**

***Theorem 1.*** **The number of partition of n into parts = 2,3,4,5,11,12,13, or 14 (mod 16) equals the number of n into even number of parts ≥ K, where 2K is the number of parts in the partition.**

**Proof:**

**Let us consider a partition of n as**

$$n = a_1 + a_2 + \cdots a_k + a_{k+1} + \cdots + a_{2k} \qquad (6)$$

$where\ a_1 \geq k,\ a_2 \geq k \ldots a_{2k} \geq k.$

$So, a_1 + a_2 + \cdots + a_{2k} \geq 2k^2.$

**Now, consider the following partition:**

$$n - 2k^2 = (a_1 - k) + (a_2 - k) + \cdots + (a_{2k} - k), \qquad (7)$$

**which is a partition of** $n - 2k^2$ $into\ at\ most\ 2k\ parts.$

**It is generated by,** $\frac{q^{2k^2}}{(q;q)_{2k}}$ $and\ so\ series = \sum_{k=0}^{\infty} \frac{q^{2k^2}}{(q;q)_{2k}}$ **gives the total number of partitions of type (7) or total number of partitions of n type (6) due to 1-1 correspondence between partitions of type (6) and (7). Hence theorem -1 proved in view of identity (1.1).**

***Theorem 2.*** **The number of partitions of n into parts** $\equiv 2, 3, 4, 5, 11, 12, 13, or\ 14\ (mod\ 16)$ **equals the number of partitions ofn into even number of parts, say 2k such that in the first half part of the partition each part** $\geq (k+1)$**, and in the second half part of partition each part** $\geq (k+1)$**.**

**Proof:**

**Let us consider a partition of n as**

$$n = a_1 + a_2 + \cdots a_k + a_{k+1} + \cdots + a_{2k} \qquad (7)$$

$where\ a_1 \geq (k+1),\ a_2 \geq (k+1), \ldots a_{2k} \geq (k-1)\ and$

$a_{k+1} \geq (k-1),\ a_{k+2} \geq (k-1), \ldots a_{2k} \geq (k-1).$

$So,\ a_1 + a_2 + \cdots + a_k + a_{k+1} + \cdots + a_{2k} \geq k(k+1) + k(-1) \geq 2k^2.$

**Now, consider the following partition:**

$$n - 2k^2 = \{a_1 - (k-1)\} + \cdots + \{a_{2k} - (k+1)\} + \{a_{k+1} - (k-1)\} + \cdots + \{a_{2k} - (k-1)\},$$

**which is a partition of** $n - 2k^2$ $into\ at\ most\ 2k\ parts.$

It is generated by, $\frac{q^{2k^2}}{(q;q)_{2k}}$ $and\ so\ series = \sum_{k=0}^{\infty}\frac{q^{2k^2}}{(q;q)_{2k}}$ gives the total number of partitions of type (7) or total number of partitions of n type (6) due to 1-1 correspondence between partitions of type (6) and (7). Hence theorem -1 proved in view of identity (1.1).

***Theorem 3.*** **The number of partitions of n into parts** $\equiv 1, 3, 4, 5, 7, 9, 11, 13, 15, 16, 17\ or\ 19\ (mod\ 20)$ **equals the number of partitions of n into even number of parts, 2k such that in the first half part of the partition each part** $\geq (k-1)$.

**Proof:**

Let us consider a partition of n as

$$n = a_1 + a_2 + \cdots a_k + a_{k+1} + a_{k+2} + \cdots + a_{2k} \quad (7)$$

$where\ a_1 \geq (k-1),\ a_2 \geq (k-1), \ldots\ a_{2k} \geq (k-1)\ and$

$a_{k+1} \geq 1,\ a_{k+2} \geq 1, \ldots\ a_{2k} \geq 1.$

$So,\ a_1 + a_2 + \cdots + a_k + a_{k+1} + \cdots + a_{2k} \geq k(k-1) + k \geq k^2.$

Now, consider the following partition:

$$n - k^2 = \{a_1 - (k-1)\} + \{a_2 - (k-1)\} + \cdots + \{a_k - (k-1)\} + \{a_{k+1} - 1\} + \cdots + \{a_{2k} - 1\},$$

which is a partition of $n - k^2\ into\ at\ most\ 2k\ parts.$

It is generated by, $\frac{q^{2k^2}}{(q;q)_{2k}}$ $and\ so\ series = \sum_{k=0}^{\infty}\frac{q^{2k^2}}{(q;q)_{2k}}$ gives the total number of partitions of type (7) or total number of partitions of n type (6) due to 1-1 correspondence between partitions of type (6) and (7). Hence theorem -1 proved in view of identity (1.2).

***Theorem 4.*** **The number of partitions of n into parts** $\equiv 1, 3, 5, 7, 9, 11, 13\ or\ 15\ (mod\ 16)$ **equals the number of partitions of n into even number of parts, say 2k +1, such that each part is** $\geq k$

**Proof:**

Let us consider a partition of n as

$$n = a_1 + a_2 + \cdots + a_{2k+1} \quad (12)$$

In which each part is $\geq k$

$where\ a_1 \geq k,\ a_2 \geq k, \ldots\ a_{2k+1} \geq k.$

$So,\ a_1 + a_2 + \cdots + a_{2k+1} \geq k(2k+1)$

Now, consider the following partition:

$$n - k(2k+1) = (a_1 - k) + (a_2 - k) + \cdots + (a_{2k+1} - k),$$

which is a partition of $n - k(2k+1) into\ at\ most\ (2k+1)\ parts.$

It is generated by, $\frac{q^{2k^2}}{(q;q)_{2k}}$ *and so series* $= \sum_{k=0}^{\infty} \frac{q^{2k^2}}{(q;q)_{2k}}$ gives the total number of partitions of type (13) or total number of partitions of n type (12) due to 1-1 correspondence between partitions of type (12) and (13). Hence theorem -4 proved in view of identity (1.3).

***Theorem 5.*** **The number of partitions of n into parts $\equiv 1, 3, 5, 7, 9, 11, 13$ *or* $15\ (mod\ 16)$ equals the number of partitions of n into even number of parts, say 2k, such that such that in the first half part of the partition each part is k and in the second half part of partition each part $\geq (k-1)$.**

**Proof:**

Let us consider a partition of n as

$$n = a_1 + a_2 + \cdots + a_{2k+1} \tag{14}$$

In which each part is $\geq k$

$where\ a_1 \geq k,\ a_2 \geq k, \ldots\ a_{2k+1} \geq k.$

$So,\ a_1 + a_2 + \cdots + a_{2k+1} \geq k(2k+1)$

Now, consider the following partition:

$$n - k(2k-1) = (a_1 - k) + (a_2 - k) + \cdots + (a_k - k) + \{a_{k+1} - (k-1)\} + \cdots + \{a_{2k} - (k-1)\}, \tag{15}$$

which is a partition of $n - k(2k-1)$ *into at most* $2k$ *parts*.

It is generated by, $\frac{q^{2k^2}}{(q;q)_{2k}}$ *and so series* $= \sum_{k=0}^{\infty} \frac{q^{2k^2}}{(q;q)_{2k}}$ gives the total number of partitions of type (15) or total number of partitions of n type (14) due to 1-1 correspondence between partitions of type (14) and (15). Hence theorem -5 proved in view of identity (1.4).

***Theorem 6.*** **The number of partitions of n into parts $\equiv 1, 4, 6, 7, 9, 10, 12$ *or* $15\ (mod\ 16)$ equals the number of partitions of n into odd number of parts, say 2k+1, such that such that in the first half part of the partition each part is k and in the second half part of partition each part is $\geq (k+1)$. Partitions are taken in ascending order.**

**Proof:**

Let us consider a partition of n as

$$n = a_1 + a_2 + \cdots + a_k + a_{k+2} + \cdots + a_{2k+1} \tag{16}$$

$where\ a_1 \geq k,\ a_2 \geq k, \ldots\ a_{2k+1} \geq k.$

$and\ a_{k+2} \geq (k+1), \ldots\ a_{2k+1} \geq (k+1).$

This gives that

$a_1 + a_2 + \cdots + a_{k+1} + \cdots + a_{2k+1} \geq 2k(k+1).$

**Now, consider the following partition:**

$$n - k(2k-1) = (a_1 - k) + (a_2 - k) + \cdots + a_{k+1} - k + \cdots + (a_{2k+1} - (k-1) , \qquad (17)$$

**which is a partition of** $n - 2k(k+1)$ *into at most* $2k+1$ *parts*.

**It is generated by,** $\frac{q^{k(2k+1)}}{(q;q)_{2k+1}}$ *and so series* $= \sum_{k=0}^{\infty} \frac{q^{2k(k+1)}}{(q;q)_{2k+1}}$ **gives the total number of partitions of type (17) or total number of partitions of n type (16) due to 1-1 correspondence between partitions of type (16) and (17). Hence theorem -6 proved in view of identity (1.5).**

# References


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