A review on basic and advance theories of classical superconductors

Veerendra Singh

Associate Professor, Department of Physics, Meerut College Meerut- 250001, Uttar Pradesh, India.

Abstract: In this article various theories of superconductivity (basic and advance) are discussed.

Index terms : BCS; Superconductivity; Critical temperature.

1. Introduction:

At the turn of eighteenth century, Helium was discovered, and in 1908, the Dutch scientist K. Onnes succeeded in liquifying helium gas at a temperature few degrees above absolute zero. This marked the turning point for measuring resistance of various metals at low temperature. In 1911 Onnes (1) observed that when mercury is cooled below 4.15 K, its resistivity abruptly falls to almost zero value, i.e., its conductivity becomes infinitely large. This phase change and the new phase of mercury was called the superconducting phase. The temperature at which this transition occurs is called the critical temperature (T_c) and this phenomenon of zero resistivity below this critical temperature was called superconductivity. It was subsequently found that a number of metals, intermetallics and alloys show this property at sufficiently low temperatures. Various theories were given to explain this superconducting behaviour in metals, intermetallics and alloys. This article gives a general overview of these theories.

2. Basic theory:

The resistance of a metal is caused by the electron scattering primarily by lattice defects and by thermal lattice vibrations (phonons) which are present down to absolute zero. It implies that zero resistance in a superconductor requires zero scattering of conduction electrons. At finite temperatures, phonon scattering is possible if there are initial and final states available for scattering process. But if a new electronic state could exist upto the critical temperature, it could provide for the impossibility of scattering by removing the final states of scattering then superconductivity could become possible.

Bardeen-Cooper-Schrieffer (BCS) gave their theory to explain this phenomenonBCS theory (2) proposed that in certain materials an unusual state of electrons near Fermi level is possible at low temperatures which results from an attraction between two electrons through a phonon interaction by overcoming the Coulomb repulsion between them. Two electrons excited slightly from the ground state of a Fermi distribution at zero temperature, could form in the presence of an attractive interaction, a real bound state. The bound state could be described a single coherent wave function. It is envisaged that at low temperature free electrons travelling through the crystal can attract neighbouring lattice ions, creating a region of excess positive charge that can in turn attract a second electron, thus forming a Cooper pair of electrons (3). This new state has an energy lower than that of the normal state of free electrons near Fermi level and separated from it by a superconducting energy gap (0.3 to 3 meV) that is larger than the energy of the phonons available for the scattering at low temperature. Scattering therefore ceases since there is no energy conserving final state to the scattering transition, the scattering relaxation time becomes infinite and resistance goes to zero.

In order to minimize the energy of the new (superconducting) ground state, the electronic states from which the Cooper pairs are to be formed must be chosen so as to maximize the binding energy of each pair and to obtain the greatest possible number of such pairs. Since in the scattering processes due to electron interactions the total momentum (or the total k vector) is in general conserved, the maximum number of pairs that can be scattered coherently is obtained by pairing states such that all the Cooper pairs have the same value of total momentum. In particular for the ground state, which carries no net current, the best possible pairing is between states of equal and opposite momentum. At the same time, since the exchange terms reduce the effective strength of interaction, the pairing of states with opposite spin, maximizes the binding energy. In this way the superconducting ground state may be formed by assuming that it can be expressed solely in terms of Cooper pairs i.e., the pair of wave vectors +k ↑ and a -k ↓ are always occupied simultaneously. One Cooper pair ($+k^1$ ↑, $-k^1$↓) can occupy another vacant position ($+k^2$↑, $-k^2$↓) by exchanging an appropriate virtual phonon. Unpaired electrons can exist simultaneously with the Cooper pairs in the superconducting state, but these are like ordinary electrons in the normal state and cannot take part in carrying supercurrent. When an electric field is applied, the superconducting energy gap at the Fermi level moves along with the electric field and stabilizes these pairs from the phonons available at the superconducting transition temperature or below it. According to the BCS theory

$$k_b T_c = 1.13\hbar\omega_D e^{-\frac{1}{[N(E_F)]V^*}}$$

Where k = Boltzman constant, w_D = characterisitic Debye frequency, $N(E_F)$ = Density of states at the Fermi level and V^* = electron - phonon coupling constant. The BCS theory could provide a reasonably good explanation for the observed isotope effect, specific heat jump, microwave absorption etc. in the classic superconductors.

3. Advance theories:

Since the inception of BCS theory (2) the theoretical efforts were directed towards the calculation of superconducting state parameters, with a view of getting reliable estimates which might serve as the basis for establishing a criterion for the occurrence of superconductivity. Two major attempts were those of D. Pines (4) and Morel and Anderson (5).

D. Pinnes attempted to develop criterion for the occurrence of superconductivity and to explain the famous Matthias regularities (6) on the basis of the original formulation of the BCS theory, in which the net electron – electron interaction which is the sum of the phonon induced attractive interaction and Coulomb repulsion, is approximated by an instantaneous square well two body interaction. Only those electrons having energy in a narrow energy shell of thickness $=\hbar\omega_{ph}$, the maximum phonon energy near the Fermi level, can participate in the interaction. The BCS criterion that the phonon induced interaction dominates over Coulomb repulsion was used. Both normal and Umklapp processes were included. The results obtained for N_0V were well below the empirical values both the transition and non-transition elements hence T_c obtained from BCS T_c equation were less than the empirical values. Nevertheless, Pines work lent substantial support to the single particle BCS model.

Morel and Anderson (MA) criticised the D. Pines's assumptions and obtained an energy gap and other parameters of the superconducting state, from BCS theory in the form as modified by Gorkov (7), Eliashberg (8), using a realistic retarded electron - electron interaction via phonons and by including Coulomb repulsion. The values of N_0V obtained by Morel and Anderson for various elements were compared with the measured values. In almost all the cases the agreement was good and as such these results were an improvement over the earlier calculations of Morel (9) but the distinction between superconductors and non-superconductors was completely missing from the work of MA.

One of the most fertile approaches to the problem of superconductivity has developed from Ginzburg – Landau (GL) theory (10). Ginzburg – Landau generalised the Landau theory (11) because Landau theory was unsuccessful in calculating the critical field for superconductors having various geometries. The generalization took the form of assumption that for a superconductor the order parameter characterizing second order phase transitions must be identified by the macroscopic wave function; this means that the order parameter is complex and secondly that in general slowly varying function of position r so that free energy now becomes a function of. Once the free energy has been expressed as a function of and the vector potential A, the equation for it, to be a minimum, give an equation for the supercurrent in terms of vector potential A. The latter has the form of London equation. Obviously, the GL theory represents a generalization of the London theory to situations in which is spatially varying.

The GL theory was developed before BCS theory. The GL equations are much simpler than the BCS theory. They are generally used when they are known to hold. The GL equations provide valuable insight into the general quantitative behaviour of superconductors. The GL theory is perhaps particularly helpful in providing a clear grasp of the relationship between the various lengths, e.g., penetration depth, coherence length involved in superconductivity.

The success of the BCS theory encouraged scientists to further investigate the role of electron – phonon interaction, in normal and superconducting metals. Migdal (12) showed that in normal metals, the electron – phonon interaction could be treated accurately to the order of $(\frac{m}{M})^{1/2}$ and this is true even for strong coupling superconductors. Eliashberg (8) and Nambu (13) have extended Migdal treatment to the superconducting state and explained that the microscopic ingredients of Eliashberg theory are the Coulomb repulsion $\mu^* = N_o V_c$ where N_0 is the single spin density of electronic states at the Fermi surface and the electron phonon spectral function $\alpha^2(\omega)F(\omega)$, which depends on the electron phonon coupling matrix element. The zero temperature Eliashberg theory determines the complex energy gap function $\Delta(\omega)$ and the finite temperature Eliashberg theory determines T_c.

The superconducting transition temperature T_c can be calculated as a function of electron – phonon and electron – electron coupling constants, within the framework of strong coupling theory. McMillan (14) using this approach, found empirical values of coupling constants and band structure densities of states for a number of metals and alloys. He predicted that electron - phonon coupling constant depends primarily on the phonon frequency rather than on the electronic properties of the metals. He also predicted a maximum superconducting transition temperature. The prime content of McMillan's work was the solutions of finite temperature Eliashberg theory so as to find out T_c for various cases and the construction, from this data, of an approximate equation relating T_c to a small number of simple parameters like θ_D (Debye temperature), λ (electron - phonon coupling strength) and μ* (e-e Coulomb pseudopotential). Later on, the prefactor of McMillan viz (θ_D /1.45) was replaces with ω/1.20 by Dynes (15) where ω is the first moment of normalized weight function g (ω) = $\frac{2}{\lambda\omega}\alpha^2(\omega)F(\omega)$.

Acknowledgement: I would like to thank the authorities of Meerut College, Meerut for providing me infrastructure which helped me in preparing this manuscript.

References:

1. H.K. Onnes, Leiden Communication, C124 (1911).

2. J. Bardeen et al, Phys. Rev.,106 162 (1957); 108 1175 (1957).

3. L.N. Cooper Phys. Rev.,104 1189 (1956).

4. D. Pines, Phys. Rev.,109 280 (1958).

5. P. Morel and P.W. Anderson, Phys. Rev., 125, 1263, (1962).

6. B. T. Matthias, Prog. in low temperature Phys., 2, 138, (1957).

7. L. P. Gorkov, Zh. Eks. Teor. Fiz., 34, 735 (1958). [English transl: Soviet Phys. JETP 7, 505 (1,958)].

8. G.M. Eliashberg, Zh. Eksp. Teor. Fiz., 38, 966 (1960); 39, 1437 (1960). [English transl: Soviet Phys. JETP 11, 696 (1960); 12, 1000, (1961)].

9. P. Morel, J. Phys. Chem. Solids. 10, 277, (1959).

10. V.L. Ginzburg et al, Zh. Eksp. Teor. Fiz., 20, 1064, (1950).

11. L.D. Landau in 'Collected papers pf L.D. landau' [The English trans. By D. Ter Haar, Gordan and Breach, New York], (1965).

12. A.B. Migdal, Zh. Eksp. Teor. Fiz., 34, 1438 (1958). [English transl: Soviet Phys. JETP, 11, 696, (1960)].

13. Y. Nambu, Phys. Rev., 117, 648, (1960).

14. W.L. McMillan, Phys. Rev., 167, 331, (1968).

15. R.C. Dynes, Solid State Commun., 10, 615, (1972).