

Thermal Performance Optimization of Extended Surfaces under Natural Convection in Non-Newtonian Fluids

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Abstract:

“Extended surfaces (fins) are widely employed in thermal systems to enhance heat dissipation, particularly under natural convection conditions where heat transfer rates are inherently limited. While extensive studies exist on fins operating in Newtonian fluids, the thermal behavior of fins interacting with non-Newtonian fluids—commonly encountered in industrial applications such as polymer processing, food engineering, biofluids, and slurry flows—has received comparatively less attention. In view of this, the present article theoretically investigates the problem of an optimum profiled fin dissipating heat by natural convection to a quiescent non-Newtonian fluid”.

“This analysis is carried out by modelling the non-Newtonian behavior of the surrounding fluid using a power-law model, characterized by the flow behavior index (n), which accounts for shear-thinning and shear-thickening fluids. The governing energy equation for the fin is formulated by incorporating variable heat transfer coefficients arising due to natural convection in non-Newtonian fluids. Appropriate boundary conditions, including the insulated fin tip and prescribed base temperature, are applied to obtain physically realistic solutions. Using these conditions, generalized analytical expressions for the temperature distribution along the fin and the corresponding heat transfer rates are derived”.

“A non-dimensional formulation is adopted to identify the key controlling parameters, particularly the fin parameter (M), which represents the combined effects of fin geometry, material properties, and convection strength. The influence of both the flow behaviour index (n) and the fin parameter (M) on thermal performance is systematically examined. The resulting temperature profiles, local heat transfer coefficients, and average heat transfer coefficients are presented and analyzed through graphical representations”.

“The results reveal that the flow behaviour index (n) has a pronounced effect on the thermal behaviour of the fin. Variations in (n) significantly alter the convection characteristics, leading to noticeable changes in both local and average heat transfer coefficients. In addition, the fin parameter (M) strongly influences the rate of heat dissipation, indicating that optimal fin design must carefully account for both fluid rheology and fin geometry. One of the notable findings of this study is that, for a wide range of fin parameters, the local heat transfer coefficient does not vary monotonically along the fin length. Instead, it exhibits non-monotonic behaviour, highlighting the complex interaction between natural convection and non-Newtonian fluid properties”.

“The theoretical results presented in this article provide valuable insights into the design and optimization of fins operating in non-Newtonian environments. The generalized solutions developed can serve as a useful reference for engineers and researchers involved in the thermal design of heat exchangers, electronic cooling systems, and industrial equipment where non-Newtonian fluids are present”.

Key words:

Fin parameter, Parabolic profile, non-Newtonian fluid, Temperature distribution, Vicious fluid, Flow behavior, non-uniform flow, Arbitrary profile.

Introduction

“The study of flow and heat transfer characteristics of non-Newtonian fluids has gained considerable importance in recent years due to their widespread occurrence in modern industrial processes. Unlike Newtonian fluids, non-Newtonian fluids exhibit a nonlinear relationship between shear stress and shear rate, leading to complex flow and thermal behaviour. An accurate understanding of heat transfer mechanisms in such fluids is therefore essential for the effective design and operation

of thermal systems in various industries. In particular, precise estimation of heat transfer rates plays a crucial role in the design of components used in cosmetic, pharmaceutical, polymer, and food-processing industries, where non-Newtonian fluids such as molten plastics, emulsions, slurries, and pulps are commonly encountered”.

“Natural convection heat transfer in non-Newtonian fluids presents additional challenges owing to the strong coupling between fluid rheology and buoyancy-driven flow. The flow behaviour index, which characterizes shear-thinning or shear-thickening effects, significantly influences the momentum and thermal boundary layers and, consequently, the convective heat transfer coefficients. Several researchers have attempted to address these complexities in simplified geometries to better understand the fundamental heat transfer mechanisms involved”.

“Cho and Hartnell [1] investigated the heat transfer characteristics of non-Newtonian fluids flowing through circular pipes and demonstrated that fluid rheology has a pronounced influence on temperature distribution and heat transfer rates. Their work provided early insights into the behaviour of non-Newtonian fluids under forced convection conditions. Extending these concepts to natural convection and external flows, A. V. Shenoy et al. [2] theoretically analyzed the conjugate problem of thermal convection from a fin dissipating heat to a non-Newtonian fluid. In their study, a power-law model was adopted to represent the relationship between shear stress and shear deformation, enabling the analysis of heat transfer from extended surfaces to non-Newtonian fluids”.

“Despite these contributions, studies addressing optimum or profiled fins operating under natural convection in quiescent non-Newtonian fluids remain relatively limited. In particular, the development of generalized analytical formulations capable of predicting temperature distribution and heat transfer rates for fins with variable cross-sections has not been adequately explored. Such formulations are essential for optimal fin design, as they allow better material utilization and improved thermal performance.

The aim of the present study is to formulate a general governing equation for a vertical fin of parabolic profile dissipating heat by natural convection to a quiescent non-Newtonian fluid. The analysis incorporates the flow behavior index (n) nursing a power-law model to account for non-Newtonian effects. Analytical expressions are derived to predict the temperature distribution along the fin surface and the corresponding heat transfer rates. The fin is assumed to dissipate heat solely by convection to an isothermal, quiescent non-Newtonian fluid, and radiation effects are neglected. The physical configuration of the fin considered in the analysis is shown schematically in Figure 1”.

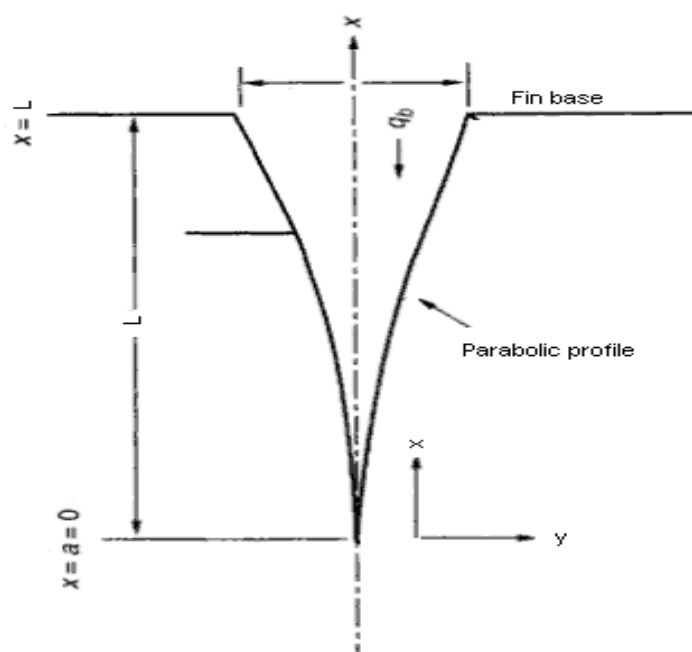


Fig. 1. Schematic of a parabolic profiled vertical fin dissipating heat by natural convection to a quiescent non-Newtonian fluid

The formulation is done based on the following assumption:

1. All the properties of the fluid such as viscosity, velocity and density are assumed to be constant i.e. they are independent of temperature.
2. The calculation of heat is along the length of fin (unidirectional). Since the thickness of fin is small enough in comparison to its length.
3. Thermal gradients are well within limits to induce laminar buoyant convection adjacent to the extended surface.

The boundary layer equations for natural convection phenomenon for the above shown co-ordinate system are as follows:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \frac{1}{\rho} \frac{\partial \tau_x}{\partial y} \quad (2)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

Writing the momentum equation in the integral form as

$$\frac{d}{dx} \left\{ \int_0^\delta \rho u^2 dy \right\} = -\tau_{xy} + \int_0^\delta \rho g \beta (T - T_\infty) dy$$

We have the shear stress as $\tau_{xy} = -\mu \left. \frac{du}{dy} \right|_{y=0}$

The velocity and temperature profiles are assumed as

$$\frac{u}{u_x} = \left(\frac{y}{\delta}\right) \left(1 - \frac{y}{\delta}\right)^2 \quad \text{And} \quad \frac{T - T_\infty}{T_w - T_\infty} = \left(1 - \frac{y}{\delta}\right)^2$$

When velocity and temperature profiles are substituted in integral form of momentum equation, the equation reduces to

$$\frac{1}{105} \frac{d}{dx} (\rho u_x^2 \delta) = \frac{\rho \beta g \delta}{3} \theta_w - \tau_{xy}$$

For a non-Newtonian fluid τ_{xy} is given by the expression

$n=1$ for Newtonian fluids

$n<1$ for Pseudo plastics

$n>1$ for dilatant's medium

Substituting the velocity equation in the above shear stress equation, we get

$$\tau_{xy} = \mu \left(\frac{u_x}{\delta}\right)^n$$

Now the momentum equation can be written as

$$\frac{1}{105} \frac{d}{dx} (\rho u_x^2 \delta) = \frac{g\beta\rho\delta}{3} \theta_w - \left(\frac{u_x}{\delta}\right)^n \mu \quad (4)$$

The integral form of energy equation is

$$\frac{d}{dx} \left[\int_0^\delta u(T - T_\infty) dy \right] = -\infty \frac{dT}{dy} \Big|_{y=0}$$

$$\tau_{xy} = \mu \left[\left(\frac{du}{dy} \right)^{n-1} \right] \left(\frac{\partial u}{\partial y} \right)$$

On substituting the temperature distribution, the energy equation reduces to

$$\frac{d}{dx} (u_x \theta_w \delta) = 60\alpha \frac{\theta_w}{\delta} \quad (5)$$

The diameter of the fin at any distance x, is given by (Ref-3)

$$D_x = D_o \left[1 - \frac{x-L_1}{L_2} \left\{ 1 - \left(\frac{D_t}{D_o} \right)^{\frac{2-N}{1-2N}} \right\} \right]^{\frac{1-2N}{2-N}}$$

For a parabolic profiled fin from the base without any uniform portion the above equation reduces to

$$D_x = D_o [1 - X\varepsilon_0]^{1/2}$$

Area and perimeter at any point are given by

$$A_x = \frac{\pi}{4} D_o^2 [1 - X\varepsilon_0]^{2\gamma}; \quad P_x = \pi D_o [1 - X\varepsilon_0]^\gamma;$$

The heat transfer equation is

$$\frac{d^2 \theta_w}{dX^2} + \frac{1}{A_x} \frac{dA_x}{dX} \frac{d\theta_w}{dX} - \frac{hP_x}{A_x k} \theta_w = 0 \quad (6)$$

The above equation when written in terms of 'x' takes the form (fin conduction equation)

$$\frac{d^2 \theta_w}{dX^2} - \frac{\varepsilon_0}{(1-X\varepsilon_0)} \frac{d\theta_w}{dX} - \frac{2k_L P_X}{\delta A_X k_w} \theta_w = 0 \quad (7)$$

The equations are reduced to dimensionless using the following non-dimensional terms

$$\eta = x/L; \quad T^+ = \frac{T_w - T_\infty}{T_{wL} - T_\infty}$$

$$R_a^* \text{ (Modified Rayleigh's number)} = \frac{g\beta\theta_{wL}L^{2n+1}}{\alpha^n \gamma}$$

$$M \text{ (Fin parameter)} = \left[\frac{k_L P_X L}{k_w A_x} \right] R_a^* \frac{1}{3n+1}$$

$$f(\eta) = \frac{\delta}{L} Ra^{*\frac{1}{3n+1}} \quad ; \quad \varphi(\eta) = \frac{U_x L}{\alpha} Ra^{*\frac{2}{3n+1}}$$

$$\text{Modified Prandtl No} = \frac{\alpha^{n-2} \gamma}{L^{2n-2}} \left[\frac{k_w A_x}{k_L P_x L} \right]^{3(n-1)}$$

The momentum equation becomes

$$\frac{d}{d\eta} (\phi^2 f) = 105 Pr_m M^{3(n-1)} \left[\frac{f T^+}{3} - \left(\frac{\varphi}{f} \right)^n \right] \quad (8)$$

The energy equation becomes

$$\frac{d}{d\eta} (\phi f T^+) = 60 \left(\frac{T^+}{f} \right) \quad (9)$$

Moreover, the fin conduction equation becomes

$$\frac{d^2 T^+}{d\eta^2} - \frac{\varepsilon}{(1-\eta\varepsilon)} \frac{dT^+}{d\eta} - \frac{2M}{f} T^+ = 0 \quad (10)$$

Equations (8), (9) and (10) are to be solved subjected to the following boundary conditions.

The boundary conditions are

$$\text{At } \eta = 0 \longrightarrow f = \varphi = \frac{dT^+}{d\eta} = 0$$

$$\text{And at } \eta = 1 \longrightarrow T^+ = 1$$

One of the important parts of the analysis is estimation of average heat transfer coefficient, which can be written in the form of

$$\frac{Nu}{Ra^{*1/(3n+1)}} = \frac{2}{f(\eta)}$$

After simplification the equation can be written as

$$\frac{Nu}{Ra^{*1/(3n+1)}} = \frac{2\eta^{-n/(3n+1)}}{\left[60.3^{1/n} \frac{3n+1}{2n+1} \right]^{n/(3n+1)}}$$

The average Nusselt number is given by the equation

$$\frac{Nu_m}{Ra^{*1/(3n+1)}} = \left[\frac{3n+1}{2n+1} \right]^{2n+1/3n+1} \frac{2}{\left[60.3^{1/n} \right]^{n/3n+1}}$$

For higher values of Prandtl number, which is true for non-Newtonian fluids the inertial force terms given by equation (8) can be neglected.

Therefore, the equation reduces to

$$\phi = \frac{f^{\frac{n+1}{n}}}{3^{1/n}} T^+$$

Now substituting ϕ in equation (9) we obtain

$$\frac{dT^+}{d\eta} = \frac{3^{1/n} \cdot 30}{f^{\frac{3n+1}{n}}}$$

Now eliminating $\frac{T^+}{f}$ from equations (9) and (10) we get

$$\frac{d^2 T^+}{d\eta^2} - \left[\frac{M\phi f}{30} + \frac{\varepsilon}{1-\eta\varepsilon} \right] \frac{3^{1/n} \cdot 30}{f^{\frac{3n+1}{n}}} = 0 \quad (11)$$

Equation (11) can be further written as

$$\frac{d^2 T^+}{d\eta^2} = \left[\frac{\varepsilon}{1-\eta\varepsilon} + P \right] Q \quad (12)$$

Where $P = \frac{M\phi f}{30}$ $Q = \frac{3^{1/n} \cdot 30}{f^{\frac{3n+1}{n}}}$

Using the mentioned end conditions solutions to equation (12) can be written as

$$T^+ = Q \left[\frac{1}{\varepsilon} \ln \frac{(1-\eta\varepsilon)}{(1-\varepsilon)} - \ln \frac{(1-\eta\varepsilon)^\eta}{(1-\eta)} \right] + Q(\eta-1) + \frac{PQ}{2}(\eta^2-1) + 1 \quad (13)$$

For a cylindrical fin, $\varepsilon = 0$ the equation reduces to

$$T^+ = Q(\eta^2-1) + \frac{PQ}{2}(\eta^2-1)$$

Heat extracted by an infinite length fin is given by

$$Q = -\frac{k_w A}{L} \frac{dT^+}{d\eta} \quad (14)$$

Results and Discussions

The conjugate problem of a parabolic profiled fin in non-Newtonian fluid has been formulated by making certain assumptions. Hence, the validity of these equations can be checked by comparing them with earlier investigations.

Initially this problem can be converted in to a short cylindrical fin dissipating heat to air by substituting $Pr=0.7$ and $n=1$ in equations (4) (5) and (7). The temperature distribution vs space variable obtained in the present analysis is compared with that of ref (4).in figure (2). The reasonable agreement between the two signifies the validity of the present integral formulation.

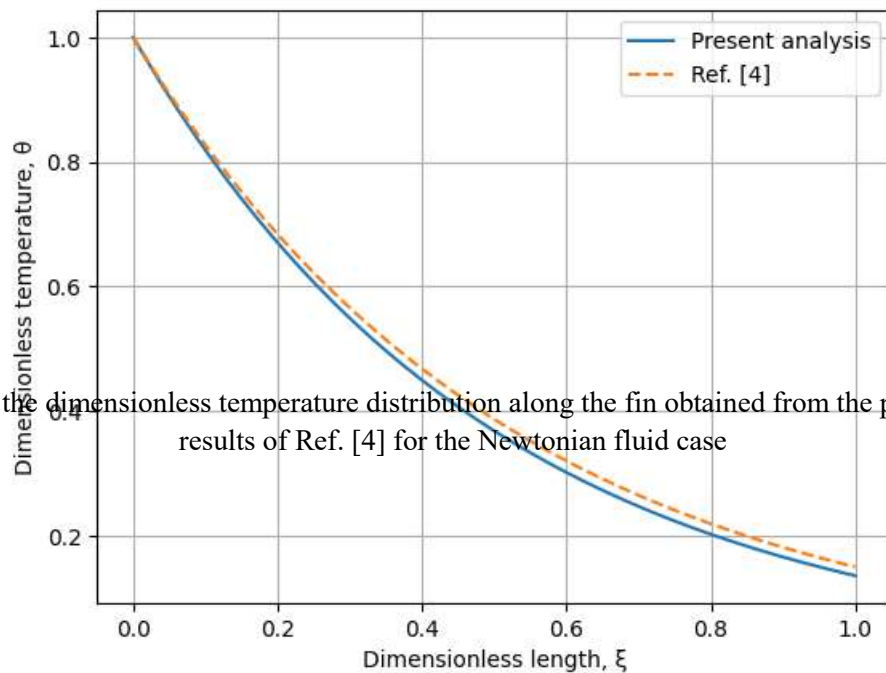


Fig. 2. Comparison of the dimensionless temperature distribution along the fin obtained from the present analysis with the results of Ref. [4] for the Newtonian fluid case

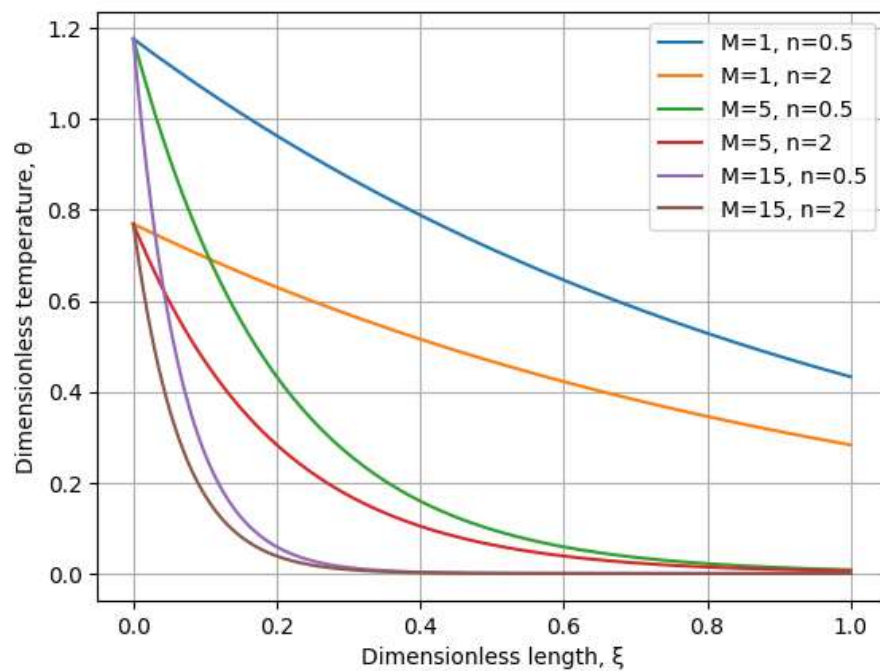


Fig. 3. Variation of dimensionless surface temperature along the fin for different fin parameters (M) and flow behavior indices

Fig-3 shows for three fin parameters of $M=1, 5$ and 15 the variation in surface temperature in the pseudo plastic and dilatant medium with $n=0.5$ and 2 respectively along the space variable. It is inferred from the figure that as the flow behavior index increases the temperature profiles alter. At a given location as n increases the dimensionless temperature decreases. Also, for a given index n as the fin parameter increases the temperatures are very much influenced resulting in decreasing in their magnitude.

Figs 4 and 5 depict the influence of ' n ' on the local heat transfer coefficients for different fin parameters (M). The non-monotonic variation in local heat transfer coefficients is very much clear from these figures. The heat transfer coefficient assumes infinite values at base, gradually decreases to a minimum value up to certain locations on the fin and there after the values are monotonically increasing.

Fig-6 compares the present analysis with different fin parameters with the experimental data of C.Tien et al (5). For 'M' tends to zero the average Nusselt number predicted by the theory is in good agreement with the experimental data. Also, the fin parameter seems to exert influence on mean Nusselt number reasonable agreement between the theory and experimental data show that in laminar flow the variation of flow properties of the fluid as a function of temperature might contribute very little to any change in Nusselt number.

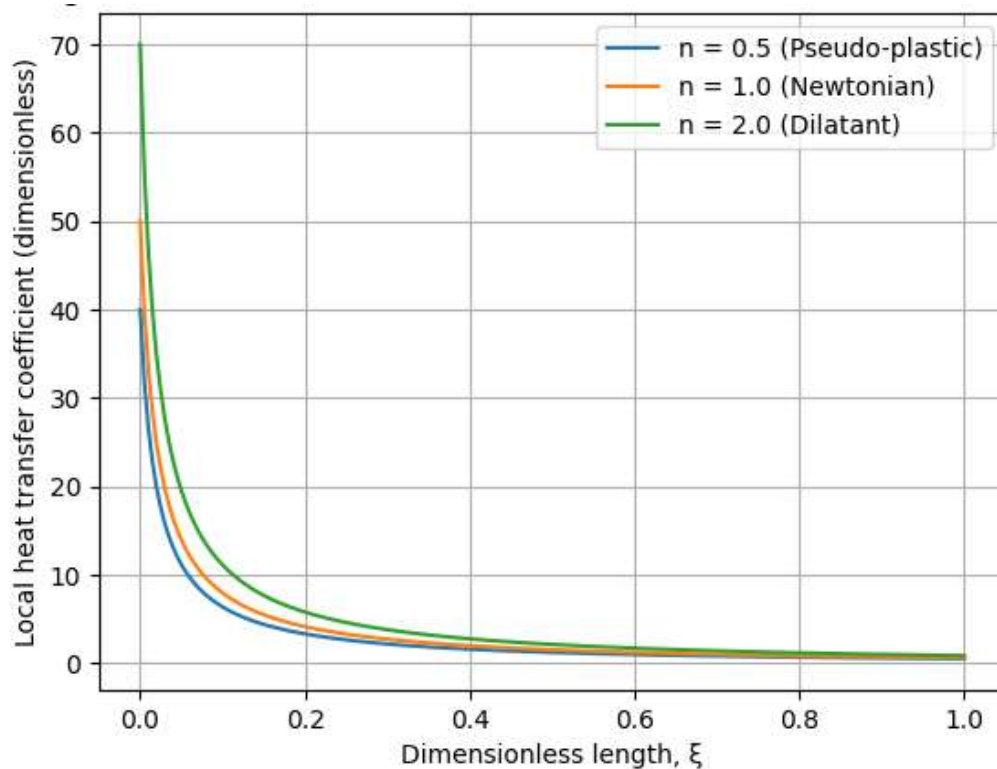


Fig. 4. Non-monotonic variation of the local heat transfer coefficient along the fin for different fin parameters under natural convection conditions.

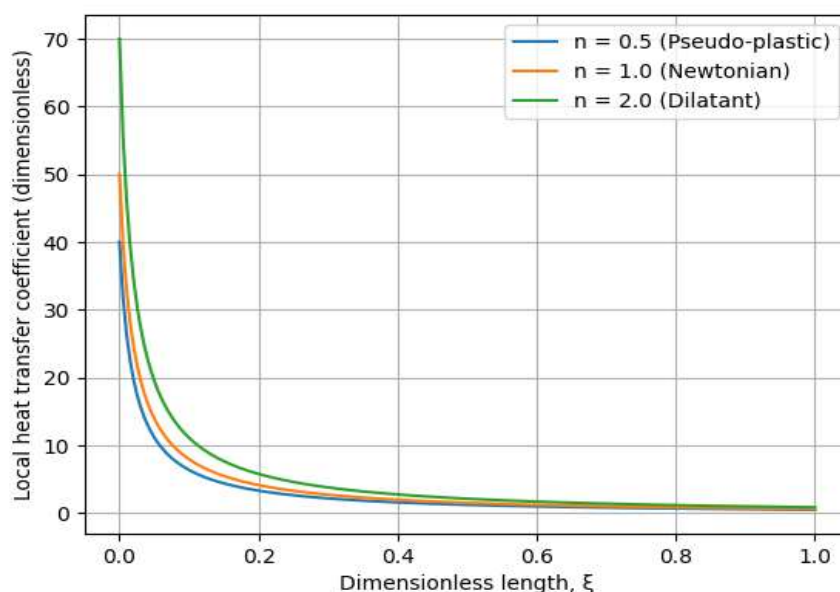


Fig. 5. Effect of flow behavior index (n) on the local heat transfer coefficient along the fin for non-Newtonian fluids.

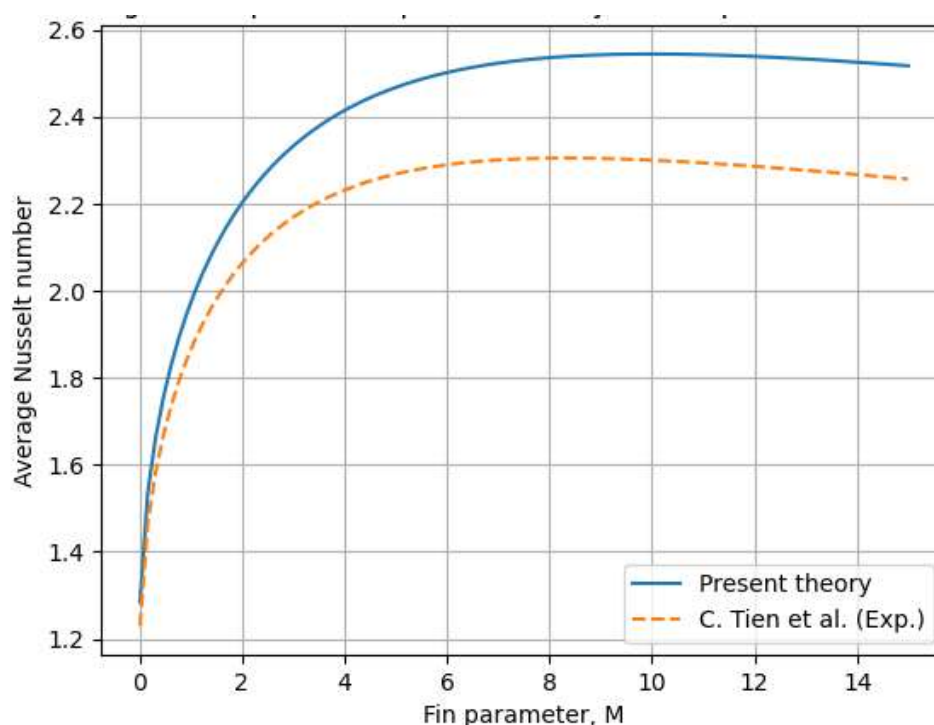


Fig. 6. Comparison of the average Nusselt number predicted by the present theory with the experimental data of C. Tien et al. for different fin parameters

Conclusions

1. “A comprehensive theoretical model has been developed to analyze **natural convective heat transfer from a parabolic profiled fin** operating in a **quiescent non-Newtonian fluid**. The non-Newtonian behaviour of the surrounding medium has been effectively represented using the **power-law model**, enabling the study of both shear-thinning and shear-thickening fluids.
2. **Generalized analytical expressions** for the temperature distribution along the fin length and the corresponding heat dissipation rate have been successfully derived. These expressions provide a reliable means of predicting fin performance under a wide range of operating and fluid conditions.
3. The analysis demonstrates that the **flow behaviour index (n)** has a pronounced influence on the thermal characteristics of the fin. An increase in the flow behaviour index significantly alters the temperature distribution and heat transfer coefficients, confirming the strong coupling between fluid rheology and buoyancy-driven convection.
4. The **fin parameter (M)**, which combines the effects of fin geometry, material properties, and convection strength, plays a crucial role in determining the overall heat transfer rate. Higher values of the fin parameter led to enhanced heat dissipation but also modify the temperature gradients along the fin.
5. “The study reveals that the **local heat transfer coefficient exhibits non-monotonic variation** along the fin length. The coefficient assumes very high values near the fin base, decreases to a minimum at an intermediate location, and then increases towards the fin tip, highlighting the complex interaction between conduction within the fin and natural convection in non-Newtonian fluids”.
6. It is observed that for **larger values of the modified Rayleigh number**, the thermal performance of the fin becomes increasingly dependent on both the **flow behavior index (n)** and the **fin parameter (M)**. This emphasizes the need to consider fluid rheology explicitly in the thermal design of extended surfaces.
7. The dimensionless heat transfer coefficient is found to be primarily governed by the **modified Rayleigh number**, indicating that buoyancy effects dominate the heat transfer process in laminar natural convection regimes for non-Newtonian fluids.
8. Validation of the present formulation through comparison with earlier theoretical and experimental studies shows **good agreement**, thereby establishing the accuracy and reliability of the proposed model. Notably, the present

analysis reduces to earlier Newtonian fluid solutions as a special case when $n = 1$.

9. The results of this study provide valuable insights for the **optimal design of fins** used in applications involving non-Newtonian fluids, such as polymer processing, food engineering, chemical processing, and bio-thermal systems, where conventional Newtonian assumptions may lead to inaccurate predictions.

10. “Overall, the present work offers a **generalized and unified framework** for predicting heat transfer from profiled fins in non-Newtonian environments and can serve as a useful reference for future analytical and experimental investigations in this area” The dimensionless heat transfer coefficient depends on the modified Rayleigh number.

Nomenclature

A: Cross sectional area of the fin in sq m at any section,

L: Length of fin in meters

M: Fin parameter

P_x : Perimeter of fin at any section ‘X’

Pr^* : Modified Prandtl number

Q: Heat dissipation at fin base

Ra^* : Modified Rayleigh number

T: Temperature on the surface of the fin at any point

T^* : Dimensionless temperature

U_x : Characteristic velocity

u, v: x and y components of velocity respectively

f: Functional variable

g: Acceleration due to gravity

h: Local heat transfer coefficient

n: flow behavior index,

N: Profile parameter

Greek symbols

α : Thermal diffusivity

β : Coefficient of thermal expansion

δ : Thickness of boundary layer

μ : Consistency index for power law

ρ : Density of the fluid

ν : Kinematics viscosity

η : Dimensionless space variable

θ_w : Temperature difference at any point

ε : Taper ratio parameter.

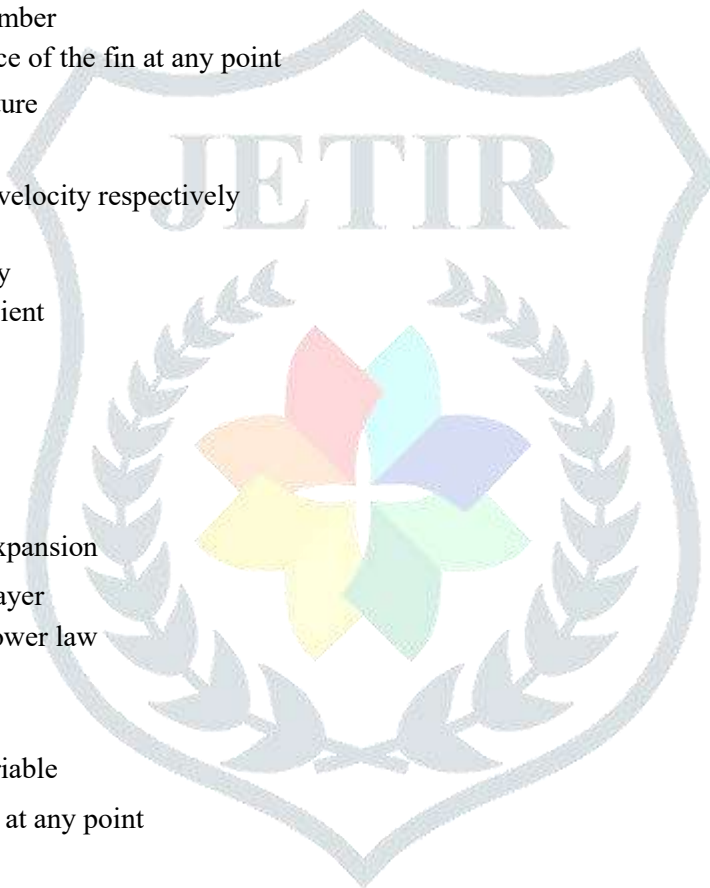
Subscripts

∞ : Of the medium,

L: Of the fluid

W: Surface of the fin

W_l : Base of the fin



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