

# Distributed Intuitionistic Fuzzy $\omega$ -Finite State Automata

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**Abstract:** In this paper the notion of distributed intuitionistic fuzzy  $\omega$ -finite state automata is introduced with different modes of acceptance along with different acceptance criteria.

**Keywords and Phrases:** Intuitionistic Fuzzy  $\omega$ -Automata, Distributed Intuitionistic Fuzzy  $\omega$ -Automata.

## I. INTRODUCTION

An  $\omega$ -language is a collection of infinite strings over a finite alphabet, an  $\omega$ -machine is any device capable of processing these input strings. The notion of intuitionistic fuzzy  $\omega$ -automata has been introduced in [8]. In section 2 some elementary and preliminary definitions are discussed. In section 3 distributed intuitionistic fuzzy  $\omega$ -finite state automata is introduced with different modes of acceptance along with different acceptance criteria. It also proved that distributed intuitionistic fuzzy  $\omega$ -finite state automata accept the same set of languages as intuitionistic fuzzy  $\omega$ -finite state automata in t-mode and the paper concludes with section 4.

## II. PRELIMINARIES

### Definition 2.1 Deterministic Finite Automaton (DFA)

A deterministic finite automaton is a quintuple  $M = (Q, X, \delta, q_0, F)$ , where,

1.  $Q$  is a finite set of states,
2.  $X$  is a finite input alphabet,
3.  $\delta : Q \times X \rightarrow Q$  is the transition function,
4.  $q_0$  in  $Q$  is the initial state, and
5.  $F \subseteq Q$  is the set of final states.

### Definition 2.2 Fuzzy $\omega$ -Automata

A fuzzy  $\omega$ -finite state automaton is an 5-tuple  $M = (Q, X, \delta, i, \text{Acc})$ , where

1.  $Q$  is a finite set of states,
2.  $X$  is finite input alphabets,
3.  $\delta$  is a fuzzy state transition function defined as  $\delta : Q \times X \times Q \rightarrow [0, 1]$ ,
4.  $i$  is an initial distribution function over  $Q$ , ie,  $i : Q \rightarrow [0, 1]$  is a function,
5.  $\text{Acc}$  is the acceptance criterion.

### Definition 2.3. Intuitionistic Fuzzy $\omega$ -Finite State Automata

An intuitionistic fuzzy  $\omega$ -finite state automaton is an 5-tuple  $M = (Q, X, i = (i_1, i_2), \delta = (\delta_1, \delta_2), \text{Acc})$ , where

1.  $Q$  is a finite set of states,
2.  $X$  is a finite input alphabets,
3.  $i = (i_1, i_2)$  is an initial distribution function over  $Q$ ,
4.  $\delta = (\delta_1, \delta_2)$  is an intuitionistic fuzzy state transition function defined as

$$\delta_1 : Q \times X \times 2^Q \rightarrow [0, 1]$$

$$\delta_2 : Q \times X \times 2^Q \rightarrow [0, 1]$$

such that  $\forall q, p \in Q, \forall x, y \in X$

$$\delta_1(q, \lambda, p) = \begin{cases} 1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

$$\delta_2(q, \lambda, p) = \begin{cases} 1 & \text{if } q \neq p \\ 0 & \text{if } q = p \end{cases}$$

$$\delta_1(q, xy, p) = \vee \{ \delta_1(q, x, r) \wedge \delta_1(r, y, p) : r \in Q \}$$
 and

$$\delta_2(q, xy, p) = \wedge \{ \delta_2(q, x, r) \vee \delta_2(r, y, p) : r \in Q \},$$

5.  $\text{Acc}$  is the acceptance criterion.

## III. DISTRIBUTED INTUITIONISTIC FUZZY $\omega$ -FINITE STATE AUTOMATA

**Definition 3.1** A Distributed intuitionistic fuzzy  $n$ - $\omega$ -finite state automaton is an 6-tuple  $M = (Q, X, i = (i_1, i_2), \delta, \mathcal{F}, \text{Acc})$ , where

1.  $Q$  is an  $n$ -tuple  $(Q_1, Q_2, \dots, Q_n)$ , where each  $Q_i$  is a set of states.
2.  $X$  is the finite set of alphabets.
3.  $\delta$  is an  $n$ -tuple  $((\delta_{11}, \delta_{21}), (\delta_{12}, \delta_{22}), (\delta_{13}, \delta_{23}) \dots)$  of state transition intuitionistic fuzzy functions where each  $\delta_{1i}, \delta_{2i}$  is defined as

$$\delta_{1i} : Q_i \times X \times 2^{Q_{union}} \rightarrow [0, 1],$$

$$\delta_{2i} : Q_i \times X \times 2^{Q_{union}} \rightarrow [0, 1], 1 \leq i \leq n$$

4.  $i = (i_1, i_2)$  is an initial distribution function over  $Q$ , that is

$$i_1 : Q \rightarrow [0, 1]$$

$i_2 : Q \rightarrow [0, 1]$  is a function.

5.  $f$  is an  $n$ -tuple  $((f_{11}, f_{21}), (f_{12}, f_{22}), (f_{13}, f_{23}) \dots)$ , where each

$f_i = (f_{1i}, f_{2i})$  is a function of the form,

$$f_{1i} : N \rightarrow [0, 1], f_{2i} : N \rightarrow [0, 1],$$

where  $N = \{1, 2, \dots, n\}$  and  $f_{1i}(i) = 1, f_{2i}(i) = 0,$

6. Acc is the acceptance criteria, where  $Q_{union} = \cup_i Q_i$

**Definition 3.2** Each of the components of the intuitionistic fuzzy  $n$ -  $\omega$ -finite state automaton is of the form  $M_i = (Q_i, X, \delta = (\delta_{1i}, \delta_{2i}))$ ,  $1 \leq i \leq n$ . Note that here  $Q_i$ 's need not be disjoint. In this system, consider many modes of acceptance depending upon the number of steps the system has to go through in each of the  $n$  -components. The different modes of acceptance are  $t$ -mode,  $*$ -mode,  $<k$  -mode,  $>k$  -mode and  $=k$  -mode.

**Definition 3.3** Consider the  $\omega$  -word  $\alpha = \alpha(0) \alpha(1) \dots$  with  $\alpha(i) \in M$ . A run  $M$  on  $\alpha$  is a sequence  $\rho = \rho(0) \rho(1) \dots \in Q_{union}^{\omega}$  such that  $\rho(0) = q_0$  for some  $q_0 \in Q_{union}$  with  $0 < i_1(q_0) + i_2(q_0) < 1$  and

$$\delta_{1j}(\rho(i), \alpha(i), \rho(i+1)) = \mu_{i+1}$$

$$\delta_{2j}(\rho(i), \alpha(i), \rho(i+1)) = \gamma_{i+1} \text{ for } i \geq 0 \text{ and } 1 \leq j \leq n, 0 \leq \mu_{i+1} + \gamma_{i+1} \leq 1.$$

**Definition 3.4** Let  $\mathcal{R}$  denote the set of all different runs of  $M$  on  $\alpha$ . Associated with each run  $\rho$  are four sequences,

$$f = f(0) f(1) \dots \in [0, 1]^{\omega}, \text{ where } f(0) = i_1(q_0), f(i) = \mu_i$$

$$g = g(0) g(1) \dots \in [0, 1]^{\omega}, \text{ where } g(0) = i_2(q_0), g(i) = \gamma_i$$

$$s = s(0) s(1) \dots \in [0, 1]^{\omega}, \text{ where } s(0) = 1, s(i) = f_{1i}(l)$$

$$\text{and } t = t(0) t(1) \dots \in [0, 1]^{\omega}, \text{ where } t(0) = 0, t(i) = f_{2i}(l) \text{ of the automaton changes component from } i \text{ to } l, 1 \leq i, l \leq n.$$

Description of each of the modes of acceptance is as follows:

**$t$  -mode Acceptance:**

Initially, the automaton which has a state  $q$ ,  $0 < i_1(q_0) + i_2(q_0) < 1$  begins processing the input string. Assume that the system starts from the component  $i$ . In component  $i$  the system follows its transition function as any "stand-alone".

$\omega$ -finite state automaton. The control is transferred from the component  $i$  to component  $j$  only if the system arrives at a state  $q \notin Q_i$  and  $q \in Q_j$  with an intuitionistic fuzzy value of  $(f_{1i}(j), f_{2i}(j))$ .

The process is repeated infinitely many times and accepted the string if the run of  $M$  over the input word satisfies Acc.

**Definition 3.5** The instantaneous description of the  $n$ -intuitionistic-fuzzy-  $\omega$  -finite state automaton (ID) is given by a 3-tuple  $(q, w, i)$ , where  $q \in Q_{union}, w \in X^{\omega}, 1 \leq i \leq n$ .

In this ID of the  $n$ -intuitionistic-fuzzy-  $\omega$  -finite state automaton,  $q$  denotes the current state of the whole system,  $w$  the portion of the input string yet to be read and  $i$  the index of the component in which the system is currently in. The transition between the ID's is defined as follows:

1.  $(q, aw, i) \vdash (q', w, i)$  iff  $\delta_{1i}(q, a, q') = \mu \in [0, 1]$  and  $\delta_{2i}(q, a, q') = \gamma \in [0, 1]$ , where  $q, q' \in Q_i, a \in X, w \in X^{\omega}, 1 \leq i \leq n$ .
2.  $(q, aw, i) \vdash (q', w, j)$  iff  $\delta_{1i}(q, a, q') = \mu' \in [0, 1]$  and  $\delta_{2i}(q, a, q') = \gamma' \in [0, 1]$ , where  $q \in Q_i, q' \in Q_j \setminus Q_i, a \in X, w \in X^{\omega}, 1 \leq i \leq n$  and  $\mu = \min\{\mu', f_{1i}(j)\}$  and  $\gamma = \max\{\gamma', f_{2i}(j)\}$ .

Let  $\vdash$  be the reflexive and transitive closure of  $\vdash$ .

**\*-mode Acceptance:**

Initially, the automaton which has a state  $q$ ,  $0 < i_1(q_0) + i_2(q_0) < 1$  begins processing the input string. Suppose the system starts the processing from the component  $i$ . Unlike the termination mode, here there is no restriction. The automaton can transfer the control to any of the components at any time if possible, that is, if there is some  $j$  such that  $q \in Q_j$  then the system can transfer the control to the component  $j$  with intuitionistic fuzzy value of  $(f_{1i}(j), f_{2i}(j))$ . The instantaneous description in \*-mode can be defined analogously.

**=  $k$  -mode ( $<k$  -mode,  $>k$  -mode) Acceptance:**

Initially, the automaton which has a state  $q$ ,  $0 < i_1(q_0) + i_2(q_0) < 1$  begins processing the input string. Suppose the system starts the processing from the component  $i$ . The system transfers the control to the other component  $j$  only after the completion of exactly  $k$  ( $k'(k' < k), k'(k' > k)$ ) number of steps in the component  $i$ , that is, if there is a state  $q \in Q_j$  then the transition from component  $i$  to the component  $j$  takes place only if the system has already completed  $k$  ( $k'(k' < k), k'(k' > k)$ ) steps in component  $i$ .

The instantaneous description of the  $n$  -intuitionistic-fuzzy-  $\omega$  -finite state automaton in the above three modes of derivations is defined as follows:

**Definition 3.6** The instantaneous description of the  $n$  -intuitionistic-fuzzy-  $\omega$  -finite state automaton (ID) is given by a 4-tuple  $(q, w, i, j)$ , where  $q \in Q_{union}, w \in X^{\omega}, 1 \leq i \leq n, j$  is a non-negative integer.

In this ID of the  $n$ -intuitionistic-fuzzy-  $\omega$  -finite state automaton,  $q$  denotes the current state of the whole system,  $w$  the portion of the input string yet to be read;  $i$  the index of the component in which the system is currently in and  $j$  denotes the number of steps for which the system has been in the  $i^{\text{th}}$  component.

- Let  $\rho' = \rho'(0) \rho'(1) \dots$  be an infinite sequence of elements of  $\rho$ , where  $\rho'(i) = q$  is such that  $(q, w, j, k), q \in Q_{union}, w \in X^{\omega}, 1 \leq j \leq n$  is an instantaneous description of  $M$ , that is,  $\rho'(0) = \rho(k), \rho'(1) = \rho(2k + 1)$  and so on.  $\rho'$  is a subsequence of  $\rho$  taking every  $k^{\text{th}}$  element of  $\rho$  starting from  $\rho(k)$ .
- Any state reached in the run  $\rho$  of the input alphabet  $\alpha$  is a candidate for a final state of the automaton working in the  $<k$  mode.
- Let  $\rho' = \rho'(0) \rho'(1) \dots$  be an infinite sequence of elements of  $\rho$ , where  $\rho'(i) = q$  is such that  $(q, w, j, l), q \in Q_{union}, w \in X^{\omega}, 1 \leq j \leq n, l \geq k$  is an instantaneous description of  $M$ , that is,  $\rho'(0) = \rho(l_0), l_0 = k, \rho'(1) = \rho(l_1), l_1 = l_0 + 1$  if the automaton continues in the same component or  $l_1 = l_0 + k + 1$  if the automaton changes its component and so on.

**Definition 3.7** Depending on  $In(\rho)$  for  $*$ ,  $t$ ,  $< k$  and  $In(\rho')$  for the  $= k$  and  $> k$  modes and have the following acceptance criteria along with the method of calculating the membership and non-membership value of the accepted string.

1. Büchi Condition:

$In(\rho) \cap F \neq \emptyset$  for an intuitionistic fuzzy final states  $F \subseteq Q_{union}$ . Then membership and non-membership value of an accepted string  $\alpha$  is calculated as follows:

$$\mu(\alpha) = \max_{\rho \in \mathfrak{R}}(\min \{ \text{distinct}_{\rho}(f), \text{distinct}_{\rho}(s), \mu_F(q), \text{ where } q \in In(\rho) \cap F \}) \text{ and}$$

$$\gamma(\alpha) = \min_{\rho \in \mathfrak{R}}(\max \{ \text{distinct}_{\rho}(g), \text{distinct}_{\rho}(t), \gamma_F(q), \text{ where } q \in In(\rho) \cap F \}).$$

2. Muller Condition:

Let  $\mathcal{F}$  be a family of intuitionistic fuzzy final states if  $\bigwedge In(\rho) = F$  where  $F \subseteq \mathcal{F}$  then the membership and non-membership value of an accepted string  $\alpha$  is calculated as follows:

$$\mu(\alpha) = \max_{\rho \in \mathfrak{R}}(\min \{ \text{distinct}_{\rho}(f), \text{distinct}_{\rho}(s), \mu_F(q), \text{ where } q \in F \}) \text{ and}$$

$$\gamma(\alpha) = \min_{\rho \in \mathfrak{R}}(\max \{ \text{distinct}_{\rho}(g), \text{distinct}_{\rho}(t), \gamma_F(q), \text{ where } q \in F \}).$$

3. Rabin Condition:

For a sequence  $\Omega$  of “accepting pairs”  $(E_1, F_1), (E_2, F_2) \dots (E_m, F_m)$  with  $E_i, F_i \subseteq Q_{union}$  being intuitionistic fuzzy sets. The membership and non-membership value of an accepted string  $\alpha$  is calculated as follows:

$$\mu(\alpha) = \max_{\rho \in \mathfrak{R}}(\min \{ \text{distinct}_{\rho}(f), \text{distinct}_{\rho}(s), \max(\min \{ \mu_{E_i}(q), \mu_{F_i}(p), \text{ where } q \in In^c(\rho) \cap E_i, p \in In(\rho) \cap F_i \}) \})$$

and

$$\gamma(\alpha) = \min_{\rho \in \mathfrak{R}}(\max \{ \text{distinct}_{\rho}(g), \text{distinct}_{\rho}(t), \min(\max \{ \gamma_{E_i}(q), \gamma_{F_i}(p), \text{ where } q \in In^c(\rho) \cap E_i, p \in In(\rho) \cap F_i \}) \})$$

4. Streett Condition:

For a sequence  $\Omega$  of pairs  $(E_1, F_1) \dots (E_m, F_m)$  with  $E_i, F_i \subseteq Q_{union}$  being intuitionistic fuzzy sets. The membership and non-membership value of an accepted string  $\alpha$  is calculated as follows:

$$\mu(\alpha) = \max_{\rho \in \mathfrak{R}}(\min \{ \text{distinct}_{\rho}(f), \text{distinct}_{\rho}(s), \min(\max \{ \mu_{E_i}(q), \mu_{F_i}(p), \text{ where } q \in (In^c(\rho) \cap E_i) \cup (In(\rho) \cap E_i),$$

$$p \in (In(\rho) \cap F_i) \cup (In^c(\rho) \cap F_i) \}) \})$$

and

$$\gamma(\alpha) = \min_{\rho \in \mathfrak{R}}(\max \{ \text{distinct}_{\rho}(g), \text{distinct}_{\rho}(t), \max(\min \{ \gamma_{E_i}(q), \gamma_{F_i}(p), \text{ where } q \in (In^c(\rho) \cap E_i) \cup (In(\rho) \cap E_i), p \in$$

$$(In(\rho) \cap F_i) \cup (In^c(\rho) \cap F_i) \}) \})$$

where  $In(\rho)$  is replaced by  $In(\rho')$  for the  $= k$  and  $> k$  modes of acceptance.

**Example 3.1** Consider the distribution intuitionistic fuzzy  $w$ -finite state automaton  $M = (Q, X, \delta, i = (i_1, i_2), \mathcal{F})$ , where

$$Q = (\{q_a, q_a'\}, \{q_b, q_b'\}, \{q_c\}, \{q_c'\}),$$

$$X = \{a, b, c\}$$

$$\delta = ((\delta_{11}, \delta_{21}), (\delta_{12}, \delta_{22}), (\delta_{13}, \delta_{23}), (\delta_{14}, \delta_{24}))$$

where  $\delta_{1i}, \delta_{2i}, 1 \leq i \leq 4$  is defined as follows:

|                                    |                                    |
|------------------------------------|------------------------------------|
| $\delta_{11}(q_a, a, q_a') = 1.0$  | $\delta_{21}(q_a, a, q_a') = 0$    |
| $\delta_{11}(q_a', a, q_a) = 1.0$  | $\delta_{21}(q_a', a, q_a) = 0$    |
| $\delta_{11}(q_a', b, q_b') = 0.9$ | $\delta_{21}(q_a', b, q_b') = 0.1$ |
| $\delta_{11}(q_a, b, q_b') = 1.0$  | $\delta_{21}(q_a, b, q_b') = 0$    |
| $\delta_{12}(q_b, b, q_b') = 1.0$  | $\delta_{22}(q_b, b, q_b') = 0$    |
| $\delta_{12}(q_b', b, q_b) = 1.0$  | $\delta_{22}(q_b', b, q_b) = 0$    |
| $\delta_{12}(q_b', c, q_c) = 0.9$  | $\delta_{22}(q_b', c, q_c) = 0.1$  |
| $\delta_{12}(q_b, c, q_c) = 1.0$   | $\delta_{22}(q_b, c, q_c) = 0$     |
| $\delta_{13}(q_c, c, q_c) = 1.0$   | $\delta_{23}(q_c, c, q_c) = 0$     |
| $\delta_{13}(q_c, c, q_c') = 1.0$  | $\delta_{23}(q_c, c, q_c') = 0$    |
| $\delta_{14}(q_c', c, q_c') = 1.0$ | $\delta_{24}(q_c', c, q_c') = 0$   |
| $\delta_{14}(q_c', c, q_c) = 1.0$  | $\delta_{24}(q_c', c, q_c) = 0$    |

$$i_1(p) = \begin{cases} 1, & \text{if } p = q_a \\ 0, & \text{otherwise} \end{cases}$$

$$i_2(p) = \begin{cases} 1, & \text{if } p \neq q_a \\ 0, & \text{otherwise} \end{cases}$$

$$F = \{q_c, q_c'\},$$

$$\mu_F(q_c) = 1, \mu_F(q_c') = 1$$

$$\gamma_F(q_c) = 0, \gamma_F(q_c') = 0.$$

The language accepted by this system in the  $\geq 2$  and  $*$  modes is given by:

$$L = \begin{cases} a^{2n}b^{2m}c^w, \mu(a^{2n}b^{2m}c^w) = 1.0, \gamma(a^{2n}b^{2m}c^w) = 0, & m, n \in Z^+ \\ a^n b^m c^w, \mu(a^n b^m c^w) = 0.9, \gamma(a^n b^m c^w) = 0.1, & \text{not both } n \text{ and } m \text{ even.} \end{cases}$$

**Theorem 3.1** For any  $n$ -intuitionistic-fuzzy- $\omega$ -finite state automaton  $M$  working in  $t$ -mode and accepting the intuitionistic fuzzy language  $L(M)$  and having a corresponding  $\omega$ -intuitionistic-fuzzy-finite state automaton accepting  $L(M)$ .

*Proof:* Let  $M = (Q, X, \delta, i = (i_1, i_2), \mathcal{F}, \text{Acc})$  be an  $n$ -intuitionistic-fuzzy- $\omega$ -finite state automaton working in  $t$ -mode, where  $\delta = ((\delta_{11}, \delta_{21}), (\delta_{12}, \delta_{22}), (\delta_{13}, \delta_{23}) \cdots (\delta_{1n}, \delta_{2n}))$ , the components have states  $Q_1, Q_2, \dots, Q_n$  and  $\mathcal{F}$  is an  $n$ -tuple  $((\mathcal{F}_{11}, \mathcal{F}_{21}), (\mathcal{F}_{12}, \mathcal{F}_{22}) \cdots (\mathcal{F}_{1n}, \mathcal{F}_{2n}))$  of intuitionistic fuzzy functions.

Consider the intuitionistic fuzzy  $\omega$ -finite state automaton

$M' = (Q', X, \delta' = (\delta'_{1i}, \delta'_{2i}), i' = (i'_1, i'_2), \text{Acc})$ , where

$Q' = \{[q, i] \mid q \in Q_{union}, 1 \leq i \leq n\} \cup Q_{union}$ .

$\delta'$  contains the following transitions: for each

$\delta'_{1i}(q_j, a, q_k) = \mu$  and  $\delta'_{2i}(q_j, a, q_k) = \gamma, q_j \in Q_i, a \in X, 1 \leq i \leq n$ .

1.  $\delta'_{1i}(q, a, [q, i]) = \mu'$  iff  $\delta_{1i}(q, a, q) = \mu'$  and  $\delta'_{2i}(q, a, [q, i]) = \gamma'$  iff  $\delta_{2i}(q, a, q) = \gamma'$ ,

2. If  $q_k \in Q_i$  then  $\delta'_{1i}([q_j, i], a, [q_k, i]) = \mu$  and  $\delta'_{2i}([q_j, i], a, [q_k, i]) = \gamma$ ,

3. If  $q_k \in Q_j \setminus Q_i$  then  $\delta'_{1i}([q_j, i], a, [q_k, j]) = \mu', \delta'_{2i}([q_j, i], a, [q_k, j]) = \gamma', 1 \leq j \leq n, \mu' = \min\{\mu, \mathcal{F}_{1i}(j)\}$  and  $\gamma' = \max\{\gamma, \mathcal{F}_{2i}(j)\}$ .

$i'_1(q) = \begin{cases} i_1(q), & \text{if } q \in Q_{union} \\ 0, & \text{otherwise} \end{cases}$   $i'_2(q) = \begin{cases} i_2(q), & \text{if } q \notin Q_{union} \\ 0, & \text{otherwise} \end{cases}$

Depending on the difference acceptance criteria and established the following:

$M$  is a Büchi automaton with  $F$  the set of final states. Then  $M'$  is a Büchi automaton with  $F' = \{[q, i] \mid q \in F \text{ and } 1 \leq i \leq n\}, \mu_{F'}([q, j]) = \mu_F(q)$  and  $\gamma_{F'}([q, j]) = \gamma_F(q), 1 \leq j \leq n$  accepting  $L(M)$ .

Consider the  $\omega$ -word  $\alpha = \alpha(0) \alpha(1) \cdots$  with  $\alpha(i) \in X$ . A run of  $M$  on  $\alpha$  is a sequence  $\rho = \rho(0) \rho(1) \cdots \in Q_{union}^w$  such that  $\delta_{1i}(\rho(i), \alpha(i), \rho(i+1)) = \mu_{i+1}$  and  $\delta_{2i}(\rho(i), \alpha(i), \rho(i+1)) = \gamma_{i+1}$  for  $i \geq 0$  and  $1 \leq i \leq n$ , with  $0 < \mu_{i+1} + \gamma_{i+1} \leq 1$ .

Let the run of  $M'$  on  $\alpha$  be  $\rho' = \rho'(0) \rho'(1) \cdots \in Q_{union}^w$ . If  $q \in In(\rho) \cap F$  then some  $j, 1 \leq j \leq n$  is such that  $[q, j] \in Q'$  and  $[q, j] \in In(\rho') \cap F'$  since  $j$  can take only finite values.

From the construction it is clear that the membership and non-membership value of the accepted string is the same as in the distributed automaton. Thus  $\alpha$  is accepted by  $M'$ .

$M$  is a Muller automaton with respect to the family  $\mathcal{F} \in 2^Q$  of final state sets. This means that the set of states assumed infinitely often in a run  $\rho$  forms a set in  $\mathcal{F}$ . For any set  $T = \{q_1, q_2, \dots, q_l\} \in \mathcal{F}$  define the collection  $S_i = 2^{\{[q_i, j] \mid 1 \leq j \leq n\}} \setminus \emptyset$ .

For any set  $s_i \in S_i, \mu_{s_i}([q_i, j]) = \mu_T(q_i)$  and  $\gamma_{s_i}([q_i, j]) = \gamma_T(q_i), 1 \leq j \leq n, 1 \leq i \leq n$  let

$$\mathcal{F}' = \bigcup_{T \in \mathcal{F}} \left\{ \bigcup_{1 \leq i \leq n} s_i \mid s_i \in S_i \right\}$$

Consider the  $\omega$ -word  $\alpha = \alpha(0) \alpha(1) \cdots$  with  $\alpha(i) \in M$ . A run of  $M$  on  $\alpha$  is a sequence  $\rho = \rho(0) \rho(1) \cdots \in Q_{union}^w$  such that  $\rho(0) = q_0$  and  $\delta_{1i}(\rho(i), \alpha(i), \rho(i+1)) = \mu_{i+1}$  and  $\delta_{2i}(\rho(i), \alpha(i), \rho(i+1)) = \gamma_{i+1}$  for  $i \geq 0$  and  $1 \leq i \leq n, 0 \leq \mu_{i+1} + \gamma_{i+1} \leq 1$ .

Now if  $\alpha$  is accepted by  $M$  then  $In(\rho)$  forms a set in  $\mathcal{F}$  say  $\{q_1, q_2, \dots, q_l\}$ . If  $\rho'$  is the run of  $\alpha$  on  $M'$  then

$In(\rho') = \{[q_1, 1], [q_1, 2], \dots, [q_1, n], [q_2, 1], [q_2, 2], \dots, [q_2, n], [q_l, 1], [q_l, 2], \dots, [q_l, n]\}$

which is clearly a set in  $\mathcal{F}'$  by construction. The membership and non-membership value of  $\alpha$  also remains the same. Thus  $\alpha$  is accepted by  $M'$ .

$M$  is a Rabin automaton with respect to the sequence  $\Omega$  of accepting pairs  $(E_1, F_1) \cdots (E_m, F_m)$  with  $E_i, F_i \in Q_{union}$ . For the automaton  $M'$  defined  $\Omega'$  as  $(E'_1, F'_1) \cdots (E'_m, F'_m)$ , where

$E'_i = \bigcup_{p \in E_i, 1 \leq j \leq n} \{[p, j]\}, \mu_{E'_i}([q, j]) = \mu_{E_i}(q), \gamma_{E'_i}([q, j]) = \gamma_{E_i}(q)$  and

$F'_i = \bigcup_{p \in F_i, 1 \leq j \leq n} \{[p, j]\}, \mu_{F'_i}([q, j]) = \mu_{F_i}(q)$  and  $\gamma_{F'_i}([q, j]) = \gamma_{F_i}(q)$ .

Let  $\rho$  and  $\rho'$  be the run of the word  $\alpha$  on the machines  $M$  and  $M'$ , respectively. If the condition  $(In(\rho) \cap E_i = \emptyset \wedge In(\rho) \cap F_i \neq \emptyset)$  holds for some  $i$  in  $\rho$  then the condition  $(In(\rho') \cap E'_i = \emptyset \wedge In(\rho') \cap F'_i \neq \emptyset)$  holds in  $\rho'$  implying that  $\bigvee_{i=1}^m (In(\rho') \cap E'_i = \emptyset \wedge In(\rho') \cap F'_i \neq \emptyset)$  holds for the sequence  $\Omega'$ . The membership and non-membership value of  $\alpha$  also remains the same. Thus  $\alpha$  is accepted by  $M'$ .

$M$  is a Streett automaton with respect to the sequence  $\Omega$  of accepting pairs  $(E_1, F_1) \cdots (E_m, F_m)$  with  $E_i, F_i \subseteq Q$ . For the automaton  $M'$  define  $\Omega'$  as  $(E'_1, F'_1) \cdots (E'_m, F'_m)$ , where

$E'_i = \bigcup_{p \in E_i, 1 \leq j \leq n} \{[p, j]\}, \mu_{E'_i}([q, j]) = \mu_{E_i}(q), \gamma_{E'_i}([q, j]) = \gamma_{E_i}(q)$  and

$F'_i = \bigcup_{p \in F_i, 1 \leq j \leq n} \{[p, j]\}, \mu_{F'_i}([q, j]) = \mu_{F_i}(q)$  and  $\gamma_{F'_i}([q, j]) = \gamma_{F_i}(q)$ .

Let  $\rho$  and  $\rho'$  be the run of the word  $\alpha$  on the machines  $M$  and  $M'$ , respectively. If the condition  $(In(\rho) \cap E_i = \emptyset \vee In(\rho) \cap F_i \neq \emptyset)$  holds for all  $i$  in  $\rho$  then the condition  $(In(\rho') \cap E'_i = \emptyset \vee In(\rho') \cap F'_i \neq \emptyset)$  holds in  $\rho'$  implying that  $\bigwedge_{i=1}^m (In(\rho') \cap E'_i = \emptyset \vee In(\rho') \cap F'_i \neq \emptyset)$  holds for the sequence  $\Omega'$ .

The membership and non-membership value of  $\alpha$  also remains the same. Thus  $\alpha$  is accepted by  $M'$ .

#### IV. CONCLUSION

In this paper, the notion of distributed intuitionistic fuzzy  $\omega$ -finite state automata is defined according to the use of their acceptance criterion.

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