

SINGLE IMAGE SUPER RESOLUTION USING WAVELET TRANSFORM & INTERPOLATION

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ABSTRACT: *The SR imaging has been one of the fundamental image processing research areas. In this work a super resolution technique based on Sparse characteristics of wavelet transform. The proposed method for resolution enhancement of image gives good qualitative, quantitative & visual results than that of some traditional & state-of-the-art methods. It has been observed that the proposed technique outperforms all other previous techniques. Hence, we proposed a wavelet based super-resolution technique, which will be of the category of interpolative methods, using sparse property of wavelets. It is based on sparse representation property of the wavelets.*

KEYWORDS: *Super Resolution, Image Reconstruction, Single Image Resolution Techniques, Resolution Enhancement, Wavelet transform, Interpolation.*

1. INTRODUCTION

Image super resolution is a image processing algorithms that produce high quality, high resolution (HR) images from a set of low quality, low resolution (LR) images or from a single image. The SR image reconstruction is useful in many practical cases where multiple frames of the same scene can be obtained, including medical imaging, satellite imaging, and video applications[17]. The basic premise for increasing the spatial resolution in SR techniques is the availability of multiple LR images captured from the same scene. The set of source low resolution (LR) images captures only a finite amount of information from a scene; the goal of SR is to extract the independent information from each image in that set and combine the information into a single high resolution (HR) image. The requirement is of SR is that each LR image must contain some information that is unique to that image[3]. The super resolution method is to take more samples of the scene so as to get some extra information which can be used, while merging the samples to get a high resolution image. These samples can be acquired by sub-pixel shifts, by changing scene, by changing the amount of blur [14]. HR means that pixel density within an image is high, and therefore an HR image can offer more details that are important in many applications, The major advantage of the super resolution approach is that it may cost less and the existing LR imaging systems can be still utilized. Synthetic zooming of region of interest (ROI) is another important application in surveillance, forensic, scientific, medical, and satellite imaging[13]. This application is most suitable for magnifying objects in the scene such as the face of a criminal or the license plate of a car [16].

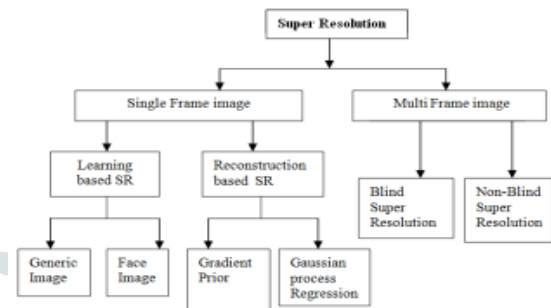


Figure 1: Classification of Super Resolution Techniques

2. VARIOUS EXISTING SUPER- RESOLUTION TECHNIQUES

2.1SR using compressive sensing based on a redundant dictionary

[1] Y. Sun Et-al, presented a compressive sensing based on a redundant dictionary has been successfully applied in super resolution imaging. However, due to the neglect of the local and nonlocal interactions of patches of a single image, the reconstructed results are not satisfactory in noise suppression and edge sharpness. In this paper, we propose an improved method by adding steering kernel regression and a nonlocal means filter as two regularization terms and use an efficient clustering sub-dictionary learning scheme. We further demonstrate better results on true images in terms of traditional image quality assessment metrics.

2.2 SR reconstruction based on the compressive sensing

[2] Y. Sun Et-al, presented an observation for medical imaging and astronomical, high-resolution (HR) images are urgently desired and required. In recent years, many researchers have proposed various ways to achieve the goal of image super-resolution (SR), ranging from simple linear interpolation schemes to nonlinear complex methods. In this paper, we deal with the SR reconstruction problem based on the theory of compressive sensing, which uses a redundant dictionary instead of a conventional orthogonal basis. We further demonstrate better results on true images in terms of peak signal-to-noise ratio (PSNR) and rootmean-square error (RMSE) and give several important improvements, compared with other methods.

Although wide variety of super-resolution literature is available, it is still an open topic to investigate. Following subsections describe some of the existing basic image super-resolution schemes.

2.3 SR via Sparse Representation

Jianchao Yang et al. [9] considered the sparse signal representation of an image. Based on previous research on image statistics the image patches can be well-represented as a sparse linear combination of elements from an appropriately chosen over-complete dictionary. Motivated by this, they proposed a sparse representation for each patch of the low-resolution input. The coefficients of this representation are used to generate the high-resolution output. Theoretical results from compressed sensing

suggest that the sparse representation can be correctly recovered from the down-sampled signals under mild conditions.

Similarity of sparse representations between the low-resolution and high-resolution images patch pair-resolution images without dictating any external training set. They proposed a framework for both magnification and deblurring using only the original low-resolution image and its blurred version. In This method, each pixel is predicted by its neighbors through the Gaussian process regression.

2.4 Nonlinear Mapping of Coherent Features

Xiao Zeng and Hua Huang presented a regression based method that can successfully recognize the identity given all these difficulties. They built a radial basis function in subspace by canonical correlation analysis to nonlinear regression models from the specific non frontal low resolution image to frontal high resolution features.

2.5 Geometric Grouplets

A. Maalouf and M. C. Larabi proposed the idea of generating a super-resolution (SR) image from a single multi-valued low-resolution (LR) input image. This problem approaches from the perspective of image geometry-oriented interpolation. They computed the grouplet transform to obtain geometry of the LR image. Geometric grouplets is constructed by orthogonal multiscale grouping with weighted Haar lifting to points grouped by association fields.

To preserve the sharpness of edges and textures SR image is synthesised by an adaptive directional interpolation using the extracted geometric information. This method showed improvements over existing geometrically driven interpolation techniques on a subjective scale, and in many cases with an improvement in psychovisual color difference.

2.6 Remotely Sensed image by Hopfield Neural Network

J Tatem Andrew et al. [12] used their idea of super-resolution for target identification in remotely sensed images. Fuzzy classification improves the accuracy of land cover target identification make robust and better for spatial representation of land cover. The Hopfield neural network converges to a minimum of an energy function, defined as a goal and several constraints. The energy minimum represents a best guess map of the spatial distribution of class components in each pixel.

They used two goal functions to make the output of a neuron similar to that of its neighboring neurons. The first goal function aims to increase the output of center neuron to 1. The second goal function aims to decrease the output of the center neuron to 0. They showed that, by using a Hopfield neural network, more accurate measures of land cover targets can be obtained compared with those determined using the proportion images alone.

2.7 Neural Network based Optimal Recovery Theory

YizhenHuang and Yangjing Long [13] proposed a optimal recoverybased neural-network Super Resolution algorithm. This method evaluated on classical SR test images with both generic and specialized training sets, and compared with other state-of-the-art methods.

Motivated by the idea that back propagation neural network are capable of learning complex nonlinear function they proposed a neural network approach that produces better results in high-frequency regions. They integrated an optimal recovery based approach with in a neural network framework and, if so, two different branches of algorithms complement each other to offer a better algorithm. Using this algorithm in a two-pass way generates visual results that are very similar regardless of the initial interpolation step, and more times of iteration only waste the computing resource but yield negligible performance gain.

2.8 Gaussian Process Regression

Wan-Chi Siu et-al, [14] addressed the problem of producing a high-resolution image from low-resolution images without dictating any external training set. They proposed a framework for both magnification and deblurring using only the original low-resolution image and its blurred version. In This method, each pixel is predicted by its neighbors through the Gaussian process regression.

They showed that, by using a proper covariance function, the Gaussian process regression can perform soft clustering of pixels based on their local structures. This algorithm can extract adequate information contained in a single low-resolution image to generate a high-resolution image with sharp edges. Compared to other edge-directed and example-based super-resolution algorithms this algorithm is superior in quality and performance.

2.9 Learning-based SR with a combining of both global and local constraints

K. Guo et al. [15] proposed a statistical learning method for SR with both global and local constraints. More specifically, they introduced a mixture model into maximum a posteriori (MAP) estimation, which combines a global parametric constraint with a patch-based local non-parametric constraint.

The global parametric constraint guarantees the super-resolved global image to agree with the sparse property of natural images, and the local non-parametric constraint is used to infer the residues between the image derived from the global constraint and the ground truth high-resolution (HR) image.

They compared it with traditional patch-based learning methods without the global constraint, and showed that this method can not only preserve global image structure, but also restore the local details more effectively.

2.10 Interpolation based SR using Multisurface Fitting

Fei Zhou et al. [16] proposed a interpolation-based method of image super-resolution reconstruction. They used the idea of multisurface fitting to take advantage of spatial structure information. Each site of low-resolution pixels is fitted with one surface, and the final estimation is made by fusing the multisampling values on these surfaces in the maximum a posteriori fashion. Figure 2 shows the flow chart of interpolation based SR using Multisurface Fitting.

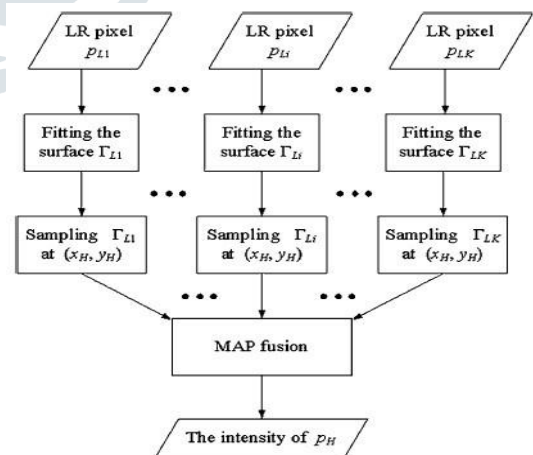


Figure 2: Flow chart of Multisurface Fitting

For the final values of high intensity pixels they used maximum a posteriori estimation on sampled surface constructed using Taylor series. They showed that, this method reconstructs high-resolution images that preserve image details effectively without any hypothesis on image prior. They extended this method to a more general noise model.

2.11 SR Based on Interpolation of Wavelet Domain

Gholamreza Anbarjafari and Hasan Demirel [17] proposed a super-resolution technique based on interpolation of the high-frequency subband images obtained by discrete wavelet transform (DWT) and the input image. They used DWT to decompose an image into different subband images. Then the high-frequency subband images and the input low-resolution image have been interpolated, followed by combining all these images to generate a new super-resolved image by using inverse DWT. Figure 3 shows the block diagram of the method proposed by Gholamreza Anbarjafari and Hasan Demirel.

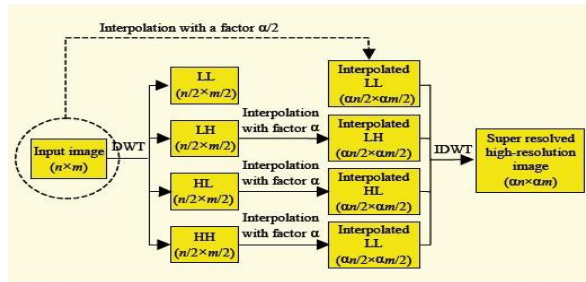


Figure 3: Block Diagram of DWT based SR

2.12 SR by Complex Wavelet Transform

Gholamreza Anbarjafari and Hasan Demirel [18] proposed a technique to enhance to resolution of satellite images based on interpolation of high-frequency subband images obtained by dual-tree complex wavelet transform (DT-CWT). This method uses DT-CWT to decompose an input low-resolution satellite image into different subband images and interpolates input images followed by combining all these images to generate high-resolution images by using inverse DT-CWT. Figure 4 shows the diagram of the method.

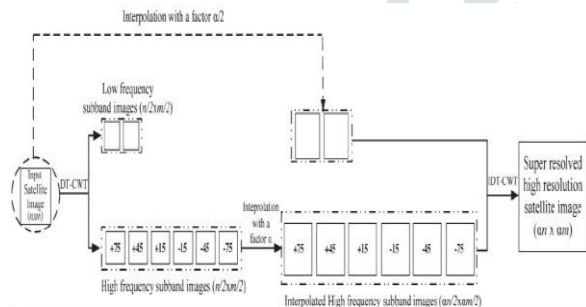


Figure 4: Block diagram of DT-CWT SR

2.13 Image Resolution Enhancement by using Discrete and Stationary Wavelet Decomposition

Gholamreza Anbarjafari and Hasan Demirel [19] proposed a image resolution enhancement technique based on interpolation of the high frequency subband images obtained by discrete wavelet transform (DWT) and the input image. The edges are enhanced by introducing an intermediate stage by using stationary wavelet transform (SWT). DWT is applied in order to decompose an input image into different subbands. Then the high frequency subbands as well as the input image are interpolated. The estimated high frequency subbands are being modified by using high frequency subband obtained through SWT. Then all these subbands are combined to generate a new high resolution image by using inverse DWT (IDWT).

3. PROPOSED METHODOLOGY

An adaptive directional image interpolation is computed by estimating sparse image mixture models in a wavelet frame. This

section describes a fast orthogonal block matching pursuit implementation. The detailed process of wavelet based super-resolution is, first the subsampled image $y(n)$ for is decomposed with wavelet transform matrix ψ whose columns are the vectors of a translation invariant wavelet frame $\psi_{d,m}$ on a single scale (the finest one). Then, it is reconstructed with a dual frame type matrix ψ , columns of this dual wavelet frames is $\psi_{d,m}$. The wavelet coefficients are;

$$c(d, m) = \{y, \psi_{d,m}\} = \psi_y(d, m)$$

The wavelet transform separates a low frequency image y_l projected over the low-frequency scaling filters $\psi_l(d, m)$ & a high-frequency image y_h projected over the finest scale wavelets in three directions $\psi_y(h, m)$.

$$y_l = \sum_{m \in G} c(l, m) \psi_{l,m}$$

$$\& y_h = \sum_{d=1}^3 c(h, m) \psi_{h,m}$$

The low frequency image has little aliasing & can, thus, be precisely interpolated with a cubic spline interpolator. Detailed process is;

Step 1: Computes a 1-D interpolations in the direction. We consider all lines of angle that intersect original image samples (crosses in Fig. 5) & we compute mid-points (circles) between image samples (crosses), with a cubic spline interpolation. This operation oversamples by a factor two either the image rows, or the image columns, or the diagonals of angle. The missing coefficients are shown as squares in Fig. 4.1.

Step 2: Calculation of new samples or dots with a cubic spline interpolation along these oversampled rows, columns or diagonals. This interpolation introduces little aliasing because of the oversampling provided by the previous step. The positions of these new samples (dots) are chosen so that any missing coefficient (square) is a mid-point between two dots on a line of angle.

Step 3: computes missing samples (squares) with a cubic spline linear interpolation along the direction from the previously calculated new samples (dots).

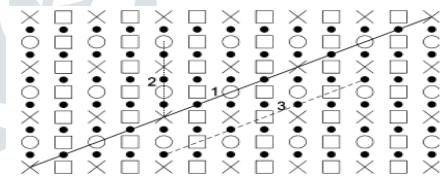


Figure 5: Directional interpolation

For every single angle, application of directional interpolator is performed for the block of wavelet components if the directional regularity factor is less in that block. Such a regularization is very much helpful, when the eigenvalues have significantly large variation, such that discrimination between regular & non-regular variations in the direction of angle, can be done. This is obtained by choosing rectangular blocks that are elongated in the direction of angle. Each block in the spatial neighborhood of is chosen to be identical in the three directions 1, 2, 3 & so on. Numerical experiments are performed with 20 angles, with blocks having a width of 2 pixels & a length between 6 & 12 pixels depending upon their orientation. Each block, thus, includes between 36 & 72 wavelet coefficients over the 1, 2, 3 directions.

An adaptive interpolation estimator by using block matching is obtained by estimating the mixing coefficients of a mixture model which minimizes the errors. The block matching technique performs the tasks of maxima finding & then energy update i.e. find the energy of blocks, if this energy is less than threshold than eliminate all blocks.

This algorithm stops when there is no sufficiently energetic block compared to a precision threshold. The minimization is, also interpreted as an optimized approximation in orthogonal bundles computed over adapted blocks of wavelet coefficients. The adaptive wavelet interpolator is derived by taking IDWT of the resulting mixing coefficients.

PROPOSED ALGORITHM

1. Take low resolution input image.
2. This subsampled image is decomposed with wavelet transform.
3. Apply cubic spline interpolation for removal of aliasing of low frequency components. This process has following sub-steps:
 - (i) First computes a 1-D interpolations in the direction.
 - (ii) Then computes new samples by Cubic Spline interpolation.
 - (iii) After that missing samples are find out by again Cubic Spline interpolation.
4. For each angle, a directional interpolator is applied over a block of wavelet coefficients if the directional regularity factor is relatively small in the block.
5. Apply adaptive interpolation estimator by using block matching, for Maxima Finding & Energy Update.
6. The adaptive wavelet interpolator is derived by taking inverse wavelet transform of the resulting mixing coefficients.
7. This will result an image having more number of pixels, compared to the input LR image, means the resultant image after application of above steps is a HR image.
8. Calculate the performance parameters like PSNR, MSE etc. and compare them with other techniques.

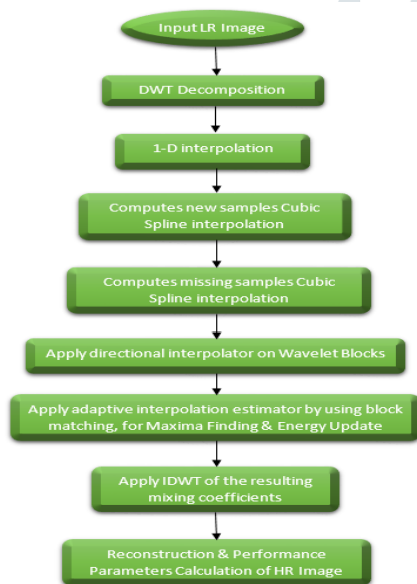


Figure 6: Proposed Wavelet based Super-Resolution Flow Chart

5. SIMULATION RESULTS

The proposed method is tested on various standard images Lena; Baboon & Pepper are taken. All the input low-resolution images interpolated with factor 2. The resolution of input image was 128 × 128. These input images are interpolated to the size of 512 × 512. For quantitative comparison of results peak signal to noise ratio (PSNR) & SSIM is used.

5.1 Performance Parameters

The performance parameters of image are MSE, PSNR, SSIM & many more. But, for image super-resolution PSNR & SSIM are highly used.

Peak Signal-to-Noise Ratio as a performance metric, which is measured in decibels (dB) for 8-bit grayscale images as; $PSNR = 10 \log_{10}(\frac{255^2}{MSE})$

Where, the mean square error (MSE), defined as:

$$MSE = \frac{1}{M \times N} \|f - \hat{f}\|^2 = \frac{1}{M \times N} \sum_{x=1}^M \sum_{y=1}^N [f(x, y) - \hat{f}(x, y)]^2$$

Where; $\| \cdot \|^2$ is the Euclidean norm.

A larger value of PSNR normally reflects the better performance for image analysis. The PSNR & MSE are generally used as the performance indices. But these are not so well matched to perceive visual quality of image directly. To resolve these problems, the structural similarity index (SSIM) [24] was proposed as another metric to compare images which correlates more appropriately with the human perception. SSIM basically maps two separate images into a single index in the interval [- 1, 1], where higher values are assigned to more similar pairs of images X & Y, calculated as;

$$SSIM = \frac{(2\mu_x\mu_y+c_1)(2\sigma_{xy}+c_2)}{(\mu_x^2+\mu_y^2+c_1)(\sigma_x^2+\sigma_y^2+c_2)}$$

Where; μ_X, μ_Y, σ_X^2 & σ_Y^2 are the averages & variances of X & Y, σ_{XY} is the covariance between X & Y, both C_1 & C_2 are predefined constants.

5.2 Simulation Results

5.2.1 Test Image Lena



Figure 7: Test image "Lena"

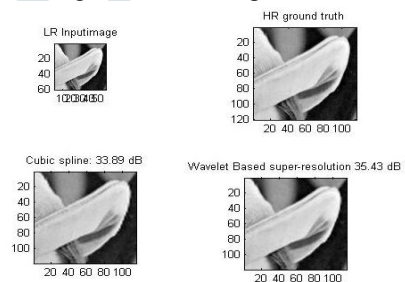


Figure 8: Simulation result for Test image "Lena"

5.2.2 Test Image Baboon

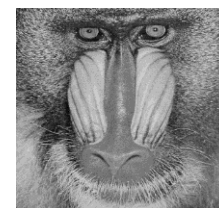


Figure 9: Test image "Baboon"

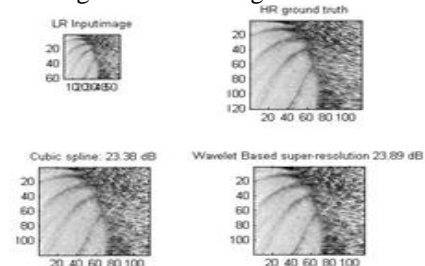


Figure 10: Simulation result for Test image Baboon"

5.2.3 Test Image Peppers



Figure 11: Test image "Peppers"

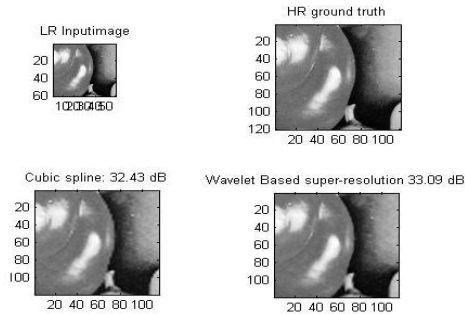


Figure 12: Simulation result for Test image "Peppers"

5.3 Simulation Results Summary

Table 5.1 shows the simulation results for various test images for PSNR and SSIM values.

Test Images	PSNR (dB)	SSIM
Lena	35.43	0.9463
Baboon	23.89	0.7359
Peppers	33.09	0.8898

Table 5.1: Simulation results summary for various test images

The quantitative comparison for PSNR of proposed technique with other methods is shown in table 5.2. The comparison shows that the proposed method gives better results.

Techniques / Images	Baboon		Lena	
	PSNR	SSIM	PSNR	SSIM
Bicubic interpolation [1]	21.58	0.4039	31.42	0.7254
SC [1]	22.36	0.4180	31.73	0.7411
CS-SR [1]	22.92	0.4808	26.23	0.8935
SRCNN [1]	23.53	0.6114	33.15	0.8631
CR-SKR-NLM-SR [1]	23.45	0.6010	32.86	0.8790
This Work	23.89	0.7359	35.43	0.9463

Table 5.2: PSNR & SSIM comparison with other techniques

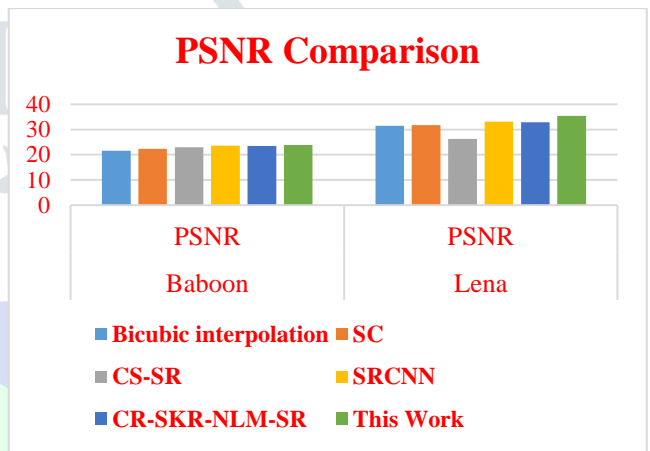


Figure 15: PSNR comparison

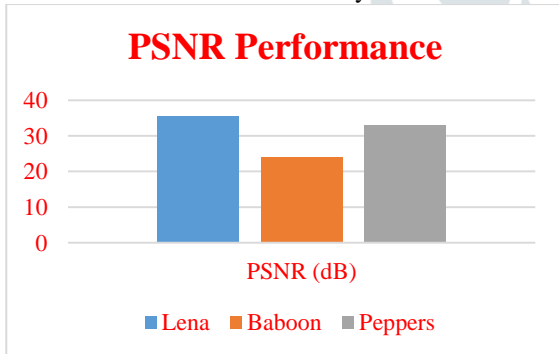


Figure 13: PSNR results summary for various test images

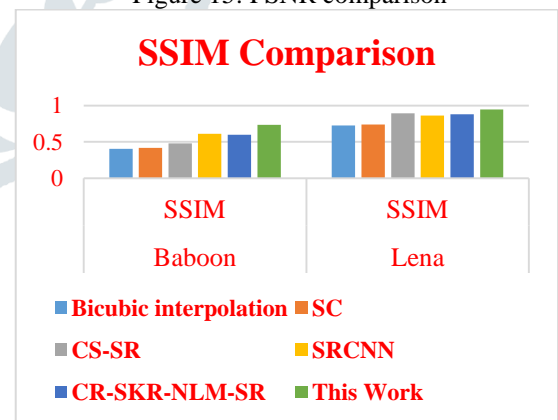


Figure 16: SSIM comparison

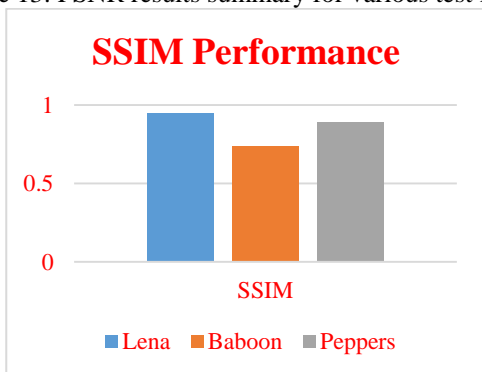


Figure 14: SSIM results summary for various test images

6. CONCLUSION

The SR imaging has been one of the fundamental image processing research areas. In this work a super resolution technique based on Sparse characteristics of wavelet transform. The proposed method for resolution enhancement of image gives good qualitative, quantitative & visual results than that of some traditional & state-of-the-art methods. It has been observed that the proposed technique outperforms all other previous techniques. Hence, we proposed a wavelet based super-resolution technique, which will be of the category of interpolative methods, using sparse

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