BASIC STUDY OF SATELLITES SYSTEM OF PARAMETRIC RESONANCES

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ABSTRACT: *In this article we studies the effect of small external dissipative and disturbing forces of general nature on the non-linear oscillation of the system of two cable connected satellites in the central gravitational field of the Earth. As satellite are considered to be material particles. The cable connecting the two satellites is assumed to be light, flexible and inextensible. The motion of each of them relative to their center of mass has been studied. It has also assumed that the center of mass moves along a given keplarian elliptical orbit and throughout the work that the satellite are subjected to absolutely non-elastic impacts when the string tightens-up The small phenomenological forces play an important role in disturbing the amplitude of the system of two cable connected satellites. The amplitude discontinuity effects are observed the parametric resonance..*

KEYWORDS: *Satellites, Resonances, Oscillations, Dissipative forces, Cable connection, disturbing forces, etc.*

I. INTRODUCTION

The cable connected satellite system is the mathematical idealization of real space system such as space vehicle and astronaught floating in space, two or multidirectional satellite system, manned space capsule attached booster by cable and spinning to provide artificial gravity for the astronaut and lastly the two satellites at the same time of rendezvous. In order to transport a man successfully to an orbiting space station. Rendezvous in outer space we must be familiar with relative notion of the satellites with respect to orbiting station and more over must predict the stable direction of approach for the rendezvous. Also, in case, the astronaut wants to float in outer space for scientific exploration. The exploration of space science resulted in the formulation of the problem of the relative motion of a system of two cable connected satellites in the central gravitational field of force.

The gravitational forces, small dissipative and disturbing forces are also present in nature. Though these force small in comparison to the gravitational force but exert their vital effects on the motion and stability of the system and produce deformation in the amplitude of the oscillation of the system. These effects are more significant when we study the motion of the system in the linear field of gravitational interaction than that in the linear one.

II. MATHEMATICAL ANALYSIS

The equation of relative motion of one of the satellites of the system in the field of central gravitation force and small external forces while the centre of mass of the system is moving along the elliptical orbit has been found as:

$$(1+e\cos V)\Psi''-2e\,\Psi'\,\mathrm{Sin}V + 3Sin\Psi.\cos\Psi = 2e\sin V + \gamma\Psi' + E\sin\upsilon V \quad \ldots.\ (1)$$

where, e = eccentricity of the orbit

V = true anomaly

Ψ = angle which the line joining the centre of mass attracting centre makes with the x-axis

Fig. 1 centre of mass the system moving along elliptical orbit

γ and E are the phenomenological parameters characterizing the dissipative and disturbing force acting on the system. Let us assume that γ and E are of the order of e. hence, our assumption is justified as such estimated are always concerned with certain model assumptions

Thus the system is described by the equation as:

(1) Moves under forced vibration due to the presence dissipative force and periodic sine forces on the R.H.S. of the equation.

(2) To our great advantage, there appears a small quantity, the eccentricity 'e', in the equation. Therefore, we can exploit the Bogoliubov, Krilov and Miltropolsky method for obtaining approximate solution of the differential equation (1), for small values of eccentricity 'e'.

We used E = e E_1 and γ = eγ_1, so, equation (1) can be written as,

$$(1+e\cos V)\,\Psi'' - 2e\,\Psi'\sin V + 3\sin\Psi.\cos\Psi =$$

$$e\{2\sin V + \gamma_1\Psi' + E_1\sin \upsilon V\} \quad \text{.......(2)}$$

Also, we put $\Psi = \frac{Z}{1+e\cos V}$

i.e. $Z = (1+e\cos V).\,\Psi$ **..... (3)**

Now the equation of the oscillatory system in the field of external forces is given by,

$$Z'' + n^2 z = e\{kz^3 + 2\sin V + 2z\cos V - 6z^3\cos V + \gamma_1 z' + E_1\sin \upsilon V\} + e^2\{-2z\cos^2 V\}$$

$$+ 12z^3\cos^2 V - \gamma_1 z'\cos V + \gamma_1 z\sin V) + e^3\{\ldots\ldots \quad \text{.... (4)}$$

On using, $2z^3 = ekz^3$ *and* $n^2 = 3$

With the help of the asymptotic method of Bogoliubov and Mitropolisky we shall obtain an approximation solution of the system equation (4), in case of parametric resonance, n = $\frac{1}{2}$

In the first approximation we have,

$$z = a\cos\left(\frac{1}{2}V + \theta\right) \quad \text{.... (5)}$$

where, the amplitude α, and phase angle θ of the oscillation are given by,

$$\frac{d\alpha}{dV} = \frac{\gamma\alpha}{2} - e\alpha.\sin 2\theta + 3e\alpha^3.\sin 2\theta$$

$$\frac{d\theta}{dV} = n - \frac{1}{2} - \frac{3\alpha^2}{4n} - e\cos 2\theta + 3e\alpha^2 . \cos 2\theta \qquad \ldots\ldots(6)$$

It is evident from equation (6) that at this overtone there is no effect of the external periodic force on the oscillation of the system while the presence of the dissipative force introduces correction in amplitude of the oscillation of the system. Now we shall examine the stationary regimes of oscillation of the system. The phase and amplitude of the stationary oscillation are defined by,

$$\frac{d\alpha}{dV} = 0 \quad \text{and} \quad \frac{d\theta}{dV} = 0$$

Now, from equation (6), we have,

$$\frac{\gamma\alpha}{2} - e\alpha . \sin 2\theta + 3e\alpha^3 . \sin 2\theta = 0$$

$$n - \frac{1}{2} - \frac{3\,\alpha^2}{4n} \quad e\cos 2\theta + 3e\alpha^2 . \cos 2\theta = 0 \qquad \ldots\ldots\ldots(7)$$

Hence, these expression can be written as,

$$\frac{\gamma\alpha}{2} = e\alpha(1 - 3e\alpha^3). \sin 2\theta, \quad \text{and}$$

$$\omega_e\ (\alpha) - \frac{1}{2} = e(1 - 3\alpha^2).(\cos 2\theta)$$

$$\text{where,} \quad \omega_e\ (\alpha) = n - \frac{3\,\alpha^2}{4n}$$

Now the oscillation other than zero amplitude oscillation in the system is given by

$$\alpha^2 \left\{ \left[\omega_e\ (\alpha) - \frac{1}{2}\right]^2 + \frac{\gamma^2}{4} \right\} = e^2\ (1 - 3\,\alpha^2)^2\, \alpha^2 \qquad \ldots\ldots\ldots.(7a)$$

To obtain the relation in neighborhood of resonance frequency, we get n=1/2+d, with this substitution in equation (7a) we have,

$$\delta = \frac{3\alpha^2}{4n} \pm \frac{e}{2}\sqrt{4\,(1 - 3\alpha^2)^2 - \frac{\gamma^2}{e^2}} \qquad \ldots\ldots\ldots\ldots(8)$$

This is a relation between the amplitude of the forced oscillation and the variation of the natural frequency about the resonance frequency. Hence, the motion of the system on a more realistic basic, we must take into account the effects of their external forces on the system.

III. EXPLANATIONS

The present research work have studied the non-linear oscillation of a cable connected system in case of parametric resonance $n = v/3$ and using, $n = \frac{1}{2}$, by Bogolivubov and Mitropolsky method. The behaviors of oscillation was studied with the help of resonance curves as with the help of relation (8), and the resonance curve has been constructed in fig.(2). The dotted line in Figure. (2) represents the skeleton curves.

$$d = \frac{3a^2}{4n}$$

Analyzing the curve, with skeleton curve we find that the branches 'AB' of the resonance curve corresponds to the stable amplitudes of the oscillation, where as the branch BC corresponds to the unstable amplitude of the oscillation. When d increase from small value, the oscillation in the system is absent until d attains the values corresponding to the point A. When d reaches the point A, oscillation appears in the system and with further increase in d; the amplitude of the oscillation goes along the branch AB of the resonance curve. At point B the oscillation losses its stability and break down. On the other hand, when d decrease from large values there is zero amplitude oscillation on the left of the point C.

When decreases the value corresponding to the point C, the amplitude of the oscillation is excited and with further decrease in d, the amplitude of the oscillation goes along the branch BA of the resonance curve. The point B and C are, therefore, the point of break and jump of the amplitude respectively.

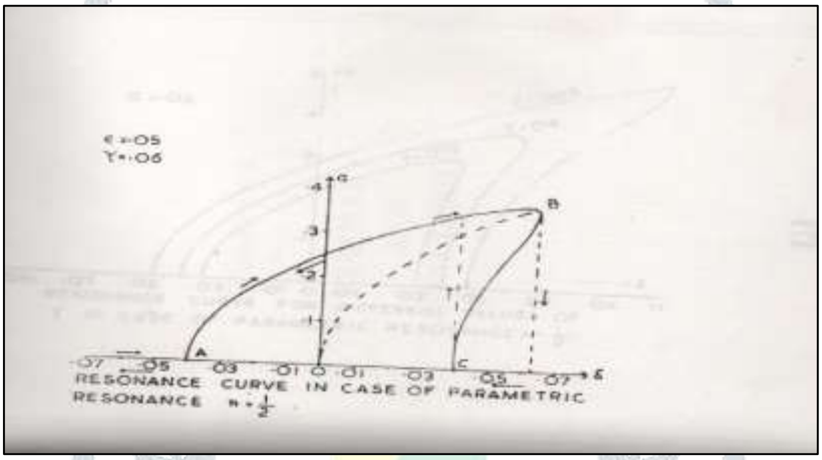

Fig. 2 resonance curve in case of parametric resonance, n $= \frac{1}{2}$

V. CONCLUSION

The present research work deals with the effects of small external dissipative and disturbing forces of general nature on the non-linear oscillation of the system of the cable connected satellites in the central gravitation field of the Earth. The satellites are considered as materials particles. The cable connecting the two satellites in supposed to be light, flexible and inextensible. The parametric excitations are found at born the overtones. But to evaluate the amplitude discontinuity effect accurately, we must know the order of magnitude of parameters, the dissipation co-efficient γ and the amplitude of the external periodic force E. It is clear that these parameters are always concerned with certain model assumptions.

Among the aforesaid probable source of their external forces, the quadropole moments and the gravitational radiations have already studies as source of periodic and dissipative force respectively. But all such estimates lead to considerable arbitrariness in evaluating these parameters. The parameters can be determined accurately by observing the system in a nearly circular orbit We observe that there is resonance behavior in the oscillation of the system near the frequency n=1/2. The break and jump in the oscillation is founded at frequencies greater than the resonance frequency. The

dissipation in the system plays a vital role in reducing the zone parametric resonance. The external periodic force is not active at this overtone.

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REFERENCES

[1]. G. W. Jr. Adams, Jan. 1970, NASA IND, pp 55-59.

[2]. F. Austin, June 1969, Grumman Aircraft. Engg. Corp., Interoffice Memo. STECH, pp 54-59.

[3]. A. M., Nikulin, , S.N. Padalka & U.S. Pronin, 1969, Mechanics of cosmic flight, Russian, pp 88- 98.

[4]. V. V. Belestky, 1960, Kosmicheskiye Isslelovania, , Russian, Vol. 7, No. 6, pp 827-840.

[5]. P. K. Bhattacharyya, S.K. Das & R.B.; Singh, Proc. Nat. Acad. Sci. India, Vol. 48, part-II.

[6]. N. N. Bogoliubov, & Y.A. Mitropolosk, 1961, Hindustan Pub. Corp., Delhi – 6.

[7]. F. L. Cherrouske, 1963, Journal of Computer Mathematics and Mathematical Physics, Russia, Vol. 3, No. 3, pp 528-538.

[8]. V. G. Dermin, 1968, Nauke, Moscow, (Russian).

[9]. P. Kulla, May 1976, Erascati, Italy, pp 349-354.

[10]. Manaziruddin & R.B. Singh, 1985, Proc. Nat. Acad. Sci., India 55 (A), 01III, pp 261-271.

[11] Krupa, M., Poth, W., Schagrl, M., Stiensl, A., Steiner, W., Treger, H., Wiedermann, G. (2006), Modelling, dynamics and central of tethered satellite systems, Non -linear Dynamics, 73-96.

[12] Liapunov, A.M. The general problem of stability of motion, Sobrania Sachimediviya, 2, In Russian.

[13] Langbort, Cedric. (2002), Bifurcation of relative equilibria in the main problem of artificial satellite theory for a prolate orbit, Celestial Me-chanics & Dynamical Astronomy, 369- 385.

[14] Markeev, A.P., (2003), Bardin, B.S. On the stability of planar oscilla-tions and rotation of a satellites in circular orbit, Celestial Mechanics & Dynamical Astronomy, 57-66.