MAGNETO HYDRODYNAMIC UNSTEADY FREE CONVECTIVE FLOW PAST VERTICAL HOT PLATE

Rekha Sharma Associate Professor
Department of Physics
Meerut College, Meerut
E-mail : drrekhasharma1961@gmail.com

ABSTRACT

Transient free convection occurs in a fluid when the temperature causes density variation which give rise to bouyancy forces. Excelent reviews of free convection problems were given by Edre A-J (27). The process of heat transfer has several Engineering applications such as in nuclear reactor considerable reduction in heat transfer occurs by applying a magnetic field. Brar [12] considered the unsteady free convection laminar flow past an infinite plate in the presence of a uniform magnetic flow. Soundelgaker [86] to [89] studied the effect a mass transfer and convection currents in M.H.D. stokes flow problems for a vertical plate. Gourla and Katoch [14] studied unsteady viscous incompressible free convection flow an electrically conducting fluid between two heated vertical plate under the presence of magnetic field perpendicular to flow. Vajrawelu [99] discussed natural convection at a heated semi infinite vertical plate with temperature dependent heat sources and sinks.

KEY WORDS :- Convection, Incompressible, Leminar, Fluid, Permeability

Consider a semi infinite heated vertical plate in an infinite fluid that is initially cold and at rest. The following assumptions were made in the analysis [1] all the fluid particles are constant exvept the density in the buoyancy force term [11]. The flow is laminar unsteady and two dimensional and the viscous dissipation and the work done by pressure are sufficiently small in comparison with the heat flow by conduction [99].

(4.2.1) $$\frac{\partial U}{\partial T} + U\frac{\partial U}{\partial x} + v\frac{\partial U}{\partial y} = \beta g(T - T_i) + v\frac{\partial U}{\partial y^2}$$

(4.2.2) $$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

(4.2.3) $$pC_p\left(\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) =$$

$$K\frac{\partial^2 T}{\partial y^2} + Q*(T - T_0)$$

Where T_0 is the initial temperature of the fluid and the other symbols have their usual meanings. Initial and boundary conditions relevant to the problem are -

(4.2.4) (a) $U = v = 0, T = T_i \; at \; x = 0$

(b) $U = v = 0, T = T_w \; at \; y = 0$

$$(c) \quad U = v = 0, T = T_i \; at \; y \to \infty$$

$$(d) \quad U = v = 0, T = T_i \; at \; z = 0$$

defining

(4.2.5) $$t = Z(g\beta\Delta T)v^{\frac{\frac{2}{3}}{\frac{1}{3}}}$$

$$x = Xg\left(\frac{\beta \, \Delta T}{v^2}\right)^{\frac{1}{3}}$$

$$y = y\left(g\,\frac{\beta \Delta T}{v^2}\right)^{\frac{1}{3}}$$

The above equations yields :-

(4.2.6) $$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + v\frac{\partial U}{\partial y} = \theta\frac{\partial^2 U}{\partial y^2}$$

(4.2.7) $$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

(4.2.8) $$P_r\left(\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) = \alpha\theta + \frac{\partial^2 \theta}{\partial y^2}$$

$$u = v = 0, \theta = 0 \, at \; x = 0$$

$$U = v = 0, \theta = 1 \, at \; y = 0$$

$$u = x = 0, \theta = 0 \, as \; y \to \infty$$

$$u = v = 0, \; \theta = 0 \, at \, t = 0$$

Where

$$\alpha = \frac{\theta^* v^{\frac{4}{3}}}{k(g\beta \, \Delta T)^{\frac{2}{3}}}$$

the non dimensional heat source or skin parameter

$P_r = \mu \dfrac{C_p}{K}$ the Pradtl no

$$\Delta T = T_\infty - T_i$$

(4.3) Mathematical Analysis of the problem

Let us now consider the unsteady flow a viscous electrically conducting liquid. Hall Effect, Electrical and Polarization are effects neglected Magnetic field is applied perpendicular to the free stream velocity. Magnetic Reynolds number is taken small so that induced magnetic field is neglected consider two heated vertical plates at a distance 2a apart. The x-axis along the parallel plates the y-axis normal to the plates passing through x-asix and the origin at the mid point between the plates.

Formulation of our problem :-

Here all physical quantities are function of y' and t' at any time t', the velocity magnetic field and the temperature distribution are given by (u', 0,0), (H'_x, H'_y, 0) and T' Respectively from div H=0, we get H'y'= constant = Initial value of the magnetic field = H_0 writing H' for $H'_{x'}$ the governing equations are as follows (1) :

(4.3.1) $$\frac{\partial u'}{\partial t'} = f_x \beta (T' - T'0) + v\frac{\partial^2 u'}{\partial y'^2} + \mu e \frac{H_0}{p'} \times \frac{\partial H'}{\partial y'}$$

(4.3.2) $$-\frac{\partial p'_1}{\partial y'} - \mu_e H' \frac{\partial H'}{\partial y'} = 0$$

(4.3.3) $$\frac{\partial H'}{\partial t'} = H_0 \frac{\partial u'}{\partial y'} + \eta_x \frac{\partial^2 H'}{\partial y'^2}$$

(4.3.4) $$\frac{\partial T'}{\partial t'} = \frac{K'}{p'C'} \frac{\partial^2 T'}{\partial y'^2}$$

p'_1, p', v', $\mu_e, \eta_d, \beta, K', p', and\ f_x$ are the pressure density, kinematic coefficient of viscosity, magnetic permeability, magnetic diffusivity, coefficient of thermal expansion, thermal conductivity, specific heat of the liquid and component of acceleration along x' direction respectively.

The initial and Boundary conditions are :

(4.3.5) (a) $$y' \geq 0, t' \leq 0, u' = 0, H' = 0, T' = T'_0,$$

(b) $$t' > 0, y' = 0, u' = A't'^m$$

$$T' = \frac{T'_0 + A_0^3}{f_x \beta v \alpha}\left(1 - e^{Ae2}\alpha t'\right)$$

(c) $$\frac{1}{\sigma}\left(\frac{\partial H'}{\partial y'}\right)_{y'=0} = \frac{1}{\sigma\,\omega'}\left(\frac{\partial H'}{\partial y'}\right)_{y'=0}$$ when t'>0

$$y' \to \infty, u' \to 0, H' \to 0, T' = T'_0$$ when t'>0

Where σ and σ_ω' are the electrical conductivity of the fluid and the plate respectively and $A_0^2 = \frac{\mu_e H_0^2}{\rho'}$; for convenience we introduce the following non dimensional equations :

(4.3.6) $$u' = \frac{u'}{A_0}, \qquad H = \frac{H'}{H_0'}$$

$$Y = \frac{\Delta_0 y'}{v}, \qquad t = \frac{A_0^2 t'}{v}$$

$$\sigma_1 = \frac{p' v C'}{K'}, \qquad \sigma_2 = \frac{v}{\eta\alpha}$$

$$G = f_x \beta v (T' - T'_0) A_0^{-3}$$

$$A = \frac{A' v^m}{A_0^{2^{m+1}}}$$

From (4.3.6) equation (4.3.1) to (4.3.4) yield to -

(4.3.7) $$\frac{\partial u}{\partial t} = G + \frac{\partial^2 u}{\partial y^2} + \frac{\partial H}{\partial y}$$

(4.3.8) $$\frac{\partial H}{\partial t} = \frac{\partial u}{\partial y} + \frac{1 \partial^2 H}{\sigma^2 \partial y^2}$$

(4.3.9) $$\frac{\partial G}{\partial t} = \frac{1}{\sigma_1} \frac{\partial^2 G}{\partial y^2}$$

The Boundary conditions are reduced to

(4.3.10) (a) $t \le 0,\ y \ge 0, u = 0, H = 0, G = 0,$

$(b)\, t \ge 0, u = A^{Tm}, G = \frac{1}{\alpha}(1 - e^{-\alpha t}),$

$\frac{1}{\sigma}\frac{\partial H}{\partial y} = 0,.\ y = 0, y \to \infty, u = 0,$

$H = 0,\ G = 0$

Now we shall use here Laplace tencniques in the above equations.

(4.3.11) $$\frac{d^2 \bar{u}}{dy^2} - p\bar{u} + \frac{d\bar{H}}{dy} = -\bar{G},$$

(4.3.12) $$\frac{d^2 H}{dy^2} - p\sigma_2 \bar{H} + \sigma_2 \frac{d\bar{u}}{dy} = 0,$$

(4.3.13) $$\frac{d^2 \bar{G}}{dy^2} - \sigma_1 p\bar{G} = 0,$$

(4.3.14) $$\bar{u}(0, p) = \frac{A}{p^{m+1}} T(m+1),$$

(4.3.15) $$\bar{G}(0, p) = \frac{1}{p(p+\alpha)}$$

(4.3.16) $$\frac{1}{\sigma}\frac{d}{dy}\bar{H}(0, p) = \sigma,$$

(4.3.17) $$\bar{u}(\infty, p) = 0, \bar{H}(\infty, p) \to 0,$$

$$\bar{G}(\infty, p) \to 0$$

Thus, we have

(4.3.18) $$\bar{G}(y,p) = \frac{1}{p(p+\alpha)} e^{-y\sqrt{p\sigma_1}}$$

Eliminating $\bar{H}$ from (4.3.11) and (4.3.12), we get -

(4.3.19) $$\frac{d^4\bar{u}}{dy^2} - \frac{d^2\bar{u}}{dy^2}(p + p\sigma_2 + \sigma_2) + G_2 P^2 \bar{u} = e \frac{-y\sqrt{-\sigma_1}(-\sigma_1+\sigma_2)_p}{p(p+\alpha)}$$

Now solving (4.3.19), we have

(4.3.20) $$\bar{u}(y,p) = A_1 e^{n1y} + A_2 e^{-n1y} + B_1 e^{n2y}$$

$$+ B_2 e^{-n2y} + L(\sigma_2 - \sigma_1) e^{-y} \sqrt{p\sigma_1}$$

Where A_1, A_2, B_1, B_2, are constants

and $$\left.\begin{matrix}\eta_1 \\ \eta_2\end{matrix}\right] = \sqrt{a+bp} \pm \sqrt{a+cp};$$

$$a = \frac{\sigma_2}{4}, b = \frac{1}{4}(1+\sqrt{\sigma_2})^2, c = \frac{1}{4}(1-\sqrt{\sigma_2})^2,$$

$$L = \frac{1}{p(p+\alpha)[1(1-\sigma_1)(\sigma_2-\sigma_1)-\sigma_1\sigma_2)]}$$

(4.3.21) Using $A_1 = B_1 = 0$

(4.3.22) $A_2 + B_2 = -\angle(\sigma_2 - \sigma_1) + \frac{A}{p^{m+1}}\Gamma(m+1)$

From above equations we have

$$\frac{d\bar{H}}{dy} = -\bar{G} + p\bar{u} - \frac{d^2\bar{u}}{dy^2},$$

$$\frac{d\bar{H}}{dy} = \frac{1}{p(p+\alpha)} e^{-y} \sqrt{p\sigma_1}$$

$$+ p(A_2 e^{-n1y} + B_2 e^{-n2y}) + \angle(\sigma_2 - \sigma_1) e^{-y} \sqrt{p\sigma_1}$$

$$- \angle(\sigma_2 - \sigma_1) p\sigma_2 e^{-y\sqrt{p\sigma_1}}$$

Finally the above equations -

(4.2.23) $$\frac{d\bar{H}}{dy} = (p - \eta_1^2) A_2 e^{-\eta 1 y}$$

$$+ (p - \eta_2^2) B_2 e^{-n2y} - e^{-y\sqrt{p\sigma_1}} \angle\sigma_1\sigma_2$$

Intergrating (4.3.23) and using $\bar{H}(\infty, p) \to 0$

(4.3.24) $$\overline{H}(y,p) = \frac{A_2\left(p-\eta_1^2\right)e^{-\eta 1 y}}{-\eta_1} + \frac{\left(p-\eta_2^2\right)B_2 e^{-\eta 2 y}}{-\eta_2} - \frac{\angle\sqrt{\sigma_1\sigma_2}\, e^{-y\sqrt{p\sigma_1}}}{-\big)\, p\sigma_1}$$

Since $\frac{1}{\sigma}\frac{d}{dy}[H(0,p)]=0]$

Therefore we have -

(4.3.25) $$A_2(p-\eta_1^2)+B_2(p-\eta_2^2)-\angle\sigma_1\sigma_2=0$$

Equation (4.3.25) and (4.3.22) yields -

If we assum $a=\frac{1}{4}, b=1, c=0$ then values are simple and we get

(4.3.26) (a) $$A_2 = \angle(1-3\sigma_1) - \frac{A\Gamma(m+1)}{p^{m+1}}(1-4p)^{1\frac{1}{2+}}$$

$$\frac{\angle(\sigma_1-1)+A\dfrac{\Gamma(m+1)}{p^{m+1}}}{2}$$

(b) $$B_2 = \frac{\angle(3\sigma_1-1)-\dfrac{A\Gamma(m+1)}{p^{m+1}}(1-4p)^{-\frac{1}{2}}}{2}$$

$$\frac{\angle(\sigma_1-1)+A\dfrac{\Gamma(m+1)}{p^{m+1}}}{2}$$

Substituting (4.3.26) in (4.3.20) we obtain-

(4.3.27) $$\bar{u}(y,p) = [e^{-y}\,\frac{\sqrt{1+4p}+1}{2}\times$$

$$\left(\frac{\angle(1-3\sigma_1)-A\dfrac{\Gamma(m+1)}{p^{m+1}}}{2}\right.$$

$$\frac{\angle(\sigma_1-1)+A\dfrac{\Gamma(m+1)}{p^{m+1}}(1+4p)^{-\frac{1}{2}}}{2}$$

$$+\frac{\angle(\sigma_1-1)-A\dfrac{\Gamma(m+1)}{p^{m+1}}}{2}$$

$$-e^{-y}\left(\frac{\sqrt{1+4p}-1}{2}\right)+\angle(1-\sigma_1)e^{-y}\sqrt{p\sigma_1}]$$

The above equation is reduced to -

$$\bar{u}(y,p)=\left\{\frac{1-3\sigma_1}{\frac{-A(m+1)}{p^{m+1}}}\right\}.^{(1+4p)-1/2}x$$

$$\left[e^{-\frac{y}{2}}\left(1+\sqrt{1+4p}\right)-e^{-\frac{y}{2}}\left(1-\sqrt{1+4p}\right)\right]$$

$$\left\{\angle(\sigma_1-1)+A\frac{\Gamma(m+1)}{p^{m+1}}\right\}$$

$$\left[e^{-y}\left(\frac{\sqrt{1+4p}}{2}\right)e^{-\frac{y}{2}}+e^{-y}\left(\frac{\sqrt{1+4p}}{2}\right)e^{\frac{y}{2}}\right]$$

or

$$\bar{u}(y,p)=(1+4p)^{-\frac{1}{2}}Sin h\frac{y}{2}\times$$

$$\left[\angle(3\sigma_1-1)-A\frac{\Gamma(m+1)}{p^{m+1}}\right]e^{-\frac{y}{2}\sqrt{1+4p}}$$

$$+\left[\angle(\sigma_1-1)+A\frac{\Gamma(m+1)}{p^{m+1}}\right]e^{-\frac{y}{2}}\sqrt{1+4p}\times$$

$$Cos h\frac{y}{2}+\angle(1-\sigma_1)e^{-y\sqrt{p\sigma_1}}$$

Solving (4.3.28), making use of Laplace techniques we have -

(4.3.29)
$$u(y,t)=(3\sigma_1-1)Sin h\frac{y}{2}1_1$$

$$+A\Gamma(m+1)/_2\,Sin h\frac{y}{2}$$

$$+(\sigma_1-1)\,Cos h\frac{y}{2}/_3+A\Gamma(m+1)/_4$$

$$(3\sigma_1-1)Cos h\frac{y}{2}+(1-\sigma_1)/_5$$

Where -

(4.3.30) (a)
$$I_1 = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} \angle e^{pt-\frac{y}{2}} \sqrt{1+4p}\ x$$
$$(1+4p)^{-1/2} dp$$

(b)
$$I_2 = \frac{1}{2\pi i}\int_{Ci\infty}^{C+i\infty} \frac{e^{pt\frac{y}{2}\sqrt{1+4p}}}{p^{m+1}} \times$$
$$(1+4p)^{-1/2} dp$$

(c)
$$I_3 = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} \angle e\ pt - \frac{y}{2}\sqrt{1+4p}\, dp$$

(d)
$$I_4 = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} \frac{e^{pt} - \frac{y}{2}\sqrt{1+4p}}{p^{m+1}} d\ p$$

(e)
$$I_5 = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} L e^{pt-y\sqrt{p\sigma_1}}$$

Substituting (4.3.20) and (4.3.26) in (4.3.24), we find

(4.3.31)
$$\overline{H}(y,p) = e^{-y\left(\frac{1+\sqrt{1+4p}}{2}\right)} \times$$
$$\left[\frac{\angle(1-3\sigma^1) - A\frac{\Gamma(m+1)}{p^{m+1}}}{2}(1+4p)^{-\frac{1}{2}+}\right.$$
$$\angle(\sigma_1 - 1) - A\frac{\Gamma(m+)}{p^{m+1}}$$
$$-e - \left(\frac{\sqrt{1+4p}}{2} - 1\right)y$$
$$\left[\frac{\angle(3\sigma_1 - 1) + A\frac{\Gamma(m+1)}{p^{m+1}}(1+4p)^{-\frac{1}{2}}}{2} + \right]$$
$$\frac{\angle(\sigma_1 - 1) + A\frac{\Gamma(m+1)}{p^{m+1}}}{2}$$
$$+\angle\sqrt{\frac{\sigma_1}{p}} e^{-y}\sqrt{p\sigma_1}$$

From inversion of the above equation, we obtain

(4.3.32)
$$H(y,t) = Sin\frac{hy}{2}(3\sigma_1 - 1)/_3$$
$$+ A\Gamma(m+1) \times Sin\frac{hy}{2}/_2$$
$$+(\sigma_1 - 1)Cos\frac{hy}{2}/_3 + A\Gamma(m+1).$$
$$Cos\frac{hy}{2}/_4\sqrt{\sigma_1}/_5$$

Where

$$/_5 = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty}\frac{e^{pt}}{\sqrt{p}}e^{-y\sqrt{p\sigma_1}}dp$$

(4.3.33) G(y,t)=I_6

Where

$$/_6 = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty}\frac{e^{pt--y\sqrt{p\sigma_1}}}{p(p+\alpha)}d\,p$$

Now shearing stress at the plate is given by

$$\tau_0 = \left(H + \frac{\partial u}{\partial y}\right)_{y=0}$$

Thus we get,

(4.3.34)
$$\tau_0 = (\sigma_1 - 1)/_3 + A\Gamma(m+1)/_4$$
$$+\sqrt{\sigma_1}/_5 + \frac{3\sigma_1 - 1}{2}(/_1)_{y=0_+}$$
$$A\Gamma\left(\frac{m+1}{2}\right)(/_2)_{y=0}(\sigma_1 - 1)\left(\frac{\partial /}{\partial y}\right)_{y=0} +$$
$$A\Gamma\left(\frac{m+1}{2}\right)\left(\frac{\partial /_4}{\partial y}\right)_{y=0} + \left(\frac{\partial /_5}{\partial y}\right)_{y=0}$$

If we draw Graph in G/α and y and also H/ and y for particular value of $\eta = 0$ & 1

(4.4) CONCLUSION

From figure 1 ti is clear that for n=0 velocity at all points are nearly the same but for $\eta = 1$ the velocity decreases as y increases. Figure 2 shows that the induced magnetic field increases from the wall for $\eta = 0$ and $\eta = 1$.

REFERENCE

1. Acharya, B.P. (1975) : Free convection flow of an elastico Indian Acad. of Sciences ; 81, 105-110.

2. Courla M.G. & Suaham, L. Kotch (1981) : Unsteady free convection M.H.D. flow between heated vertical plates. Ganita 42, 143-153.

3. Bodica, J.R. & Osterle, J.F. (1962) : The development of free convection between heated vertical plates. Jr. of Heat & Transfer A.S.M.E. Ser. C. Vol. 84, 65-70.

4. Brar, G.S. (1978) : J. De Mechanique, 17, 231-240.

5. Goldstein, R.J. & Biggs, D.G. (1964) : ASME, J. Heat transfer 86, 490-500.

6. Soundelgakar, V.M. & R.M. Lahirikar & S.G. Pohansekar (1993) : Free convection effects on the flow through a porous medium in the viscosity of an infinite vertical oscillatory plates Proc. Math. Soc. B.H.U., 9, 69-81.

7. Soundelgaker, V.M. Das, U.N. & Deka, R. (1996) : Unsteady hydro-magnetic free and forced convective flows through a porous medium past an infinite vertical plate with variable plate temperature. Proc. Math. Soc. 12, 15-19.

8. Vidyamdhi, V., Rao Dhananjaya, V & Vasudeva, V. Murthy (1984) : Free convection effects on steady flow past an vertical plate in a porous medium. Acta Ciencia India. Xm, 217-224 and (1983) IXm, 55-62.

9. Yadav, Ramu Raj (1991) : Steady flow of a viscoelastic fluid past an infinite plate with uniform suction and constant heat flux. Jr. of P.A.S. Janpur, 2, 57-62.