STUDY ON SEMI-CLASSICAL TREATMENT OF ZEEMAN EFFECT BASED ON QUANTIZATION RULES OF BOHR'S SOMMERFIELD

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Abstract: Zeeman proposed that when atomic spectra are placed in an external magnetic field splits into a number of components which is called after him Zeeman splitting. In this paper, I have tried to produce an explanation of this effect on the basis of semi-classical mechanics and the quantization rule of Bohr-Sommerfield.

KEY WORDS: Quantization rule, Magnetic field, Semi-classical mechanics.

I. INTRODUCTION

According to classical theory, the electron revolves around the nucleus in a fixed orbit due to which the orbital angular momentum is perpendicular to the plane of the orbit. The orbital angular momentum state is not found in the atom is left unperturbed due to the presence of degenerate states. Now a coupling between the magnetic field and the orbital angular momentum occurs when the atom is placed in an external magnetic field. So, several components lines are observed corresponding to different energies due to the splitting of degeneracy.

II. THEORY

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The Zeeman effect can be explained based on semi-classical mechanics by using the classical Hamiltonian theory for a charged particle placed in a uniform magnetic field B and the Bohr-Sommerfield quantization rule. The canonical momentum for the charged particle in a magnetic field is given by

$$\mathrm{M} \rightarrow \mathrm{M} - \frac{Q}{C} \cdot \mathrm{A} \tag{1}$$

Where A is the vector potential and it satisfies the Maxwell's equation

$$\nabla \cdot B = 0 \tag{2}$$

It means that

$$\mathrm{B} \rightarrow \nabla \times A \tag{3}$$

Since A is not unique so let us consider

$$A' \rightarrow \mathrm{A} + \nabla \cdot \mathrm{A}$$

Where A' is another vector potential that is different by a gradient from A.

The magnetic field will be the same when taking curl on both sides. for convenient let's consider Coulomb gauge then we have $\nabla \cdot A = 0$ then the vector potential A becomes

$$\mathrm{A} = -\frac{1}{2}\, \mathrm{r} \times B \tag{4}$$

Equation (4) will satisfy the Coulomb gauge. The classical Hamiltonian for a charged particle is given by

$$\mathrm{H} = \frac{1}{2m}\left(\mathrm{M} - \frac{Q}{C} \cdot \mathrm{A}\right)^2 \tag{5}$$

Expanding equation (5) by using Binomial expansion we have

$$\mathrm{H} = \frac{1}{2m} M^2 - \frac{2QMA}{2mc} + \frac{Q^2A^2}{2mC^2} \tag{6}$$

Since M and A are classical quantities they follow the commutation rule

$$\mathrm{M} \cdot A = A \cdot M \tag{7}$$

Using equations (4) and(7) in equation (6) we get

$$\mathrm{H} = \frac{1}{2m} M^2 - \frac{2QM\,(r \times B)}{2mc} + \frac{Q^2\,(r \times B) \cdot (r \times B)}{8mC^2} \tag{8}$$

In the region far from electric charge, the scalar potential becomes zero. Now using the scalar triple product and quadruple product of a vector as given below

$\mathrm{A} \cdot (B \times C) = B \cdot (C \times A) = \mathrm{C} \cdot (A \times B)$ and $(A \times B) \cdot (C \times D) = (A \cdot C) \cdot (B \cdot D) = (A \cdot D) \cdot (B \cdot C)$

If A= C and B= D then the above equation becomes

$$(A \times B) \cdot (C \times D) = A^2B^2 - (A \cdot B)^2$$

Now putting A → r and B → M in equation (8) we get

$$H = \frac{1}{2m} M^2 - \frac{Q\,(r \times M)\cdot B}{2mc} + \frac{Q^2 [r^2 M^2 - (r\cdot M)^2]}{2mC^2} \quad (9)$$

But L = r × M so the magnetic field in the z-direction then B = B $\hat{z}$ and coupling will be the z-component of the angular momentum L_z. So we have

$$H = \frac{1}{2m} M^2 - \frac{Q\,L_z \cdot B}{2mc} + \frac{Q^2\,B^2 [x^2 + y^2]}{2mC^2} \quad (10)$$

Now according to the Bohr-Sommerfield quantization rule, we have

$$\oint P_\varphi\, d\varphi = m_i h \quad (11)$$

Where h is Planck's constant, m_i is an integer (m_i = 0,1,2, 3,…, …) which will be either positive or negative including zero and P_φ is the conical momentum of φ.

The Zeeman effect can be obtained when net angular momentum will add to zero for atoms of closed-shell configuration. The Zeeman effect exhibits only when a single electron is occupied in the outmost orbit of the atom.

Now if P_φ= L_z then $\oint d\varphi = 2\pi$ and it has its projection either parallel or anti-parallel to the magnetic field B = B $\hat{z}$ so we have

$$L_z = m_i \frac{h}{2\pi} = m_i \hbar \quad (12)$$

Hence the final expression for Hamiltonian is

$$H = \frac{1}{2m} M^2 - \frac{Q\, m_i \hbar \cdot B}{2mc} + \frac{Q^2\,B^2 [x^2 + y^2]}{2mC^2} \quad (13)$$

The angular momentum along the z-direction in the magnetic field is known as the Zeeman term and the last term of R.H.S. represents the diamagnetic term.

III. CONCLUSION:

The Zeeman effect cannot be explained based on semi-classical mechanics satisfactorily because there is no classical analogy to spin. Quantum mechanics is only applicable to explain the Zeeman effect satisfactorily because the contribution of the spin angular momentum is also included. Zeeman effect is a powerful tool in spectrum analysis.

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