COMPUTATION OF NOISE SPECTRAL DENSITY IN SWITCHED CAPACITOR CIRCUITS

Dhananjay Kumar

Department of Physics, Jai Prakash University,Chapra-841301, Bihar, India.

ABSTRACT

In this paper presents a time-domain method to obtain the average noise power spectral density is proposed. In this paper, we us MFT along with the method proposed to obtain the noise spectral density in switched capacitor circuits. For each value of $w_{m,}$ we show that me method requires only two integrations over a clock cycle. This paper discusses the algorithm for computation of the noise spectral density, the application of the MFT method to noise spectral density computation and contains the results of the computations performed. In this paper, the possibility of using the MFT technique to speed up computations of the noise spectral density was explored.

Key Words: Spectral Density, MFT Technique, Switched Capacitor Circuit.

INTRODUCTION

In this paper we have studied the Computation of noise spectral density in switched capacitor circuits which as received considerable attention. It would be advantageous to have an efficient time domain technique that can easily be integrated into a standard circuit simulator such as SPICE. A step in this direction was taken by Demir *et* al [11]. They obtained a set of ordinary differential equations for the time-varying covariance matrix. A time-domain method to obtain the average noise power spectral density is proposed. It is based on the fact that the power spectral density is the limiting value of the expected energy spectral density per unit time.

When applied to periodically varying circuits, this formulation requires efficient techniques for computation of the quasi-periodic steady state solution of a set of linear time-varying ordinary differential equations. Other than direct integration, two methods that are commonly used to get steady state solutions are the shooting Newton technique and the mixed frequency/time method(MFT) . The shooting Newton technique is applicable to circuits with a periodic steady state response. In this method, transient simulations have to be performed for at least one period of the output signal to get the corrected initial state. This has some disadvantages in the case of switched capacitor circuits, where the clock frequency is usually much greater than the signal hand width. A single output period could very easily include a few hundred clock periods. The output period also depends on the relationship between the frequency at which the PDS is desired, ω_m, and the clock frequency, ω_m. Therefore, the computation of the noise spectrum using this method could mean long transient simulations. The MFT method can be used to circumvent some of these difficulties. In this method, the transient solution, computed over a few selected clock cycles, can be used to construct the quasi-periodic output signal accurately. This method has been demonstrated to be very efficient for computation of the steady state output signal as well as distortion in large switched capacitor filter circuits [13, 16].

In this paper, we us MFT along with the method proposed to obtain the noise spectral density in switched capacitor circuits. For each value of w_m, we show that me method requires only two integrations over a clock cycle. Moreover, due to the nature of the differential equations involved, the MFT method can be used to obtain the noise spectrum for all frequencies, including the clock frequency and its harmonics, At the clock frequency and its harmonics, it degenerates to the shooting Newton method.

This paper discusses the algorithm for computation of the noise spectral density, the application of the MFT method to noise spectral density computation and contains the results of the computations performed.

ALGORITHM FOR COMPUTATION OF THE NOISE SPECTRAL DENSITY

As with all other techniques, noise is treated as a perturbation. Only thermal noise sources are considered. This includes noise due to the switches and Op-Amps In this paper, we use the state variable form of the equations, however, the modified nodal equations can also be used.

The state variable form of the circuit equations without noise sources can be written as:

$$\frac{dx(t)x}{dt} = F(x(t), v(t)) \qquad (1)$$

Where $\mathbf{x}(t)$ is the vector containing the state variables of the circuits and v(f) is the vector of large signal excitations to the circuit. This can be solved to get the large signal steady state solution of the circuit, $\mathbf{x}_s(t)$. With the noise sources added, the solution is assumed to be of the form:

$$\mathbf{x}(t)= \mathbf{X}_3\,(t) + \mathbf{x}_n(t) \qquad (2)$$

Where $\mathbf{x}_n(t)$ represents the noise voltages. Since noise is treated as a perturbation, a linearized form of the state equations along with additive noise sources can be used for noise computations. The large signal input sources are set to zero. The linearized equations can be written as:

$$\frac{dx(t)x}{dt} = A(t) = x_n(t) + B(t)\, u(t) \qquad (3)$$

Here, A(t) is the Jacobian of F(.) computed at steady state. B(*t*) is a matrix containing the spectral intensity of the noise sources and u(*t*) is the vector containing the various noise sources, all of which are assumed to be standard Gaussian white noise processes uncorrelated with each other.

Assume that the noise spectral density is to be determined at node '*N*' of the circuit. Let $x_{nN}(t)$ (t) be the noise waveform at this node. For convenience, it is assumed that $x_{nN}(t)$ is also a state variable. We define:

$$X(t,\omega) = \int_0^t x\, nN(\tau) e^{-jwr} d\tau \qquad (4)$$

X(t,ω) is essentially the Fourier transform of a "t-segment" of the noise waveform. It is then possible to define the expected energy spectral density of this finite segment of the waveform, E(t,w) as:

$$E(t,\omega) = \{/X(t,\omega)/^2\}$$

Here E{a-} denotes the expectation operator. The expected power spectral density is given by [14]:

$$\text{PDS} = \lim_{t\to\infty} \frac{\varepsilon(t,\omega)}{t} \qquad (5)$$

Using the methodology of stochastic differential equations, it can be shown that the energy spectral density and the cross-spectral density, K’(t), are given

$$\frac{d\mathcal{E}(t,\omega)}{dt} = E\ \{(\ x n_N\ (t)\ X(t.\omega)\ e^{-jwt} + E\ x^{*} nN\ (t) X\ (t,\omega)\ \} e^{-jwt} \quad (6)$$

$$\frac{dK\prime(t)}{dt} = A\ (t)\ K'(t) + E\ \{x_n\ (t)\ x^{*} nN(t)\ e^{-jwt}\} \quad (7)$$

Where **K**’(t) is a vector with:

$$K'(t) = E\{\ x_{ni}\ (t) X(t,\ \omega^{*})\}$$

The second term in equation (3,6) is a vector containing the time-varying variance at the output node and its cross correlation with other nodes in the circuit. This can be obtained by solving the differential equations for the time-varying covariance matrix, given as [11]:

$$\frac{dK(t)}{dt} = K\ (t)\ A\ (t)^{*T} + A\ (t)\ K\ (t) + B\ (t)\ B(t)^{T} \quad (8)$$

Where

$$K_{ij}\ (t) = E\{x_{ni}(t) x^{*}_{nj}(t)\}$$

A more detailed derivation of the equations and its application to switched capacitor can be found in [17].

Equations (5), (6) and (7) have to be solved in order to get the output noise spectral density. In the next section, efficient techniques to perform this computation are discussed.

APPLICATION OF THE MFT METHOD

Since noise is regarded as a perturbation, equations (6) and (7) are linear time-varying equations, Moreover, since both the input vector [consisting of the noise sources] and the state matrix vary periodically with the clock, the steady state covariance matrix is a periodic function of the lock. The shooting Newton method can therefore be used very efficiently to get the periodic steady state covariance matrix. The equations for the cross-spectral densities, however, contain both the "measurement" frequency, Wm and the clock frequency, Wc. The "input" vector in (6) is effectively a product of a tone of wm and the vector, KN(t), containing the output variance and cross-correlations with other nodes, Since $K_N(t)$ is a periodic function of the clock, its elements can be expanded as a Fourier series in terms of the clock. Therefore, equation (6) can be written as:

$$\frac{dK'(t)}{dt}_k = A_k K' \{(t) + e^{-jwtmt} \sum_{n=-\infty}^{+\infty} C ne^{-jnwct} \qquad (9)$$

Where k represents the k^{th} clock phase and A_k is the state matrix in the k^{th} clock phase. The solution K'(t) will therefore contain components at frequencies wm_}ne_c.

Both the shooting Newton and MFT technique were used to obtain the output PSD. In the shooting Newton technique, starting with an initial guess for K' (0), the equations have to be integrated for a complete period of K' (t). After integration for one period, the state transition matrix can be used to get the correction to K'(0). Using these corrected values, the equations have to be integrated once more for a complete period to get the power spectral density. In the MFT technique, the signal K' (t) is sampled at the clock frequency to obtain a discrete signal that is independent of the clock fundamentals. The number of samples required depends upon the number of Fourier coefficients needed to represent the envelop. This method it is assumed that the initial guess is zero and trapezoidal rule is used for the integration.

In order to do this integration, the set of differential equations (6) have to be integrated once more for one clock cycle using the corrected initial state. Once d_0 is obtained, it is seen from equations (4) and (5) that the PSD is given by

$$PSD = do + d^*_0 = 2Re(d_0) \quad (10)$$

Inspection of equation reveals why this technique is very efficient for spectral density computations. Other than the clock fundamentals, K'(t) contains only the fundamental at ω_m. As a result, the number of equations to be solved remains the same as the original set of differential equations. We need to integrate only over one clock cycle to get the corrected value of K'(0). Therefore, irrespective of ω_m the MFT method requires only two integrations over a clock period to compute the PSD.

RESULTS:

The noise spectrum of a switched capacitor low pass and a bandpass filter was computed using the method described. For the low pass filter, computed and experimental results are available in [21]. The experimental results are obtained with an external white noise source with a PSD of -61.5dB connected to the non-inverting input of the op-amp. For the bandpass filter, the noise spectral density was computed by Toth et al [22] and Yuan et al [15]. In order to compare with published results. We have used the same macro models and element values that were used to obtain the published results. Besides direct integration, both the shooting Newton and MFT techniques were implemented for these circuits. The code was written in Python, a public domain scripting language. It is run on a 1.7GHz Intel PIV processor running Linux. Figure 1 shows a comparison with experimental results for the low pass filter. Both the shooting Newton and MFT give virtually identical results for the power spectral densities. The time taken by the two methods is shown in Table 1. It is seen that MFT takes less than a fourth of the time taken by the shooting Newton technique. The output noise

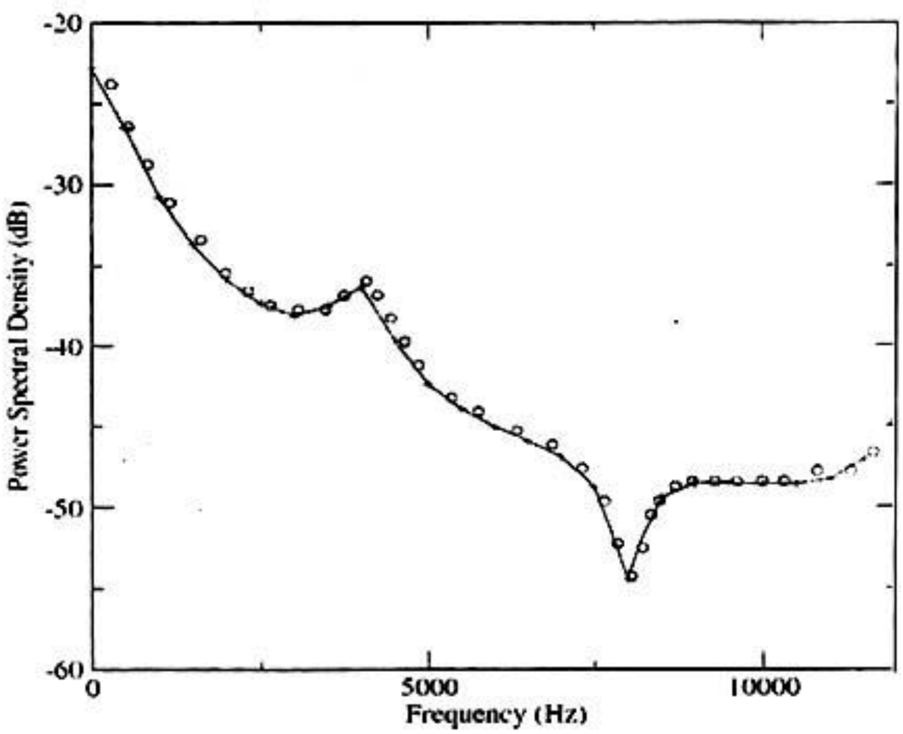

Figure 1: Comparison between experimental and simulated results. : experimental data [11], Solid line: MFT method, + : Shooting Newton.

Spectrum of the bandpass filter, computed using both the shooting Newton and MFT technique. Almost indentical results for the PSD are obtained using

Table 1: Time taken by the two methods for the lowpass (LPF) and bandpass (BPF) filter

Circuit	Method	Number of frequencies	Total time (seconds)	Time/freq (seconds)
LPF	MFT	26	464.42	17.86
	S-N	28	2250.87	80.38
BPF	MFT	53	1011	19.07
	S-N	54	45731	846.87

Both techniques. As expected, the MFT technique is far more efficient. Results are summarized in Table 1. The time taken by MFT per frequency remains almost the same as for the low pass filter, whereas the shooting Newton method takes an order of magnitude longer. Direct integration of the equations takes more than a day.

In the table, the time taken per frequency is an average time, For the MFT method, this also represents approximately the time taken at each frequency. In the case of the shooting Newton technique, the time taken for computation depends strongly. In the case of the shooting Newton technique, the time taken for computation depends strongly on the measurement frequency. For example, in the two cases simulated, it takes twice the amount of time to get the PSD at 0.5 kHz than at 1 kHz.

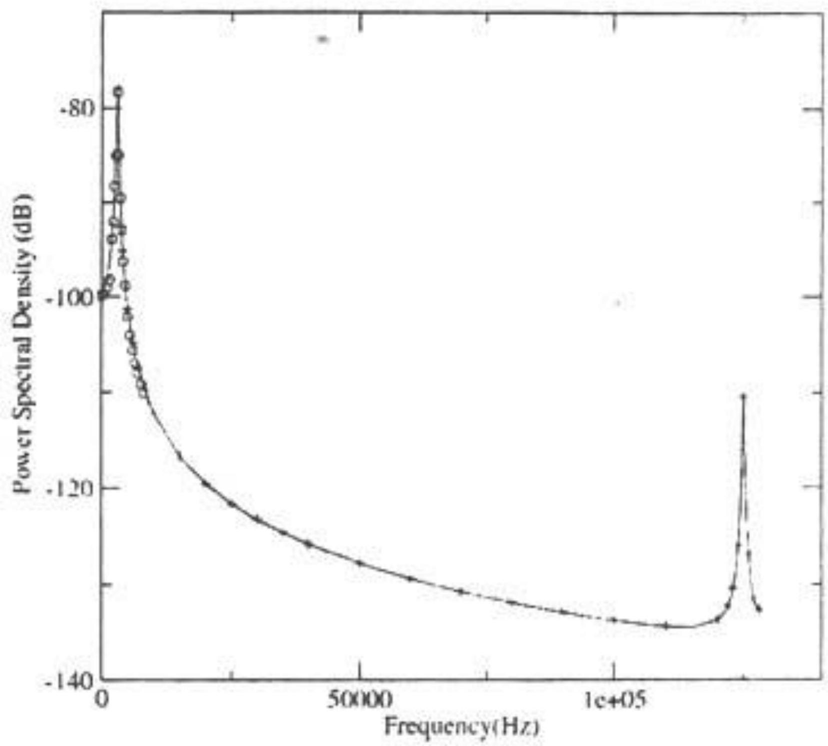

Figure 2: Output noise spectral density of the bandpass filter.

DISCUSSION

In this paper, the possibility of using the MFT technique to speed up computations of the noise spectral density was explored. It turns out that it is extremely efficient and requires only two integrations over a clock period for each frequency at which the spectrum is desired. Unlike the shooting Newton technique, the time required is also relatively independent of the measurement frequency.

REFRENCES:

[1] D.A Johna and K. Martin, *Analog Integrated Circuit Design*. New York: Wiley, 1997

[2] C. Enz and G.C Temes, “Circuit techniques for reducing the effects of op-amp imperfections: Autozeroing, correlated double sampling, and

chopper stabilization," *Proc. IEEE*, vol. 84, no. 11, pp. 1584-1614, Nov. 1996.

[3] R. Gregorian and G.C.Temes, *Analog MOS Integrated Circuits for Signal Processing*. New York: Wiley, 1986, sec. 7.4.

[4] R.Schreier and G.C Temes, *Understanding Delta-Sigma Data Converters*. Piscataway, NJ: IEEE Press/Wiley, 2005. Appendix C.

[5] J. Steensgaard. "Noise sources in switched-capacitor delta-sigma modulators," in *Lacture Notes for "Practical Approach to Delta-Sigma Design" EPFL/MEAD Microelectronics Short Course,* Lausanne, Switzerland, Jun. 21-25, 20004, [Online]: Available

http://www.steensgaard.org/jesper/publications. htm# DSM_ NIOSE_2004_06_23.

[6] W.Ki and G.C Temes "Offset-compensated switched-capacitor integrator," in *Proc. IEEC Int. Symp. Cicuits and Systems*, May 1990, pp.2829-2832.

[7] A. Heck, *Introduction to Maple*. New York: Springer-Verlag, 2003.

[8] K. Kundert. Simulating switched-capacitor filters with Spectre RF. [Online]. Avilable:http//www.designers-guide.com/Analysis//scfulters. Pdf

[9] L. Toth, I. Yusim, and K. Suyama, "Noise analysis of ideal switched capacitor network," *IEEE Trans. Circuits syst. I, Fundam. Theory Appl.*, vol. 46, no. 3, pp. xx-xx. Mar. 1999. Rechard Schreier (S'89-M'91) received the Ph.D. degree from the University to Toronto, Toronto, Canada, in 1991.

[10] A. Demir, E.W.Y.Liu, and A.L.Sangiovanni- Vincentelli. Time domain non-monte carlo noise simulation for nonlinear dynamic circuits with arbitrary excitations. *IEEE. Trans. Computer Aided Design*, 15(5):493-505,May 1996.

[11] C.Gobet and A.Knob. Noise analysis of switched capacitor networks. *IEEE Trans. Circuits and Syst.* CAS-30(1):37-43,Jan 1983.

[12] D. Feng and J. Phillips and K. Nabors and K. Kundert and J. White Efficient computation of quasi-periodic circuit operating conditions via a mixed frequency / time approach. In *Proceedings of the Design Automation conference* Pages 635-640, 1999.

[13] D.G. Lampard. Generalization of the weiner-khintchine theorem to nostationary processes. *J.Appl. Physice*, 25 (6):802-803, 1952.

[14] F.Yuan and A.Opal. Adjoint network of periodically switched linear circuits with application to noise analysis. *IEEE Trans. Circuits and Syst.-I*, 48(2):139-151, Feb. 2012.

[15] J.Chen and S. Kang. A Mixed Frequency-Time Approach For Quasi-Periodic Steady-State Simulation of Multi-Level Modeled Circuits. Pages VI-194-197, 1999.

[16] J. Goette and C.Gobet. Exact noise analysis of sc circuits and an approximate computer implementation. *IEEE Trans. Circuits and Syst.*, 3(4):508-521, April 1989.

[17] W.A Gardner. Introduction to Random Processes: with application to signals and systems. New York: McGraw-Hill, 1989.