

# TWO UNIT COLD STANDBY SYSTEM WITH WARRANTY

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## ABSTRACT

The Present paper deals with the analysis of a two unit standby system with warranty concept. In this system there is an assumption that the operating unit fails under warranty period due to bad assembling then it is sent for repair. But if it fails due to failure of any component in the warranty period then it is sent for replacement. Failure time distributions of operating unit under warranty and after warranty are exponential. Repair and replacement time distributions are general. Using the regenerative point technique in Markov renewal process, the various reliability characteristics of the system model under study are obtained.

**Kew words :** Markov Renewal Process, Warranty, Availability, Busy period.

## INTRODUCTION

Various authors [ 2, 5, 7, 10 ] have analysed several reliability models in which failed units are repaired. But in the present study the idea of warranty period and replacement is incorporated . Keeping this idea in mind we develop an engineering system which consist of two units with configuration that one is operative and other as

cold standby. If the operative unit fails in the warranty period due to bad assembling, then it is sent for repair. But if operative unit fails due to failure of any component in the warranty period then it is sent for replacement which is free of cost. If operative unit fails due to any types after warranty period, then it is sent for repair. A single repair facility is used for repair and replacement policies and using regenerative point technique in Markov renewal process, the following reliability measures are obtained.

- (i) Transient and steady state transition probabilities.
- (ii) Mean sojourn time
- (iii) Mean time to system failure
- (iv) Pointwise and steady state availability of the system
- (v) Expected busy period of the repairman in time interval  $(0, t]$
- (vi) Expected number of visits by the repairman in  $(0, t]$
- (vii) Profit analysis of the system.

## MODEL DESCRIPTION AND ASSUMPTIONS

- (i) The system consists of two identical units initially one unit is operative and the other as cold standby.
- (ii) Upon a failure of an operative unit, cold standby unit takes a random amount of time to become operative.
- (iii) If the operative unit fails due to bad assembling, it is sent for repair.
- (iv) If the operative unit fails due to failure of any component in the warranty period, it is sent for replacement.

- (v) If the operative unit fails due to failure of any component after warranty period, it is sent for repair.
- (vi) The repair and replacement time distribution for failed unit are general.
- (vii) Failure rate of operative unit are constant.
- (viii) A single repairman facility is used for repair and replacement.

## NOTATIONS AND STATES

$\alpha_1, \alpha_2, \alpha_3$	constant failure rates of the operative unit when it fails due to bad assembling, component fails in warranty period and after warranty period respectively.
$f(.), F(.)$	P.d.f and c.d.f. of time to complete the repair of bad assembling.
$g(.), G(.)$	P.d.f. and c.d.f of time to complete the replacement.
$h(.), H(.)$	P.d.f. and c.d.f of time to complete the repair after warranty period.
$m_1, m_2, m_3$	Mean time for repair of bad assembling, replacement and repair after warranty period respectively.
$N_o$	Normal unit kept as operative.
$N_s$	Normal unit kept as cold standby.
$F_{r_1}$	Failed unit caused by bad assembling is under repair.
$F_{r_2}$	Failed unit caused by failure of any component in the warranty period is under replacement.
$F_{r_3}$	Failed unit caused by failure of any component after warranty period is under repair.
$F_{wr_1}$	Failed unit caused by bad assembling is waiting for repair.

- $F_{Wr_2}$  Failed unit caused by failure of any component in the warranty period is waiting for replacement.
- $F_{Wr_3}$  Failed unit caused by failure of any component after warranty period is waiting for repair.
- $F_{R_1}$  Repair of failed unit caused by bad assembling is continued from earlier state.
- $F_{R_2}$  Replacement of failed unit caused by failure of any component in the warranty period is continued from earlier state.
- $F_{R_3}$  Repair of failed unit caused by failure of any component after warranty period is continued from earlier state.

Using above notations and assumptions the possible states of the system are.

### UP States

$$S_0 : (N_o, N_s) \quad , \quad S_1 : (N_o, F_{r_1})$$

$$S_2 : (N_o, N_{r_2}) \quad , \quad S_3 : (N_o, F_{r_3})$$

### Down States

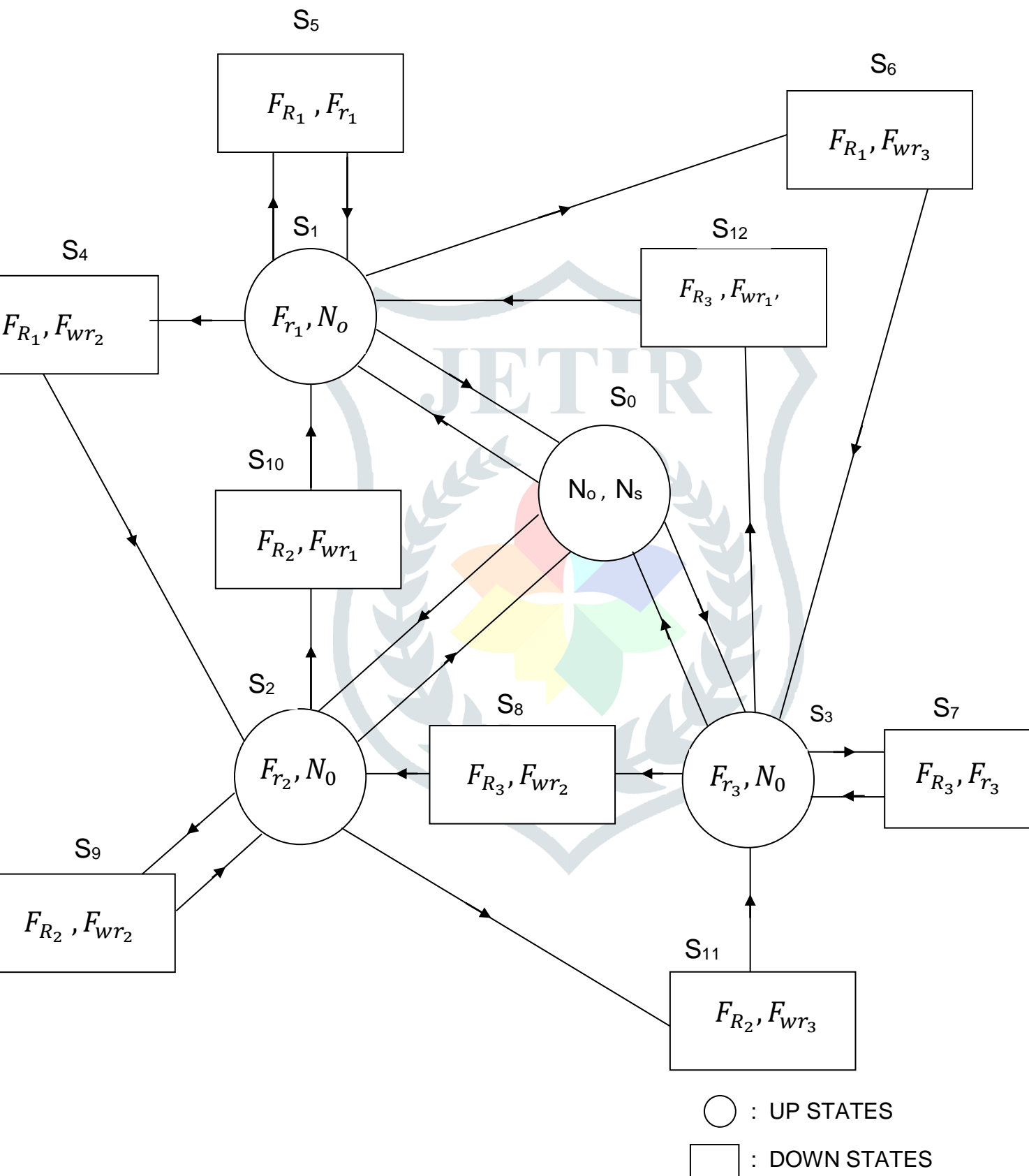
$$S_4 : (F_{R_1}, F_{r_2}) \quad , \quad S_5 : (F_{R_1}, F_{Wr_1}) \quad , \quad S_6 : (F_{R_1}, F_{Wr_3})$$

$$S_7 : (F_{R_3}, F_{Wr_3}) \quad , \quad S_8 : (F_{R_3}, F_{Wr_2}) \quad , \quad S_9 : (F_{R_2}, F_{Wr_2})$$

$$S_{10} : (F_{R_2}, F_{Wr_1}) \quad , \quad S_{11} : (F_{R_2}, F_{Wr_3}) \quad , \quad S_{12} : (F_{R_3}, F_{Wr_1})$$

The states  $S_0, S_1, S_2$  and  $S_3$ , are regenerative while  $S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}$  and  $S_{12}$ , are non-regenerative states. The possible states and transitions between them are shown in the following figure.

# Transition Diagram



## TRANSITION AND STEADY STATE PROBABILITIES

The non zero elements of the transition probability,  $P = (P_{ij})$  are given as under

$$P_{01} = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_3} = \frac{\alpha_1}{\sum \alpha_i}, \quad \because \sum \alpha_i = \alpha_1 + \alpha_2 + \alpha_3$$

$$P_{02} = \frac{\alpha_2}{\sum \alpha_i}, \quad P_{03} = \frac{\alpha_3}{\sum \alpha_i}, \quad P_{10} = f^*(\sum \alpha_i)$$

$$P_{14} = \frac{\alpha_2}{\sum \alpha_i} [1 - f^*(\sum \alpha_i)] = P_{12}^{(4)}$$

$$P_{15} = \frac{\alpha_1}{\sum \alpha_i} [1 - f^*(\sum \alpha_i)] = P_{11}^{(5)}$$

$$P_{16} = \frac{\alpha_3}{\sum \alpha_i} [1 - f^*(\sum \alpha_i)] = P_{13}^{(6)}$$

$$P_{20} = g^*(\sum \alpha_i), \quad P_{29} = \frac{\alpha_2}{\sum \alpha_i} [1 - g^*(\sum \alpha_i)] = P_{22}^{(10)}$$

$$P_{2,10} = \frac{\alpha_1}{\sum \alpha_i} [1 - g^*(\sum \alpha_i)] = P_{21}^{(10)}$$

$$P_{2,11} = \frac{\alpha_3}{\sum \alpha_i} [1 - g^*(\sum \alpha_i)] = P_{23}^{(11)}$$

$$P_{30} = h^*(\sum \alpha_i), \quad P_{37} = \frac{\alpha_3}{\sum \alpha_i} [1 - h^*(\sum \alpha_i)] = P_{33}^{(7)}$$

$$P_{38} = \frac{\alpha_2}{\sum \alpha_i} [1 - h^*(\sum \alpha_i)] = P_{32}^{(8)}$$

$$P_{3,12} = \frac{\alpha_1}{\sum \alpha_i} [1 - h^*(\sum \alpha_i)] = P_{31}^{(12)}$$

$$P_{42} = P_{51} = P_{61} = P_{73} = P_{82} = P_{92} = 1$$

$$P_{10,1} = P_{11,3} = P_{12,1} = 1$$

The above probabilities satisfies the following relations :

$$P_{01} + P_{02} + P_{03} = 1$$

$$P_{10} + P_{14} + P_{15} + P_{16} = 1 = P_{10} + P_{11}^{(5)} + P_{12}^{(4)} + P_{13}^{(6)}$$

$$P_{20} + P_{29} + P_{2,10} + P_{2,11} = 1 = P_{20} + P_{21}^{(10)} + P_{22}^{(9)} + P_{23}^{(11)}$$

$$P_{30} + P_{37} + P_{38} + P_{3,12} = 1 = P_{30} + P_{31}^{(12)} + P_{32}^{(8)} + P_{33}^{(7)}$$

### Mean Sojourn Time

Mean sojourn time  $\mu_i$  in state  $S_i \in E$  is defined as the expected time for which the system stays in state  $S_i$  before transiting to any other state. If  $T_i$  denotes the sojourn time in state  $S_i$ , then the mean sojourn time is given by

$$\mu_i = E[T] = \int P [T_i > t] dt$$

Thus, the mean sojourn time in regenerative state  $i$  are,

$$\mu_0 = \frac{1}{\sum \alpha_i} , \mu_1 = \frac{1}{\sum \alpha_i} [1 - f^*(\sum \alpha_i)]$$

$$\mu_2 = \frac{1}{\sum \alpha_i} [1 - g^*(\sum \alpha_i)]$$

$$\mu_3 = \frac{1}{\sum \alpha_i} [1 - h^*(\sum \alpha_i)]$$

The conditional mean sojourn time in state  $S_i$ , given that the system is to transit to state  $S_j$ , we mathematically define as

$$m_{ij} = \int tdQ(t)$$

Thus, we have

$$m_{01} = \frac{\alpha_1}{(\sum \alpha_i)^2}, m_{02} = \frac{\alpha_2}{(\sum \alpha_i)^2}, m_{03} = \frac{\alpha_3}{(\sum \alpha_i)^2},$$

$$m_{10} = \int te^{-(\sum \alpha_i)t} dF(t)$$

$$m_{14} = \frac{\alpha_2}{\sum \alpha_i} \left[ \frac{1}{\sum \alpha_i} - \int te^{-(\sum \alpha_i)t} dF(t) - \frac{f^*(\sum \alpha_i)}{\sum \alpha_i} \right]$$

$$m_{15} = \frac{\alpha_1}{\sum \alpha_i} \left[ \frac{1}{\sum \alpha_i} - \int te^{-(\sum \alpha_i)t} dF(t) - \frac{f^*(\sum \alpha_i)}{\sum \alpha_i} \right]$$

$$m_{16} = \frac{\alpha_3}{\sum \alpha_i} \left[ \frac{1}{\sum \alpha_i} - \int te^{-(\sum \alpha_i)t} dF(t) - \frac{f^*(\sum \alpha_i)}{\sum \alpha_i} \right]$$

$$m_{20} = \int te^{-(\sum \alpha_i)t} dG(t)$$

$$m_{29} = \frac{\alpha_2}{\sum \alpha_i} \left[ \frac{1}{\sum \alpha_i} - \int te^{-(\sum \alpha_i)t} dG(t) - \frac{g^*(\sum \alpha_i)}{\sum \alpha_i} \right]$$

$$m_{2,10} = \frac{\alpha_1}{\sum \alpha_i} \left[ \frac{1}{\sum \alpha_i} - \int te^{-(\sum \alpha_i)t} dG(t) - \frac{g^*(\sum \alpha_i)}{\sum \alpha_i} \right]$$

$$m_{2,11} = \frac{\alpha_3}{\sum \alpha_i} \left[ \frac{1}{\sum \alpha_i} - \int te^{-(\sum \alpha_i)t} dG(t) - \frac{g^*(\sum \alpha_i)}{\sum \alpha_i} \right]$$

$$m_{30} = \int te^{-(\sum \alpha_i)t} dH(t)$$

$$m_{37} = \frac{\alpha_3}{\sum \alpha_i} \left[ \frac{1}{\sum \alpha_i} - \int te^{-(\sum \alpha_i)t} dH(t) - \frac{h^*(\sum \alpha_i)}{\sum \alpha_i} \right]$$

$$m_{38} = \frac{\alpha_2}{\sum \alpha_i} \left[ \frac{1}{\sum \alpha_i} - \int te^{-(\sum \alpha_i)t} dH(t) - \frac{h^*(\sum \alpha_i)}{\sum \alpha_i} \right]$$



$$m_{3,12} = \frac{\alpha_1}{\sum \alpha_i} \left[ \frac{1}{\sum \alpha_i} - \int t e^{-(\sum \alpha_i)t} dH(t) - \frac{h^*(\sum \alpha_i)}{\sum \alpha_i} \right]$$

It can be easily verified that

$$m_{01} + m_{02} + m_{03} = \mu_0$$

$$m_{10} + m_{14} + m_{15} + m_{16} = \mu_1$$

$$m_{10} + m_{11}^{(5)} + m_{12}^{(4)} + m_{13}^{(6)} = m_1$$

$$m_{20} + m_{29} + m_{2,10} + m_{2,11} = \mu_2$$

$$m_{20} + m_{21}^{(10)} + m_{22}^{(9)} + m_{23}^{(11)} = m_2$$

$$m_{30} + m_{37} + m_{38} + m_{3,12} = \mu_3$$

$$m_{30} + m_{31}^{(12)} + m_{32}^{(8)} + m_{33}^{(7)} = m_3$$

## MEAN TIME TO SYSTEM FAILURE

To investigate the distribution function  $\pi_i(t)$  of the time to system failure with starting state  $S_i$ , we regard the failed states as absorbing. By using the probabilistic arguments, the recursive relations among  $\pi_i(t)$  are :

$$\pi_0(t) = Q_{01}(t)\pi_1(t) + Q_{02}(t)\pi_2(t) + Q_{03}(t)\pi_3(t)$$

$$\pi_1(t) = Q_{10}(t)\pi_0(t) + Q_{14}(t) + Q_{15}(t) + Q_{16}(t)$$

$$\pi_2(t) = Q_{20}(t)\pi_0(t) + Q_{29}(t) + Q_{2,10}(t) + Q_{2,11}(t)$$

$$\pi_3(t) = Q_{30}(t)\pi_0(t) + Q_{37}(t) + Q_{38}(t) + Q_{3,12}(t)$$

( 1 – 4 )

Taking Laplace – Stieltjes transform of ( 1 – 4 ) and solving them for  $\tilde{\pi}_0(s)$  by omitting the arguments 's' for brevity, we get

$$MTSF = E(T) = \frac{d\tilde{\pi}_0(s)}{ds} \Big|_{s=0} = \frac{N_1}{D_1} \quad (5)$$

Where

$$N_1 = \mu_0 + P_{01} \mu_1 + P_{02} \mu_{02} + P_{03} \mu_3$$

and

$$D_1 = 1 - (P_{01}P_{10} + P_{02}P_{20} + P_{03}P_{30}) \quad (6-7)$$

## AVAILABILITY ANALYSIS

If  $M_i(i)$  defines the probability that the system is in a regenerative state,  $S_i$  remains up at least for time t without transiting to any other regenerative state, then

$$M_0(t) = e^{-(\sum \alpha_i)t}, \quad M_1(t) = e^{-(\sum \alpha_i)t} \bar{F}(t)$$

$$M_2(t) = e^{-(\sum \alpha_i)t} \bar{G}(t), \quad M_3(t) = e^{-(\sum \alpha_i)t} \bar{H}(t)$$

By using the arguments of the theory of regenerative process, the pointwise availability  $A_i(t)$  is satisfy the following recursive relations :

$$A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) + q_{03}(t) \odot A_3(t)$$

$$A_1(t) = M_1(t) + q_{10}(t) \odot A_0(t) + q_{11}^{(5)}(t) \odot A_1(t) + q_{12}^{(4)}(t) \odot A_2(t)$$

$$+q_{13}^{(6)}(t) \odot A_3(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \odot A_0(t) + q_{21}^{(10)}(t) \odot A_1(t) + q_{22}^{(9)}(t) \odot A_2(t)$$

$$+q_{23}^{(11)}(t) \odot A_3(t)$$

$$A_3(t) = M_3(t) + q_{30}(t) \odot A_0(t) + q_{31}^{(12)}(t) \odot A_1(t) + q_{32}^{(8)}(t) \odot A_2(t)$$

$$+q_{33}^{(7)}(t) \odot A_3(t)$$

( 8 - 11)

Taking the Laplace Transform of the above relation ( 8 – 11 ) and solving for  $A_0^*(s)$ . By omitting the arguments 's' for brevity, we have

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)} \quad (9)$$

The steady state availability of the system is given by

$$A_0 = \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_2(0)}{D_2'(0)} = \frac{N_2}{D_2} \quad (10)$$

Where

$$\begin{aligned} N_2 = & \mu_0 \left[ \left(1 - P_{21}^{(5)}\right) \left\{ \left(1 - P_{22}^{(9)}\right) \left(1 - P_{33}^{(7)}\right) - P_{23}^{(11)} P_{32}^{(8)} \right\} - P_{12}^{(4)} \times \right. \\ & \left. \left\{ P_{21}^{(10)} \left(1 - P_{33}^{(7)}\right) - P_{23}^{(11)} P_{31}^{(12)} \right\} - P_{13}^{(6)} \left\{ P_{32}^{(8)} P_{21}^{(10)} + P_{31}^{(12)} \left(1 - P_{22}^{(9)}\right) \right\} \right] \\ & + \mu_1 \left[ P_{01} \left\{ \left(1 - P_{22}^{(9)}\right) \left(1 - P_{33}^{(7)}\right) - P_{23}^{(11)} P_{32}^{(8)} \right\} \right. \\ & \left. + P_{02} \left\{ P_{21}^{(10)} \left(1 - P_{33}^{(7)}\right) - P_{23}^{(11)} P_{31}^{(12)} \right\} \right] \end{aligned}$$

$$\begin{aligned}
& + P_{03} \left\{ P_{32}^{(8)} P_{21}^{(10)} + P_{31}^{(12)} (1 - P_{22}^{(9)}) \right\} \\
& + \mu_2 \left[ P_{01} P_{12}^{(4)} (1 - P_{33}^{(7)}) + P_{01} P_{13}^{(6)} P_{32}^{(8)} \right. \\
& + P_{02} (1 - P_{11}^{(5)}) (1 - P_{33}^{(7)}) + P_{02} P_{13}^{(6)} P_{31}^{(12)} \\
& + P_{03} (1 - P_{11}^{(5)}) P_{32}^{(8)} + P_{03} P_{31}^{(12)} P_{12}^{(4)} \left. \right] \\
& + \mu_3 \left[ P_{01} P_{12}^{(4)} P_{23}^{(11)} + P_{01} P_{13}^{(6)} (1 - P_{22}^{(9)}) \right. \\
& + P_{02} P_{23}^{(11)} (1 - P_{11}^{(5)}) + P_{02} P_{13}^{(6)} P_{21}^{(10)} \\
& + P_{03} (1 - P_{11}^{(5)}) (1 - P_{22}^{(10)}) - P_{03} P_{12}^{(4)} P_{21}^{(10)} \left. \right]
\end{aligned}$$

and

$$\begin{aligned}
D_2 = & \mu_0 \left[ (1 - P_{33}^{(7)}) \left\{ P_{10} (1 - P_{22}^{(9)}) + P_{12}^{(4)} P_{20} \right\} \right. \\
& + P_{13}^{(6)} \left\{ P_{30} (1 - P_{22}^{(9)}) + P_{32}^{(8)} P_{20} \right\} \\
& + P_{23}^{(11)} \left( P_{12}^{(4)} P_{30} - P_{10} P_{32}^{(8)} \right) \left. \right] \\
& + m_1 \left[ (1 - P_{22}^{(9)}) (1 - P_{33}^{(7)} - P_{03} P_{30}) \right. \\
& - P_{02} P_{20} (1 - P_{33}^{(7)}) - P_{32}^{(8)} (P_{23}^{(11)} + P_{03} P_{20}) - P_{02} P_{23}^{(11)} P_{30} \left. \right] \\
& + m_2 \left[ (1 - P_{33}^{(7)}) (P_{12}^{(4)} + P_{02} P_{10}) \right. \\
& + P_{03} (P_{10} P_{32}^{(8)} - P_{12}^{(4)} P_{30}) + P_{13}^{(6)} (P_{32}^{(8)} - P_{02} P_{03}) \left. \right]
\end{aligned}$$

$$\begin{aligned}
& + m_3 \left[ \left(1 - P_{22}^{(9)}\right) \left(P_{13}^{(6)} + P_{03} P_{10}\right) \right. \\
& \left. + P_{12}^{(4)} \left(P_{23}^{(11)} + P_{03} P_{20}\right) + P_{02} \left(P_{10} P_{23}^{(11)} - P_{13}^{(6)} P_{20}\right) \right]
\end{aligned}$$

## BUSY PERIOD ANALYSIS

$B_i(i)$  is defined as the probability that the repairman is busy at epoch  $t$  starting from  $S_i \in E$ . From elementary probabilistic arguments, we get  $B_0(t) = q_{01}(t) \odot B_1(t) + q_{02}(t) \odot B_2(t) + q_{03}(t) \odot B_3(t)$

$$\begin{aligned}
B_1(t) &= W_1(t) + q_{10}(t) \odot B_0(t) + q_{11}^{(5)}(t) \odot B_1(t) + q_{12}^{(4)}(t) \odot B_2(t) \\
& \quad + q_{13}^{(6)}(t) \odot B_3(t)
\end{aligned}$$

$$\begin{aligned}
B_2(t) &= W_2(t) + q_{20}(t) \odot B_0(t) + q_{21}^{(10)}(t) \odot B_1(t) + q_{22}^{(9)}(t) \odot B_2(t) \\
& \quad + q_{23}^{(11)}(t) \odot B_3(t)
\end{aligned}$$

$$\begin{aligned}
B_3(t) &= W_3(t) + q_{30}(t) \odot B_0(t) + q_{31}^{(12)}(t) \odot B_1(t) + q_{32}^{(8)}(t) \odot B_2(t) \\
& \quad + q_{33}^{(7)}(t) \odot B_3(t)
\end{aligned}$$

( 11 – 14 )

Where

$$W_1(t) = e^{-(\Sigma \alpha_i)t} \bar{F}(t) \quad , \quad W_2(t) = e^{-(\Sigma \alpha_i)t} \bar{G}(t)$$

$$W_3(t) = e^{-(\Sigma \alpha_i)t} \bar{H}(t)$$

Taking the Laplace Transform of the above equations ( 11 – 14 ) and solving for  $B_0^*(s)$ . By omitting the argument 's' for brevity, we obtain

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)} \quad (15)$$

The steady state busy period, when the system starts from  $S_i$ , is obtained as follows

$$B_0 = \lim_{s \rightarrow 0} s B_0^*(s) = \frac{N_3(o)}{D_2(o)} = \frac{N_3}{D_2} \quad (16)$$

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$$\begin{aligned} N_3 = & \mu_1 \left[ \left(1 - P_{22}^{(9)}\right) \left\{ \left(1 - P_{33}^{(7)}\right) - P_{03}P_{30} \right\} - P_{02}P_{20} \left(1 - P_{33}^{(7)}\right) \right. \\ & \left. - P_{32}^{(8)} \left( P_{23}^{(11)} + P_{03}P_{20} \right) - P_{02} P_{30} P_{23}^{(11)} \right] \\ & + \mu_2 \left[ \left(1 - P_{33}^{(7)}\right) \left( P_{12}^{(4)} + P_{02}P_{10} \right) + P_{32}^{(8)} \left( P_{13}^{(6)} + P_{10}P_{03} \right) \right. \\ & \left. + P_{30} \left( P_{02} P_{13}^{(6)} - P_{03} P_{12}^{(4)} \right) \right] \\ & + \mu_3 \left[ \left(1 - P_{22}^{(9)}\right) \left( P_{13}^{(6)} + P_{03}P_{10} \right) + P_{23}^{(11)} \left( P_{12}^{(4)} + P_{02}P_{10} \right) \right. \\ & \left. + P_{20} \left( P_{03} P_{12}^{(4)} - P_{02} P_{13}^{(6)} \right) \right] \end{aligned}$$

And  $D_2$  is defined as availability analysis.

### EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

$V_i(t)$  is defined as the expected number of visits by the repairman in  $(0, t]$ , given that the system initially starts from regenerative state  $S_i$ . Using Probabilistic arguments, we get the following recursive equations:

$$V_0(t) = Q_{01}(t) \$ [1 + V_1(t)] + Q_{02}(t) \$ [1 + V_2(t)] + Q_{03}(t) \$ [1 + V_3(t)]$$

$$V_1(t) = Q_{10}(t) \$ V_0(t) + Q_{11}^{(5)}(t) \$ V_1(t) + Q_{12}^{(4)}(t) \$ V_2(t) + Q_{13}^{(6)}(t) \$ V_3(t)$$

$$V_2(t) = Q_{20}(t) \$ V_0(t) + Q_{21}^{(10)}(t) \$ V_1(t) + Q_{22}^{(9)}(t) \$ V_2(t) + Q_{23}^{(11)}(t) \$ V_3(t)$$

$$V_3(t) = Q_{30}(t) \$ V_0(t) + Q_{31}^{(12)}(t) \$ V_1(t) + Q_{32}^{(8)}(t) \$ V_2(t) + Q_{33}^{(7)}(t) \$ V_3(t)$$

( 17 – 20 )

Taking Laplace – Stieltjes transform of ( 17 – 20 ) and solving them for  $\tilde{V}_0(s)$  by omitting 's' for brevity, we get

$$\tilde{V}_0(s) = \frac{N_4(s)}{D_2(s)} \quad (21)$$

In steady state, the number of visits per unit time is given by

$$V_0 = \lim_{t \rightarrow \infty} \left( \frac{V_0(t)}{t} \right) = \lim_{s \rightarrow 0} s \tilde{V}_0(s) = \frac{N_4}{D_2} \quad (22)$$

Where

$$\begin{aligned} N_4 = & \left(1 - P_{11}^{(5)}\right) \left[ \left(1 - P_{22}^{(9)}\right) \left(1 - P_{33}^{(7)}\right) - P_{23}^{(11)} P_{32}^{(8)} \right] \\ & - P_{12}^{(4)} \left[ P_{21}^{(10)} \left(1 - P_{33}^{(7)}\right) + P_{23}^{(11)} P_{31}^{(12)} \right] \\ & - P_{13}^{(6)} \left[ P_{32}^{(8)} P_{21}^{(10)} + P_{31}^{(12)} \left(1 - P_{22}^{(9)}\right) \right] \end{aligned}$$

and

$D_2$  is the same as in availability analysis.

## PROFIT ANALYSIS

The profit obtained to the system model in steady state can be obtained as

$$P = K_0 A_0 - K_1 B_0 - K_2 V_0 \quad (23)$$

## Where

$K_0$  = Revenue per unit up time of the system

$K_1$  = Cost per unit time for which the repairman is busy

$K_2$  = Cost per visit by the repairman.

## CONCLUSION

In the previous study it was assumed that the operative unit fails then it was sent for repair. But in the present study the concept of warranty period is used. For making the system more effective, it is considered that the operative unit fails due to failure of any component under warranty period then it is sent for replacement in stead of repair. The optimum results are obtained which are in the equation (5) , (10) , (16), (22), and (23).

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