Fluctuating magneto hydrodynamic convective flow over a vertical porous plate with slip flow and oscillating boundary conditions

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ABSTRACT:

The study investigates the unsteady free convective viscous incompressible flow over an infinite vertical porous flat plate with variable heat and mass transfer in a slip-flow regime. Assuming variable suction at the porous plate, approximate solutions for velocity, skin friction, temperature, and species concentration are derived. The effects of G_r, G_c, S_c, A, and β\beta for $P_r = 0.71$ (air) are presented graphically and discussed qualitatively. It is observed that an increase in the Grashof number leads to a decrease in fluid velocity. Conversely, an increase in the modified Grashof number generally results in an increase in velocity. As the Schmidt number increases, the velocity initially increases within the boundary layer region but subsequently decreases. Magnetic intensity is found to suppress fluid velocity. Additionally, an increase in pore size and Prandtl number causes a decrease in fluid velocity. The suction parameter significantly influences temperature; as suction increases, the temperature field decreases. Similarly, fluid concentration decreases with increasing suction. For a fixed magnetic intensity, an increase in the Grashof number results in increased skin friction. However, as magnetic intensity increases, skin friction decreases. The influence of amplitude on velocity profiles is examined, showing that increased amplitude leads to decreased fluid velocity and occasional backward flow near the boundary layer, which diminishes farther from the boundary

Key words: Free Convection, In compressible Fluid, Heat and Mass Transfer.

1. INTRODUCTION

Free convection is applicable in various industrial and atmospheric applications involving transport processes driven by temperature and chemical differences. Heat transfer in porous media occurs in natural phenomena and engineered processes, influenced by the thermal properties and volume fractions of the medium. Free convection flow can be enhanced by adding oscillating temperatures to a mean plate temperature. Transient free convective flow acts as a cooling device in situations requiring efficient heat and mass transfer. It is also important in the early stages of melting near a heated surface Magneto convection is important in controlling mountain iron flow, industrial liquid metal cooling in nuclear reactors, and magnetic separation of molecular semi-conducting materials. An example is the separation of Uranium U_{235} from U_{238} by gas diffusion in nuclear power stations. When mass transfer occurs in a fluid at rest, it is driven purely by molecular diffusion due to concentration gradients. The extensive technological and industrial applications of heat and mass transfer in convection flows have generated considerable interest in this field of study. The temperature of the surrounding stationary fluid and the plate were considered to be constant in all of the aforementioned presentations. Nevertheless, many industrial applications are not that straightforward, and at high temperatures, the plate temperature frequently begins to oscillate around a non-zero mean temperature. The particle next to a solid surface no longer takes on the surface's velocity in many real-world applications. It "slips" nearly along the surface since the particles at the surface always have a finite tangential velocity. Because of this, the flow regime is known as the slip-flow regime, and it is impossible to ignore this effect. Therefore, to make the effect more realistic and the suitability of the industrial and scientific applications.

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2.MATHEMATICAL FORMULATION

An unsteady free magneto hydrodynamic (MHD) convective flow of a viscous, in compressible fluid past an infinite vertical porous flat plate in a slip-flow regime, with periodic temperature and concentration, is considered. The variable suction velocity distribution $V^* = V_0 - A^{ei\omega t}$ fluctuates with respect to time.A coordinate system is employed with the wall lying vertically in the x^* y^* plane. The x^* -axis is taken vertically upward along the vertical porous plate. Since the plate is considered infinite in the $x*$ -direction, all physical quantities are independent of $x*$. Under these assumptions, the physical variables are purely functions of y^* and t. By neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq's approximation), the problem can be governed by the following set of equations.

$$\frac{\partial u^*}{\partial t^*} - V_0^*\left(1 + \varepsilon A e^{i\omega^* t^*}\right)\frac{\partial u^*}{\partial y^*} = g\beta(T^* - T_\infty^*) + g\beta^0(C^* - C_\infty^*) + v\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma\beta_0^2}{\rho}u^* - \frac{v}{K^*}u^* \tag{1}$$

$$\rho C_P\left[\frac{\partial T^*}{\partial t^*} - V_0^*\left(1 + \varepsilon A e^{i\omega^* t^*}\right)\frac{\partial T^*}{\partial y^*}\right] = k\frac{\partial^2 T^*}{\partial y^{*2}} \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} - V_0^*\left(1 + \varepsilon A e^{i\omega^* t^*}\right)\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \tag{3}$$

The boundary conditions of the problem are:

$$\left.\begin{aligned} &u^* = B^* e^{i t}, \; T^* = T_w^* + \varepsilon(T_w^* - T_\infty^*)e^{i\omega^* t^*}, \; C^* = C_w^* + \varepsilon(C_w^* - C_\infty^*)e^{i\omega^* t^*} \text{ at } y^* = 0 \\ &u^* \to 0, \; T^* \to T_\infty^*, \; C^* \to C_\infty^* \text{ as } y^* \to \infty \end{aligned}\right\} \tag{4}$$

We now introduce the following non-dimensional quantities into Eqs. (1) to (4)

$$y = \frac{y^* V_0^*}{v},\; t = \frac{t^* V_0^{*2}}{4v},\; u = \frac{u^*}{V_0^*},\; \omega = \frac{4v\omega^*}{V_0^{*2}},\; \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*},\; C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}.$$
$$Gr = \frac{g\beta v(T_w^* - T_\infty^*)}{V_0^{*3}},\; Gr = \frac{g\beta^\circ v(C_w^* - C_\infty^*)}{V_0^{*3}},\; \Pr = \frac{\mu C_p}{k} = \frac{v\rho C_p}{k},\; Sc = \frac{V}{D}$$
$$M = \frac{\sigma\beta_0^2 v}{\rho V_0^{*2}},\; K = \frac{K^* V_0^{*2}}{V^2},\; B = \frac{B^*}{V_0^*}.$$

All physical variables are defined in nomenclature. The $(*)$ stands for dimensional quantities. The subscript (∞) denotes the free stream condition. Then equations (1) to (3) reduce to the following non-dimensional form:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y} = Gr\theta + GcC + \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{K} \tag{5}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{i\omega x}\right)\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{6}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{7}$$

The boundary conditions to the problem in the dimensionless form are:

$$\left.\begin{array}{l} u = Be^{i\mu t},\ \theta = 1 + \varepsilon e^{i\omega t},\ C = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0 \\ u \to 0,\ \theta \to 0,\ C \to 0 \text{ as } y \to \infty \end{array}\right\} \tag{8}$$

3.METHOD OF SOLUTION:

Assuming the small amplitude oscillations ($\varepsilon < 1$), we can represent the velocity u , temperature θ and concentration C near the plate as follows:

$$u(y,t) = u_0(y) + \varepsilon u_1(y) e^{i\omega t} \tag{9}$$
$$\theta(y,t) = \theta_0(y) + \varepsilon \theta_1(y) e^{i\omega t} \tag{10}$$
$$C(y,t) = C_0(y) + \varepsilon C_1(y) e^{i\omega t} \tag{11}$$

Substituting (9) to (11) in (5) to (7), equating the coefficients of harmonic and non harmonic terms, neglecting the coefficients
of ε^2, we get

$$u_0'' + u_0' - \left(M + \frac{1}{K}\right) u_0 = - Gr\theta_0 - GcC_0 \tag{12}$$

$$u_1'' + u_1' - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right) u_1 = - Gr\theta_1 - GcC_1 - Au_0' \tag{13}$$
$$\theta_0'' + \Pr \theta_0' = 0 \tag{14}$$
$$\theta_1'' + \Pr \theta_1' - \frac{i\omega \Pr \theta_1}{4} = - A\Pr \theta_0' \tag{16}$$
$$C_0'' + ScC_0' = 0 \tag{17}$$
$$C_1'' + ScC_1' - \frac{i\omega ScC_1}{4} = - AScC_0' \tag{18}$$

The corresponding boundary conditions reduce to:

$$\left.\begin{array}{l} u_0 = 0,\ u_1 = \frac{B}{\varepsilon} e^{i(\psi-\omega)t},\ \theta_0 = 1,\ \theta_1 = 1,\ C_0 = 1,\ C_1 = 1 \text{ at } y = 0 \\ u_0 = 0,\ u_1 = 0,\ \theta_0 = 0,\ \theta_1 = 0,\ C_0 = 0,\ C_1 = 0 \text{ as } y \to \infty \end{array}\right\} \tag{19}$$

where primes denote differentiation with respect to 'y'. Solving the Eqns. (12) to (17) under the boundary conditions (18) we get:

$$\theta_0(y) = e^{-\Pr y} \tag{20}$$

$$C_0(y) = e^{-Scy} \tag{21}$$

$$\theta_1(y) = m_5 e^{-m_6 y} + m_7 e^{-\Pr y} \tag{22}$$

$$C_1(y) = m_8 e^{-m_9 y} + m_{10} e^{-Scy} \tag{23}$$
$$u_1(y) = m_{11} e^{-m_{12} y} - m_{13} e^{-m_6 y} - m_{14} e^{-m_9 y} + m_{15} e^{-m_2 y} + m_{16} e^{-Pry} + m_{17} e^{-Scy} \tag{24}$$

where

$$M_1 = M + \frac{1}{K}, \qquad m_1 =- m_3 - m_4, \qquad m_2 = \frac{1 + \sqrt{1 + 4M_1}}{2},$$

$$m_3 =- \frac{Gr}{\mathrm{Pr}^2 - \mathrm{Pr} - M_1}, m_4 =- \frac{Gc}{Sc^2 - Sc - M_1}, \quad m_5 = 1 - \frac{4iA\mathrm{Pr}}{\omega}, \quad m_6 = \frac{\mathrm{Pr} + \sqrt{\mathrm{Pr}^2 + i\omega \mathrm{Pr}}}{2},$$

$$m_7 = \frac{4iA\mathrm{Pr}}{\omega},$$

$$m_8 = 1 - \frac{4iASc}{\omega}, \qquad m_9 = \frac{Sc + \sqrt{Sc^2 + i\omega Sc}}{2},$$

$$m_{10} = \frac{4iASc}{\omega}, \qquad m_{11} = \frac{B}{\varepsilon} e^{i(\psi - \omega)t} + m_{13} + m_{14} - m_{15} - m_{16} - m_{17},$$

$$m_{12} = \frac{1 + \sqrt{1 + 4M_1 + i\omega}}{2}, \qquad m_{13} = \frac{m_5 Gr}{m_6{}^2 - m_6 - (M_1 + i\omega/4)},$$

$$m_{14} = \frac{m_8 Gc}{m_9{}^2 - m_9 - (M_1 + i\omega/4)}, \qquad m_{15} = \frac{m_1 m_2 A}{m_2{}^2 - m_2 - (M_1 + i\omega/4)},$$

$$m_{16} = \frac{m_3 A\mathrm{Pr} - m_7 Gr}{\mathrm{Pr}^2 - \mathrm{Pr} - (M_1 + i\omega/4)}, \qquad m_{17} = \frac{m_4 ASc - m_{10} Gc}{Sc^2 - Sc - (M_1 + i\omega/4)}.$$

The important characteristics of the problem are the skin-friction and heat transfer at the plate.

Skin Friction:

The dimensionless shearing stress on the surface of a body, due to a fluid motion, is known as skin-friction and is defined by the Newton's law of viscosity

$$\tau^* = \mu \left(\frac{\partial u^*}{\partial y^*}\right) \tag{25}$$

Substituting equations (21) and (24) into (9) we can calculate the shearing stress component in dimensionless form as

$$\tau = \frac{\tau^*}{\rho V_0^{*2}} = \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{26}$$

$$=- m_1 m_2 - m_3 \mathrm{Pr} - m_4 Sc + \varepsilon e^{i\omega t}(- m_{11} m_{12} + m_6 m_{13} + m_9 m_{14} - m_2 m_{15} - m_{16} \mathrm{Pr} - m_{17} Sc)$$

FIGURS

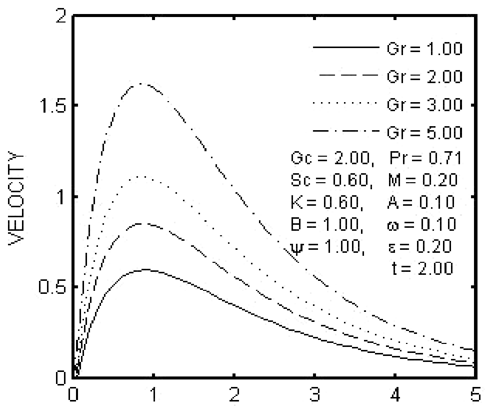

Fig.1:- Effect of Gr on velocity profiles

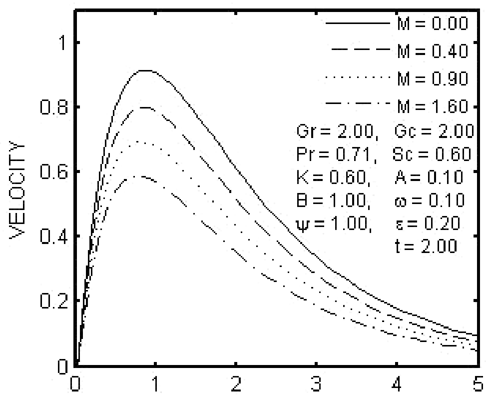

Fig. 4:- Effect of Magnetic field on velocity

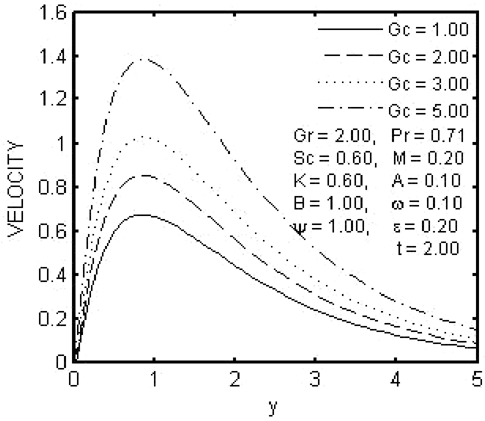

Fig2:effective of gc on velocity profiles

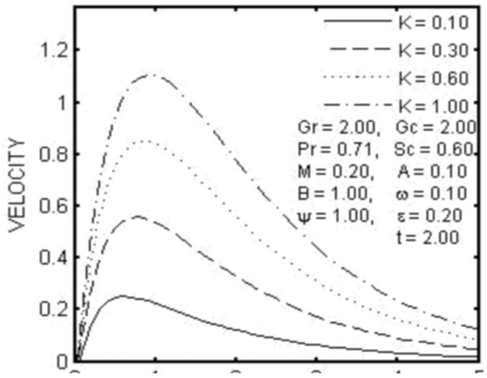

Fig5:effect on permeability on velocity

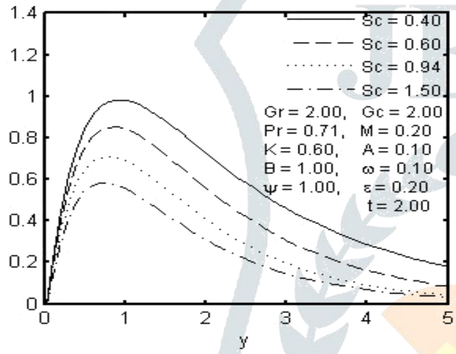

Fig3:Effect sc on velocity profile

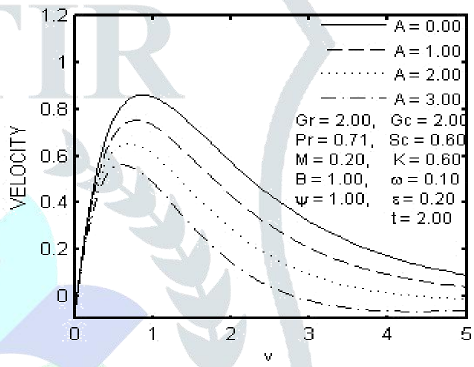

Fig 6:effect of suction parameter velocity

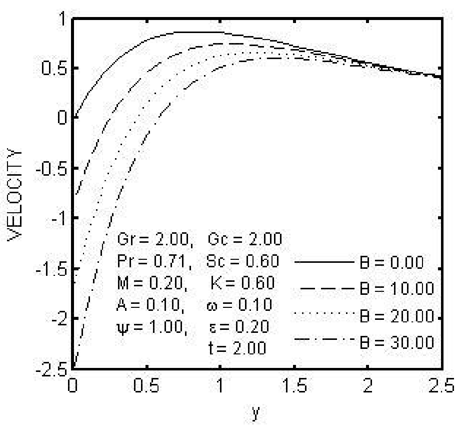

Fig. 8:- Effect of Pr on velocity profiles

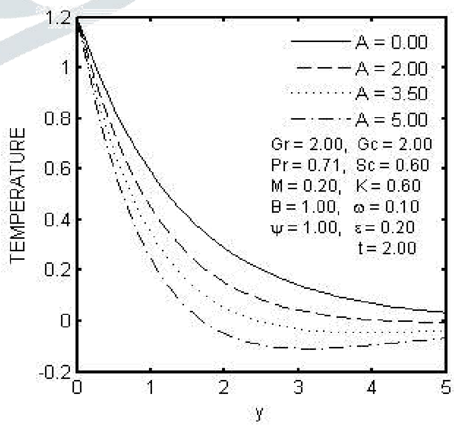

Fig.11:- Effect of Sc on concentration

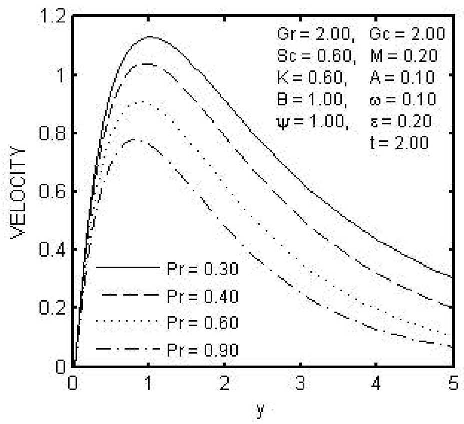

Fig. 8:- Effect of Pr on velocity profiles

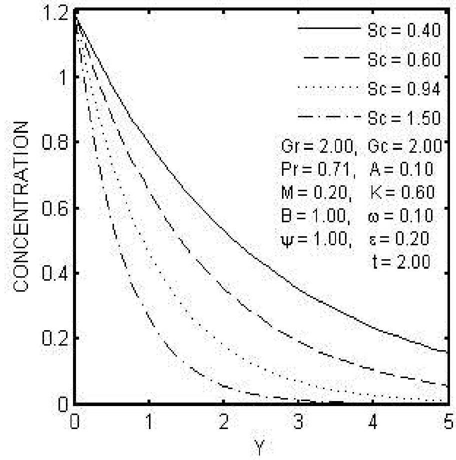

Fig.11:- Effect of Sc on concentration

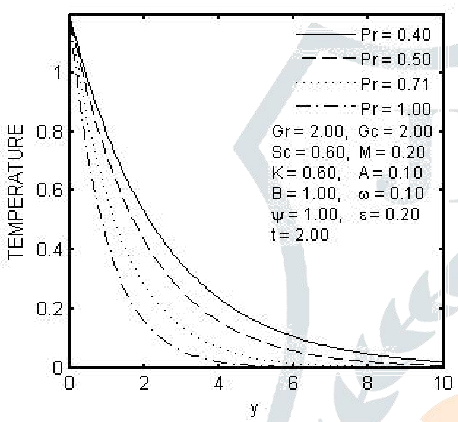

Fig. 9 :- Effect of Pr on temperature profiles

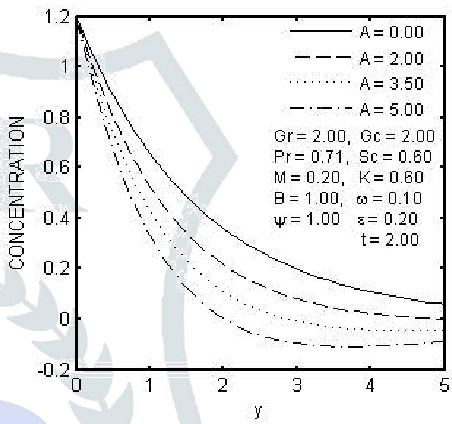

Fig.12:- Effect of Suction parameter on concentration

4. RESULT AND CONCLUSION

The effect of the Grashof number on velocity profiles is illustrated in Fig. 1. It is observed that an increase in the Grashof number contributes to an increase in the fluid's velocity. Additionally, in the boundary layer region, the velocity increases and then decreases. Far from the plate, the Grashof number does not have a significant effect.

The variation of velocity with respect to the modified Grashof number (Gc) is shown in Fig. 2. As Gc increases, the velocity generally increases. Similar to the previous case, an increase in velocity is noticed in the boundary layer region before it decreases.The influence of the Schmidt number on velocity profiles is shown in Fig. 3. As the Schmidt number increases, the velocity decreases. The effect of the Schmidt number is not significant at the boundary but becomes more distinct further away from the plate, where the dispersion due to the Schmidt number is more noticeable.The impact of magnetic intensity on the velocity field is illustrated in Fig. 4. As the magnetic intensity increases, the fluid velocity decreases. This observation aligns with the realistic situation where magnetic intensity suppresses fluid velocity.The influence of the porosity of the fluid bed on velocity profiles is shown in Fig. 5. As the pore size of the fluid bed increases, the velocity decreases. This is consistent with real-life situations, where an increase in pore size traps fluid in the pores, reducing fluid velocity.The variation in fluid velocity with respect to the suction parameter is shown in Fig. 6. As the suction parameter increases, the fluid velocity decreases. At the boundary, the suction parameter shows no influence, but its effect becomes more pronounced further from the plate.The influence of amplitude on velocity profiles is shown in Fig. 7. As the amplitude increases, the fluid velocity decreases, and sometimes a backward flow is observed. This backward flow is due to fluid percolation into the boundary. However, far from the bounding surface, the effect of the backward flow is negligible, and the influence of amplitude is not seen.The influence of the Prandtl number on velocity profiles is illustrated in Fig. 8. As the Prandtl number increases, the fluid velocity decreases. The effect of the Prandtl number is not significant at the boundary but becomes more significant further from the bounding surface.Fig. 9 shows the influence of the Prandtl number on the temperature field. As the Prandtl number increases, the temperature decreases. The Prandtl number's effect is not significant at the boundary or further away from the plate.The influence of the suction parameter on the temperature field is shown in Fig. 10. As the suction parameter increases, the temperature field decreases. The increase in the suction parameter contributes to the parabolic nature of the temperature profiles. Further from the plate, the suction parameter loses its significance.Fig. 11 illustrates the effect of the Schmidt number on concentration profiles. As the Schmidt number increases, the concentration decreases. The increase in the Schmidt number contributes to the parabolic nature of the concentration profiles. Dispersion in concentration becomes more distinctive further from the bounding surface.The influence of the suction parameter on the concentration of the fluid medium is shown in Fig. 12. As the suction parameter increases, the concentration of the fluid medium decreases. Further from the plate, the effect of the suction parameter diminishes rapidly.

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