A Study of symmetric Kenmotsu Manifold

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Abstract

The object of the present paper is to A Study of symmetric Kenmotsu Manifoldand some properties of quarter-symmetric non-metric connection.

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1 Introduction

Let (M_n, g) be a Riemannian manifold of dimension n. A linear connection ∇ in (M_n, g), whose torsion tensor T of type $(1, 2)$ is defined as

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y], \tag{1.1}$$

for arbitrary vector fields X and Y, is said to be torsion free or symmetric if T vanishes, otherwise it is non-symmetric. If the connection ∇ satisfy $\nabla g = 0$ in (M_n, g), then it is called metric connection, otherwise it is non-metric.

In 1972, K. Kenmotsu [21] introduced a class of contact Riemann manifold known as Kenmotsu Manifold. He studied that if a Kenmotsu manifold satisfy the condition $R(X, Y)Z = 0$, then the manifold is of negative curvature -1, where R is the Riemannian curvature tensor of type (1, 3) and $R(X, Y)Z$ is derivative of tensor algebra at each point of the tangent space. Several properties of Kenmotsu Manifold have been studied by De and Pathak [6], De [7], Sinha and Srivastav, De and Pathak [6]and many others. Ozgur studied generalised recurrent Kenmotsu manifold and proved that if M be a generalised recurrent Kenmotsu manifold and generalised Ricci Recurrent Kenmotsu manifold then $\beta = \alpha$ holds on M. Sular studies the generalised recurrent and generalised Ricci recurrent Kenmotsu manifol with respect to semi symmetric metric connection and proved that $\beta = 2\alpha$ where α and β are smooth functions and M is generalised recurrent and generalised Ricci recurrent Kenmotsu manifold admitting a semi-symmetric connection. In the present chapter, we studied the properties of semi-symmetric non-metric connection in Kenmotsu manifold.

2 Preliminaries

An n-dimensional Riemannian manifold (M_n, g) of class C^∞ with a 1-form η, the associated vector field ξ and a $(1, 1)$ tensor field φ satisfying

$$\varphi^2 X + X = \eta(X)\xi, \tag{2.1}$$

$$\varphi\xi = 0, \quad \eta(\varphi X) = 0, \quad \eta(\xi) = 1, \tag{2.2}$$

for arbitrary vector field X, is called an almost contact manifold. This system (φ, ξ, η) is called an almost contact structure to M_n [3]. If the associated Riemannian metric g in M_n satisfy

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.3}$$

for arbitrary vector fields X, Y in M_n, then (M_n, g) is said to be an almost contact metric manifold. Putting ξ for X in (2.3) and using (2.2), we obtain

$$g(\xi, Y) = \eta(Y). \tag{2.4}$$

Also,

$$\phi(X, Y) \overset{\text{def}}{=} g(\varphi X, Y) \tag{2.5}$$

gives

$$\phi(X, Y) + \phi(Y, X) = 0. \tag{2.6}$$

where $\phi = d\eta$ is 2-form.

If moreover

$$(D_X\varphi)(Y) = g(\varphi X, Y)\xi - \eta(Y)\varphi X, \tag{2.7}$$

$$D_X\xi = X - \eta(X)\xi, \tag{2.8}$$

hold in (M_n, g), where D being the Levi-Civita connection of the Riemannian metric g, then (M_n, g) is called a Kenmotsu manifold [21]. Also the following relations hold in a *Kenmotsu manifold*

$$K(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.9}$$

$$K(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{2.10}$$

$$S(X, \xi) = -(n - 1)\eta(X), \tag{2.11}$$

$$(D_X\eta)(Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.12}$$

for arbitrary vector fields X and Y, where K and S denote the Riemannian curvature and Ricci tensors of the connection D respectively.

Theorem 2.1. Let M_n be an *n*-dimensional Kenmotsu manifold equipped with a semi- symmetric non-metric connection ∇, then the scalar curvature with respect to semi- symmetric non-metric connection is equal to scalar curvature with respect to Levi-Civita connection.

References

[1] **Agashe N.S. and Chafle M.R.**, A *semi-symmetric non-metric connection in a Riemannian manifold*, Indian J. Pure Appl. Math., **23**(1992), 399–409.

[2] **Augre P., Blair D.E. and Carriazo, A.**, *Generalized Sasakian Space Form*, Isreal J. Math **141** (2004), 157-183.

[3] **Blair D.E.**, *Contact manifolds in Riemannian geometry*, Lecture Note in Mathe- matics, 509, Springer-Verlag Berlin, 1976.

[4] **Chaubey S.K. and Ojha R.H.**, *On the m-projective curvature tensor of a Kenmotsu manifold*, Diff. Geometry-Dynamical system, Vol. **12**(2010), 52-60.

[5] **De U.C. and Sengupta J.**, *Quarter-symmetric metric connection on Sasakian manifold*, Common Fac. Sci. Univ. Ank. Series Al Vol. **49**(2000), 7–13.

[6] **De U.C. and Sengupta J.**, *On three dimensional Kenmotsu manifolds*, Indian Jour. Pure Appl. Math., **35**(2004) 159-165.

[7] **De U.C.**, *On φ-symmetric Kenmotsu manifolds*, Int. Elect. Jour. of Geometry, **1**(2008), 33-38.

[8] **De U. C. and Sheikh A. A.**, *Differential Geometry of manifolds*, Narosa Publishing House, 2009.

[9] **De U.C. and Sarkar A.**, *On the quasi-conformal curvature tensor of a (k,μ)contact metric manifold*, Math Reports **14(64)**(2012), 115–129.

[10] **Duggal K.L. and Bejancu A.**, *Lightlike submanifolds of Semi-Riemannianian Manifold and Applications*, Kluwar Academic. Publisher, 1996.

[11] **Duggal K.L. and Jin D.H.**, *Null curves and hypersurfaces of semi-Riemannian manifolds*, World Scientific, 2007.

[12] **Duggal K.L. and Sahin B.**, *Differential Geometry of Lightlike submanifolds* Frontier Mathematics, Birkhauser Vorlag A.G., Germany, 2010.

[13] **Golab S.**, *On semi-symmetric and quarter-symmetric connections*, Tensors N.S. **29**(1975), 249-254.

[14] **Gray, A.**, *Some global properties of contact structure*, Ann., Math, **69**(1959), 421- 450.

[15] **Guines R., Sahin B. and Kilic E.**, *On Lightlike hypersurfaces of a semi- Riemannian Space form*, Turk. J. Math. **27**(2003), 283-297.

[16] **Imai T.**, *Hypersurfaces of a Riemannian manifold with semi-symmetric metric connection*, Tensor N.S., **23**(1972), 300-306.

[17] **Jeffery Lee M.**, *Manifold and Differentiable Geometry, Graduate studies in Mathematics*, American Mathematical Soc., Vol 107, 2009.

[18] **Jin D.H.**, *Einstien Half-lightlike submanifolds of a Lorentzian space from with a semi-symmetric non-metric connection*, Journal of inequalities and application, **1**(2013), 403.

[19] **Jin D.H.**, *Non-tangential Half-lightlike submanifolds of semi-Riemannian manifolds with semi-symmetric non-metric connections*, Journal of Korean Math. Soc. **51**(2014)no.2, 311-323.

[20] **Jin D.H.**, *Lightlike hypersurfaces of a trans-Sasakian manifold with a quarter- symmetric connection*, Applied mathematical Sciences, Hikari Ltd, Vol. **9**(2015), no. 28, 1393-1406.

[21] **Kenmotsu K.**,*A class of almost contact Riemannian manifolds*, Tohoku Math. J., **24**(1972), 93-103.

[22] **Klic E. and Bahadir O.**, *Lightlike hypersurface of a product manifold and quarter- symmetric non-metric connections*, Int. Jouranal of Maths. and Mathematical Sci. Hindawi. Volume (2012), Art I.D. 178390, 17 pages.

[23] **Matsumoto K.**, *On Lorentzian para-compact manifold*, Bull. of Yamagata Univ., Nat. Sci. **12**(1989), 151-156.

[24] **Mihai and Rosca R.**, *On Lorentzian P-Sasakian manifolds*, Classical Analysis, World Scientific Publication 1992, 155-169.

[25] **Mishra R.S. and Pandey S.N.**, *On quarter-symmetric F-connection*, Tensor N.S. 34(1980), 1–7.

[26] **Mishra R.S.**, *Structures on differentiable manifold and their applications*, Chan- drama Prakashan, Allahabad, India, 1984.

[27] **Mondal A.K. and De U.C.**, *Some properties of a quarter-symmetric connection on Sasakian manifold*, Bull. Math. Anal. Appl. issue 3(2009), 99–108.

[28] **Özgür C.**, *On weakly symmetric Kenmotsu manifolds*, Differential Geometry- Dynamical Systems, **8**(2006), 204-209.

[29] **Özgür C.**, *On generalized recurrent Kenmotsu manifolds*, World Applied Sciences J., **2**(2007), no. 1, 29-33.

[30] **Rastogi S.C.**, *On quarter symmetric metric connections*, Tensor N.S. **14**(1987), 133–141.

[31] **Rastogi S.C.**, *A note on quarter-symmetric metric connections*, Indian J. Pure Appl. Math 18 **12**(1987), 1107–1112.

[32] **Sengupta J. and Biswas B.**, *Quarter symmetric non-metric connection on a Sasakian Manifold*, Bull. Cal., Math. Soc, **2**(2003), 169-176.

[33] **Sinha B.B. and Srivastava A.K.**, *Curvature on Kenmotsu manifold*, Indian J. Pure and Appl. Math. 22, **1**(1991),23-28.

[34] **Sular S., Ozgur C. and De U.C.**, *Quarter symmetric connection in a Kenmotsu Manifold*, Sut. J. Math.,44 **2**(2008), 297-306.

[35] **Thorpe J.A.**, *Elementary Topics in Differential Geometry*, Springer-Verlag, New York, 1979.

[36] **Yano K., Kon M.**, *Structures on Manifolds*, World Scientific, 1984.

[37] **Yaser E., Coken A. C. and Yucesan A.**, *Lightlike hypersurfaces in Semi- Riemannian Manifolds with semi-symmetric non-metric connection*, Math, Scand., **102**(2008), no. 2, 253-264.

[38] **Yucesan A. and Yasar E.**, *Lightlike submanifolds of a semi-Riemannian manifold with a semi-symmetric non-metric connections*, Bull., Korean Math., Soc, **47**(2010), no.5, 1089-1103.

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