Min-Min Operation on Intuitionistic Fuzzy Matrix

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Abstract: In this paper Min-Min operation on IFMs and study conditions for convergence powers of transitive IFM are introduced.
Keywords and Phrases: Intuitionistic fuzzy set (IFS), Intuitionistic fuzzy implication operator (IFIO), Intuitionistic fuzzy matrix (IFM)

I. INTRODUCTION

Since Zadeh [11] introduction of fuzzy sets. Atanassov [1] generalized the concept of fuzzy sets into intuitionistic fuzzy set (IFS) A in X (universal set) is defined as an object of the following from $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$ where the functions: $\mu_A(x): x \to [0,1]$ and $\gamma_A(x): X \to [0,1]$ define the membership function and non-membership function of the element $x \in X$ respectively and for every $x \in X: 0 \le \mu_A(x) + \gamma_A(x) \le 1$. Xu, Yager [10] defined an Intuitionistic Fuzzy Matrix (IFM). Aas$A = [\langle a_{ij_\mu}, a_{ij_\gamma}\rangle]$ where a_{ij_μ} and a_{ij_γ} denote the membership and non-membership value respectively.

After the introduction of Fuzzy Matrix (FM) theory using Max-Min algebra by Thomson [9], Bhowmik and Pal [3] studies the convergence of the Max-Min of an IFM by Hashimoto [4] and several others have studied the convergence of power of a fuzzy transitive matrix. Further, the Max-Min operation has been extended to IFM. Atanassov [2] used implication operators in IFSs. Sriram and Murugadas [8] used $\leftarrow$ implication operator for IFM and studied concept of g-inverse and semi-inverse of an IFM which was a generalization of FM studied. Murugadas and Lalitha [5] used hook implication operator $\leftharpoondown$ for IFS as well as IFM. Muthuraji, Sriram and Murugadas [6] used min-min composition of IFM. Riyaz Ahmad padder and Murugadas [7] Max-Max operation on Intuitionistic fuzzy Matrix.

In this paper we introduce Min-Min operation directly to IFMs which is more relevant than Max-Min operation. For example, consider two IFMs A and B such that

$$A = \begin{pmatrix} \langle .3,.2\rangle & \langle .4,.1\rangle \\ \langle .1,.5\rangle & \langle .1,.8\rangle \end{pmatrix} \text{and} B = \begin{pmatrix} \langle 0.3,0.4\rangle & \langle 0.5,0.2\rangle \\ \langle 0.2,0.5\rangle & \langle 0.3,0.6\rangle \end{pmatrix}$$

Then Max-Min $AB = \begin{pmatrix} \langle 0.3,0.4\rangle & \langle 0.3,0.2\rangle \\ \langle 0.1,0.5\rangle & \langle 0.1,0.5\rangle \end{pmatrix}$

Then Min-Min $AB = \begin{pmatrix} \langle 0.2,0.5\rangle & \langle 0.3,0.6\rangle \\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle \end{pmatrix}$

Thus Min-Min $AB \le$ Max-Min AB.

II. PRELIMINARIES

Let $\langle x, x'\rangle, \langle y, y'\rangle \in$ IFS then $\langle x, x'\rangle \vee \langle y, y'\rangle = \langle Min\{x, y\}, Max\{x', y'\}\rangle$

For any two comparable elements $\langle x, x'\rangle, \langle y, y'\rangle \in$ IFS the operation $\langle x, x'\rangle \leftarrow \langle y, y'\rangle$ is defined as

$$\langle x, x'\rangle \leftarrow \langle y, y'\rangle = \begin{cases} \langle 1,0\rangle & \text{if} \langle x, x'\rangle \ge \langle y, y'\rangle \\ \langle x, x'\rangle & \text{if} \langle x, x'\rangle < \langle y, y'\rangle \end{cases}$$

For $n \times n$ intuitionistic fuzzy matrices $A = \langle a_{ij}, a_{ij}'\rangle$ and $P = [\langle p_{ij}, p_{ij}'] $ then

$$A \wedge P = (\langle a_{ij} \wedge p_{ij}, a_{ij}' \vee p_{ij}'\rangle)$$
$$A \vee P = (\langle a_{ij} \vee p_{ij}, a_{ij}' \wedge p_{ij}'\rangle)$$

Here $A \vee P, A \wedge P$ are equivalent to $A + P, A \odot P$ the component wise additional and component wise multiplication A, P respectively.

$A \times P = (a_{il}, a_{il}' \wedge p_{lj}, p_{lj}') \vee (a_{i2}, a_{i2}' \wedge p_{2j}, p_{2j}') \vee \cdots \vee (a_{in}, a_{in}' \wedge p_{nj}, p_{nj}')$

$A \overset{c}{\leftarrow} P = \langle a_{ij}, a_{ij}'\rangle \overset{c}{\leftarrow} \langle p_{ij}, p_{ij}'\rangle$.

Here $\overset{c}{\leftarrow}$ represents component wise comparison of A, P using $\leftarrow$.

$A^\circ = I = (\delta_{ij}, \delta_{ij}')$ where $\langle \delta_{ij}, \delta_{ij}'\rangle = \langle 1,0\rangle$ if $i = J$ and $\langle \delta_{ij}, \delta_{ij}'\rangle = \langle 0,1\rangle$ if $i \ne j$.

$A^{k+1} = A^k \times A, k = 0,1,2, \cdots$

$A \le P (P \ge A)$ if and only if $\langle a_{ij}, a_{ij}'\rangle \le \langle p_{ij}, p_{ij}'$ for all i, j.

If $A \ge I_n$, then A is reflexive IFM where in the $n \times n$ identity IFM. $A = \langle a_{ij}, a_{ij}'\rangle$ is weakly reflexive IFM if and only if $\langle a_{ij}, a_{ij}'\rangle \ge \langle a_{ij}, a_{ij}'\rangle$ for all $i, j = 1,2, \cdots, n$.

Throughout we deal with intuitionistic fuzzy matrices. A matrix A is transitive if $A^2 \le A$. This matrix represents a intuitionstic fuzzy transitive relation. The above definition of transitivity is equivalent to what is called Max-Min transitivity. That is, matrix $A = \langle a_{ij}, a_{ij}'\rangle$ is transitive if and only if $\min(\langle a_{ik}, a_{ik}'\rangle, \langle a_{kj}, a_{kj}'\rangle) \le \langle a_{ij}, a_{ij}'\rangle$, for all k. This definition is most basic and seems to be convenient when intuitionistic fuzzy matrices are generalized to certain matrices over other algebras.

III. SOME RESULTS

I define Min-Min operation on IFM and exhibit some interesting results. In the following, let $A = [\langle a_{ij}, a_{ij}'\rangle], P = [\langle p_{ij}, p_{ij}'\rangle]$ be IFM of order $n \times n$ and the entries in A and P are comparable.

Definition 3.1 *For* IFMs$A$and$P$define, the Min-Min product of$A$and$P$as

$$A \bullet P = \left(\bigwedge_{k=1}^{n} \langle a_{ik} \wedge p_{kj}, \bigvee_{k=1}^{n} \langle a_{ik}' \vee p_{kj}'\rangle \right)$$

Let $A \bullet P$ denote the Min-Min product of the IFMs A and P.

Clearly $A \bullet P$ is also an IFM, $\bullet$ is associative and $\bullet$ is distributive over addition $(+)$. Also the set of all IFM under $+$ and $\bullet$ from a semi-ring.

Theorem 3.2 If A is an $n \times n$ transitive matrix, then $(A \overset{c}{\leftarrow} (A \times P))^n = (A \overset{c}{\leftarrow} (A \times P))^{n+1}$ for any $n \times n$ IFMP.

Proof. Let $S = \langle s_{ij}, s_{ij}' \rangle = A \overset{c}{\leftarrow} (A \times P)$, that is

$$\langle s_{ij}, s_{ij}' \rangle = \langle a_{ij}, a_{ij}' \rangle \overset{c}{\leftarrow} \left(\bigvee_{k=1}^{n} (a_{ik} \wedge p_{kj}), \bigwedge_{k=1}^{n} (a_{ik}' \vee p_{kj}') \right)$$

1. Assume that there exist indices $l_1, l_2, \cdots, l_{n-1}$ such that

$$\langle s_{il_1}, s_{il_1}' \rangle \vee \langle s_{l_1 l_2}, s_{l_1 l_2}' \rangle \vee \cdots \vee \langle s_{l_{n-1} j}, s_{l_{n-1} j}' \rangle = \langle f, f' \rangle < \langle 1,0 \rangle$$

Let $l_o = i$ and $l_n = j$. Then $l_a = l_b$ for some a and $b (a > b)$. We define $\langle h, h' \rangle$ by

$$\langle h, h' \rangle = \langle a_{l_a l_{a+1}}, a_{l_a l_{a+1}} \rangle \vee \langle r_{l_{a+1} l_{a+2}}, r_{l_{a+1} l_{a+2}} \rangle \vee \cdots \vee \langle a_{l_{b-1} l_b}, r_{l_{b-1} l_b} \rangle$$

where $a > m \geq b$

Then $\langle h, h' \rangle = \langle a_{l_{m-1} l_m}, a_{l_{m-1} l_m}' \rangle < \left(\bigvee_{k=1}^{n} \langle a_{lmk} \wedge p_{klm} \rangle, \bigwedge_{k=1}^{n} \langle a_{lmk}' \vee p_{klm}' \rangle \right)$

If $\langle a_{l_m l_m}, r_{l_m l_m}' \rangle \geq \left(\bigvee_{k=1}^{n} \langle a_{lmk}' \wedge p_{klm} \rangle, \bigwedge_{K=1}^{n} \langle a_{lmk}' \vee p_{klm}' \rangle \right)$

$$\langle h, h' \rangle \geq \langle a_{l_m l_m}, a_{l_m l_m}' \rangle \geq \langle a_{l_m k}, \wedge p_{k_1 l m}, a_{lmk_1}', a_{l_m k_1}' \vee p_{k_1 l_m}' \rangle$$
$$= \langle a_{l_a l_{a+1}}, a_{l_a l_{a+1}}' \rangle \wedge \langle a_{l_{a+1}} l_{a+2}, a_{l_{a+1}} l_{a+2} \rangle \wedge \cdots \wedge \langle a_{l_{b-1} l_b}, a_{l_{b-1} l_b}' \rangle$$

for some k_1. Since $\langle a_{l_{m-1} l_{m_1}} a_{l_{m-1} l_m}' \rangle = \langle h, h' \rangle$ we have

$$\langle a_{l_{m-1} k-1}, a_{l_{m-1} k_1} \rangle \leq \langle a_{l_{m-1} l_m}, a_{l_{m-1} l_m}' \rangle \wedge \langle a_{l_m k_1}, a_{l_m k_1}' \langle a_{l_m k_1}, a_{l_m k_1}' \rangle = \langle h, h' \rangle$$

Thus,

$$\left(\bigvee_{k=1}^{n} \langle a_{l_{m-1} k} \wedge p_{k l_m} \rangle, \bigwedge_{k=1}^{n} \langle a_{l_{m-1} k}', \vee p_{k l_m}' \rangle \right) \leq \langle a_{l_{m-1} k_1}, a_{l_{m-1} k_1}' \rangle \wedge \langle p_{k_1 l_m}, p_{k l_m}' \rangle \leq \langle h, h' \rangle$$

which is contradiction. So,

$$\langle a_{l_m l_m}, a_{l_m l_m}' \rangle < \left(\bigvee_{k=1}^{n} \langle a_{l_m k} \wedge p_{k l_m} \rangle, \bigwedge_{k=1^n} \langle a_{l_m k}' \vee p_{k l_m}' \rangle \right).$$

Hence $\langle s_{l_m l_m}, s_{l_m l_m}' \rangle \leq \langle h, h' \rangle \leq \langle g, g' \rangle$

Therefore $\langle s_{ij}^{n+1}, s_{ij}^{n+1} \rangle \leq \langle g, g' \rangle$.

2. Assume that there exist indices $l_1, l_2, \cdots, l_n$ such that

$$\langle s_{il_1}, s_{il_1}' \rangle \vee \langle s_{l_1 l_2}, s_{l_1 l_2}' \rangle \vee \cdots \vee \langle s_{lnj}, s_{lnj}' \rangle = \langle g, g' \rangle < \langle 1,0 \rangle.$$

Let $l_o = i$ and $l_{n+1} = j$

(a) Assume $l_a = l_b = l_c$ where $a > b > c$. Then we have

$$\langle s_{l_m l_m}, s_{l_m l_m}' \leq \langle g, g' \rangle, a > m \geq b$$

Thus,

$$\langle s_{il_m}^{(m)}, s_{il_m}'^{(m)} \rangle \vee \langle s_{l_m l_m}^{(c-b-c)}, s_{l_m l_m}'^{(c-b-c)} \rangle \vee \langle s_{l_m l_b}^{(b-m)}, s_{l_m l_b}'^{(b-m)} \vee \langle s_{l_c j}^{(n+1-c)}, s_{l_c j}'^{(n+1-c)} \rangle \leq \langle g, g' \rangle$$

so $\langle s_{ij}^n, s_{ij}'^n \rangle \leq \langle g, g' \rangle$

(b) Assume $l_a = l_b$ and $l_c = l_d$

(i) If $a > b > c > d$ then $\langle s_{l_m l_m}, s_{l_m l_m}' \rangle \leq \langle g, g' \rangle, a > m \geq b$ for some l_m.

Thus,

$$\langle s_{il_m} m, s_{il_m}' m \rangle \vee \langle s_{l_m l_m} (d - c - 1), s_{l_m l_m}' (d - c - 1) \rangle \vee \langle s_{l_m l_c} (c - m), s_{l_m l_c}' (c - m) \rangle \vee$$
$$\langle s_{l_d j} (n + 1 - d), s_{l_d j}' (n + 1 - d) \rangle \leq \langle g, g' \rangle$$

So $\langle s_{ij} n, s_{ij}' n \rangle \leq \langle g, g' \rangle$.

(ii) If $a > c > b > d$ then $\langle s_{l_m l_m}, s_{l_m l_m}' \rangle \leq \langle h, h' \rangle \leq \langle g, g' \rangle, a > m \geq b$ for some l_m

where

$$\langle h, h' \rangle = \langle a_{l_{m-1} l_m}, a_{l_{m-1} l_m}' \rangle$$
$$= \langle a_{l_n l_{n+1}}, a_{l_n l_{n+1}}' \rangle \vee \cdots \vee \langle a_{i_{b+1} l_b}, a_{l_{b-1} l_b}' \rangle$$

Since it is clear that $\langle s_{ij} n, s_{ij}' n \rangle \leq \langle g, g' \rangle$ for $m \geq c$, suppose that $m > c$. If

$$\langle a_{l_a l_m}, a_{l_a l_m}' \rangle \geq \left(\bigvee_{k=1}^{n} \langle a_{l_a K} \wedge p_{k l_m} \rangle, \bigwedge_{k=1}^{n} \langle a_{l_a k}' \vee p_{k l_m}' \rangle \right)$$

Then

$\langle g, g' \rangle \geq \langle h, h' \rangle \geq \langle a_{l_a l_m}, a_{l_a l_m}' \rangle \geq \langle a_{l_n k_1}, a_{l_n k_1}' \rangle \wedge \langle p_{k_1 l_m}, p_{k_1 l_m}' \rangle$ for some k_1.

Thus

$$\langle a_{l_{m-1} K_1}, a_{l_{m-1} k_1} \rangle \leq \langle a_{l_{m-1} l_m}, a_{lm-1 l_m}' \rangle \vee \langle a_{l_m l_n}, a_{l_m l_n}' \rangle \wedge \langle a_{l_a k_1}, a_{l_a k_1}' \rangle = \langle h, h' \rangle$$

we have

$$\left(\bigvee_{k=1}^{n} \langle a_{l_{m-1} k} \wedge p_{k l_m} \rangle, \bigwedge_{k=1}^{n} \langle a_{l_{m-1} k}' \vee p_{k l_m}' \rangle \right) \leq \langle a_{l_{m-1} k_1}, a_{l_{m-1} k_1}' \rangle \vee \langle p_{k_1 l_m}, p_{k l_m}' \rangle \leq \langle h, h' \rangle$$

which contradicts the fact that

$$\langle h, h' \rangle = \langle s_{l_{m-1} l_m}, s_{l_{m-1} l_m} \rangle < 0$$

So $\langle s_{l_m l_m}, s_{l_m l_m}' \rangle \leq \langle g, g' \rangle$.

Hence $\langle s_{il_a}(a), s_{il_a}'(a) \rangle \vee \langle s_{l_a l_m}, s_{l_a l_m}' \rangle \vee \langle s_{l_m l_m}^{(m-a-2)}, s_{l_m l_m}'^{(m-a-2)} \rangle \vee \langle s_{l_m j}^{(n+1-m)}, s_{l_m j}'^{(n+1-m)} \rangle \leq \langle g, g' \rangle$.

3. If $a > c > d > b$ then $\langle s_{l_m l_m}, s_{l_m l_m}' \rangle \leq \langle g, g' \rangle, a > m \geq b$ for some....

It is clear that $\langle s_{ij}(n), s_{ij}'(n) \rangle \leq \langle g, g' \rangle$ for $m \geq c$ (or) $d \geq m$. Suppose that $c \geq m \geq d$.

By the same argument as in (ii) we have

$\langle s_{l_m l_m}, s_{l_m l_m}' \rangle \leq \langle g, g' \rangle$ then

$$\langle s_{il_a}(a), s_{il_a}'(a)\rangle \vee \langle s_{l_al_m}, s_{l_al_m}'\rangle \vee \langle s_{l_ml_n}^{(m-a-2)}, s_{l_ml_n}'^{(m-a-2)}\rangle \vee \langle s_{l_mj}^{(n+1-m)}, s_{l_mj}'^{(n+1-m)}\rangle \leq \langle g, g'\rangle.$$

□

Example 3.3 $A = \begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}$ and $P = \begin{pmatrix} \langle 0.1,0.6\rangle & \langle 0.5,0.2\rangle \\ \langle 0.4,0.5\rangle & \langle 0.3,0.2\rangle \end{pmatrix}$

$$A \times P = \begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}\begin{pmatrix} \langle 0.1,0.6\rangle & \langle 0.5,0.2\rangle \\ \langle 0.4,0.5\rangle & \langle 0.3,0.2\rangle \end{pmatrix}$$

$$A \times P = \begin{pmatrix} \langle 0.1,0.5\rangle & \langle 0.5,0.2\rangle \\ \langle 0.4,0.5\rangle & \langle 0.3,0.2\rangle \end{pmatrix}$$

$$A^2 = A \cdot A = \begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}\begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$A^2 = \begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$S = A \overset{c}{\leftarrow} (A \times P)$$

$$= \begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix} \overset{c}{\leftarrow} \begin{pmatrix} \langle 0.1,0.5\rangle & \langle 0.5,0.2\rangle \\ \langle 0.4,0.5\rangle & \langle 0.3,0.2\rangle \end{pmatrix}$$

$$S = \begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$S^2 = S \cdot S = \begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}\begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}$$

Then $S^3 = S^2 \times S = \begin{pmatrix} \langle 1,0\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.5\rangle & \langle 1,0\rangle \end{pmatrix}$

Thus we have $S^3 = S^2$

From Theorem 3.2, we get the following two results.

Corollary 3.4 If A is an $n \times n$ transitive matrix, then $(A \overset{c}{\leftarrow} (P \times A))^n = (A \overset{c}{\leftarrow} (P \times A))^{n+1}$ for any $n \times n$ matrix P.

Corollary 3.5 Let A be an $n \times n$ transitive matrix, then $A^n = A^{n+1}$.

We now consider conditions under which our $n \times n$ transitive matrix A fulfills the relationship $A^{n-1} = A$, where $n \geq 2$.

Theorem 3.6 Let A be an $n \times n$ matrix if $A \wedge I \geq P \geq A$ and the Min-Min product $A \bullet A^T \leq (\langle a_{ij}, a_{ij}'\rangle)$ for some j then $P^{n-1} = p^n$.

Proof. First we know that $P^{n-1} \leq P^n$ suppose that $\langle p_{ij}(n-1), p_{ij}'(n-1)\rangle = \langle c, c'\rangle \leq \langle 1,0\rangle$

Then there exist indices $k_1, k_2, \cdots, k_{n-2}$ such that $\langle p_{ik_1}, p_{ik_1}'\rangle \vee \langle p_{k_1k_2}, p_{k_1k_2}'\rangle \vee \cdots \vee \langle p_{k_{n-2}j}, p_{k_{n-2}j}'\rangle = \langle c, c'\rangle$

Thus $\langle a_{ik_1}, a_{ik_1}'\rangle \vee \langle a_{k_1k_2}, a_{k_1k_2}'\rangle \vee \cdots \vee \langle a_{k_{n-2}j}, a_{k_{n-2}j}'\rangle \leq \langle c, c'\rangle$

Let $k_0 = i$ and $k_{n-1} = j$

(a) If $k_a = k_b$ for some a and $b (a > b)$, then $\langle p_{k_ak_a}(a-b), p_{k_ak_a}'(a-b)\rangle \leq \langle c, c'\rangle$.

Thus

$$\langle a_{k_ak_a}, a_{k_ak_a}'(a-b)\rangle \leq \langle c, c'\rangle, \langle a_{k_ak_b}, a_{k_ak_b}'\rangle \leq \langle c, c'\rangle$$
$$\langle p_{k_ak_a}, p_{k_ak_a}'\rangle \leq \langle c, c'\rangle.$$

So

$$\langle p_{ik_1}, p_{ik_1}'\rangle \vee \langle p_{ik_1k_2}, p_{ik_1k_2}'\rangle \vee \cdots \vee \langle p_{ka_1k_a}, p_{k_{a-1}k_a}'\rangle$$
$$\vee \langle p_{k_ak_a}, p_{k_ak_a}'\rangle \vee \langle p_{kak_{a+1}}, p_{k_ak_{a+1}}'\rangle \vee \cdots \vee \langle p_{kn-2j}, p_{k_{n-2}j}'\rangle \leq \langle c, c'\rangle.$$

Hence $\langle p_{ij}(n), p_{ij}'(n) \leq \langle c, c'\rangle$

(b) Suppose that $k_a \neq k_b$ for all $a \neq b$. By hypothesis,

$\langle \bigwedge_{k=1}^{n} \langle a_{lkm} \wedge a_{kml}\rangle, \bigvee_{k=1}^{n} \langle a_{lkm}' \vee a_{kml}'\rangle\rangle \geq \langle a_{a_{km}km}, a_{k_mk_m}'\rangle$ for some m.

Then $\langle a_{k_mk_n}, r_{k_mk_m}'\rangle \leq c, c'\rangle$

$$\langle p_{k_mk_m}, p_{k_mk_m}'\rangle \leq \langle c, c'\rangle$$

Thus

$$\langle p_{ik_j}, p_{ik_j}'\rangle \vee \langle p_{k_1k_2}, p_{k_1k_2}'\rangle \vee \cdots \vee \langle p_{k_{m-1}}, p_{k_{m-1}k_m}'\rangle \vee \langle p_{k_mk_m}, p_{k_mk_m}'\rangle$$
$$\vee \langle p_{k_mk_{m+1}}, p_{kk_{m+1}}'\rangle \vee \cdots \vee \langle p_{k_{n-2j}}, p_{k_{n-2j}}'\rangle \leq \langle c, c'\rangle$$

So $\langle p_{ij}(n), p_{ij}'(n)\rangle \leq \langle c, c'\rangle$

(2) Next we show that $p^n p^{n-1}$

Let $\langle p_{ij}(n), p_{ij}'(n)\rangle = \langle c, c'\rangle < \langle 1,0\rangle$

Then there exists indices $k_1, k_2, \cdots, k_{n-1}$ such that $\langle p_{ik_1}, p_{ik_1}'\rangle \vee \langle p_{k_1k_2}, p_{k_1,k_2}'\rangle \vee \cdots \vee \langle p_{k_{n-1j}}, p_{k_{n-1j}}' = \langle c, c'\rangle$

Let $k_0 = i$ and $k_n = j$. Then $k_a = k_b$ for some a and $b (a > b)$. Thus $\langle p_{k_ak_a}(a-b), p_{k_ak_a}(a-b)\rangle \leq \langle c, c'\rangle$

So

$$\langle a_{k_ak_a}(a-b), a_{k_ak_a}(a-b)\rangle \leq \langle c, c'\rangle$$
$$\langle a_{k_ak_a}, a_{k_ak_a}'\rangle \leq \langle c, c'\rangle$$
$$\langle p_{k_ak_a}, p_{k_ak_a}'\rangle \leq \langle c, c'\rangle$$

Hence

$$\langle p_{ik_1}, p_{ik_1}'\rangle \vee \langle p_{k_1k_2}, p_{k_1,k_2}'\rangle \vee \cdots \vee \langle p_{k_{a-1}k_a}, p_{k_{a-1}k_a}' \vee \langle p_{k_ak_a}^{(}b-a-1\big), p_{k_ak_a}'\rangle \vee \langle p_{k_bk_{b+1}}, p_{k_bk_{b+1}}'\rangle \vee \cdots$$
$$\vee \langle p_{k_{n-1j}}, p_{k_{n-1j}}'\rangle \leq \langle c, c'\rangle$$

Therefore $\langle p_{ij}^{(}n-1), p_{ij}'^{(}n-1)\rangle \le \langle c, c'\rangle$.

Example 3.7 $A = \begin{pmatrix} \langle 0.1,0.2\rangle & \langle 0,0.3\rangle \\ \langle 0.3,0,2\rangle & \langle 0,0.3\rangle \end{pmatrix}, P = \begin{pmatrix} \langle 0.3,0.2\rangle & \langle 0,0.2\rangle \\ \langle 0.4,0.1\rangle & \langle 0.1,0.2\rangle \end{pmatrix}$

$$A^2 = \begin{pmatrix} \langle 0.1,0.2\rangle & \langle 0,0.3\rangle \\ \langle 0.3,0,2\rangle & \langle 0,0.3\rangle \end{pmatrix}\begin{pmatrix} \langle 0.1,0.2\rangle & \langle 0,0.3\rangle \\ \langle 0.3,0,2\rangle & \langle 0,0.3\rangle \end{pmatrix}$$

$A^2 = \begin{pmatrix} \langle 0.1,0.2\rangle & \langle 0,0.3\rangle \\ \langle 0.1,0.2\rangle & \langle 0,0.3\rangle \end{pmatrix} \le A$ (A is transitive)

$$A^3 = \begin{pmatrix} \langle 0.1,0.2\rangle & \langle 0,0.3\rangle \\ \langle 0.1,0.2\rangle & \langle 0,0.3\rangle \end{pmatrix} = A^2$$

$$P^2 = \begin{pmatrix} \langle 0.3,0.2\rangle & \langle 0,0.2\rangle \\ \langle 0.4,0.1\rangle & \langle 0.1,0.2\rangle \end{pmatrix}\begin{pmatrix} \langle 0.3,0.2\rangle & \langle 0,0.2\rangle \\ \langle 0.4,0.1\rangle & \langle 0.1,0.2\rangle \end{pmatrix}$$

$=\begin{pmatrix} \langle 0.3,0.2\rangle & \langle 0,0.2\rangle \\ \langle 0.3,0.2\rangle & \langle 0.1,0.2\rangle \end{pmatrix}$

$$P^2P = \begin{pmatrix} \langle 0.3,0.2\rangle & \langle 0,0.2\rangle \\ \langle 0.3,0.2\rangle & \langle 0.1,0.2\rangle \end{pmatrix}\begin{pmatrix} \langle 0.3,0.2\rangle & \langle 0,0.2\rangle \\ \langle 0.4,0.1\rangle & \langle 0.1,0.2\rangle \end{pmatrix}$$

$$P^3 = \begin{pmatrix} \langle 0.3,0.2\rangle & \langle 0,0.2\rangle \\ \langle 0.3,0.2\rangle & \langle 0.1,0.2\rangle \end{pmatrix} = P^2$$

Theorem 3.8 *If A is an $n \times n$ transitive matrix, $A \wedge I \ge P \ge A$ and $p \bullet p^T \le \langle p_{ij}, p_{ij}'\rangle$ for some j, then $p^{n-1} = p^n$.*

As a special care of Theorem 3.6 or Theorem 3.8 we obtain the following corollary where A is a transitive IFM.

Corollary 3.9 If$A$is an$n \times n$transitive intuitionistic fuzzy matrix.

$$A \bullet A^T \le \langle a_{ij}, a_{ij}'\rangle \text{for some} j \text{then} A^{n-1} = A^n.$$

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