Strong Split Line Block Domination of a Graph

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Abstract— ***In this paper, we introduce the concept of strong split line block domination of a graph. We establish some properties of this domination parameter in terms of elements of a graph G and its relationships with other domination parameters of the graph G.***

KEY WORDS:

Domination number, strong split domination number, line block graph.

I. INTRODUCTION

By a graph here we mean a finite, undirected graph without loops or multiple edges. For any undefined terms or notation, we refer [1].

The line block graph $L_b(G)$ of a graph G is the graph whose vertex set is the union of the set of edges and the set of blocks of G in which two vertices are adjacent if the corresponding blocks are adjacent or one corresponds to a block of G and other to an edge incident with it. This concept was introduced by Kulli [3].

The block cutpoint graph bc (G) of a graph is the graph whose vertex set is the union of set of blocks and the set of cut vertices of G in which two vertices are adjacent if the corresponding blocks are adjacent or the corresponding cutvertices are incident with the blocks. This concept was first studied by Harary in [1] and was studied in [7].

The middle graph $M(G)$ of a graph G is the graph whose vertex set is the union of set of vertices and edges of G with two vertices are adjacent if they are adjacent edges of G on one corresponds to a vertex and the other to an edge incident with it. This concept was introduced in [2] and was studied by Kulli, Patil and Biradar in [4, 5, 6]

A set $S \subseteq V(G)$ is said to be a dominating set of G, if every vertex in $V - S$ is adjacent to some vertex in S. The minimum cardinality of vertices in such a set is called the domination number of G and is denoted by $V(G)$. A dominating set $S \subseteq V(G)$ is a strong split dominating set, if the induced subgraph $\langle V - S\rangle$ is totally disconnected with at least two vertices. The strong split domination number $\gamma_{ss}(G)$ of G is the minimum cardinality of a strong split dominating set of G. This concept was well studied in [7, 8, 9, 10]. A set $D \subseteq V[L_b(G)]$ is said to be strong split line block dominating set if the induced subgraph. $\langle V[L_b(G)] - D\rangle$ is totally disconnected with at least two vertices. The strong split line block domination number $\gamma_{sslb}(G)$ of $L_b(G)$ is the minimum cardinality of a strong split line block dominating set of G. In this paper, we study the theoretic properties of $\gamma_{sslb}(G)$ and many bounds are obtained in terms of elements of G and its relationship with other domination parameters are found.

We need the following results for our further results.

Theorem A [11] : For any connected (p, q) tree T, $\gamma_{ssbc}(T) = n$.
Theorem B [10] : For any connected (p, q) tree T, $\gamma_{sstb}(T) = q + 1$.
Theorem C [9] : For any graph G, $\gamma_{ssm}(G) = q$.
Theorem D [7] : For any (p, q) connected graph G, $\gamma_{ss}(G) = \alpha_0(G)$.

II. RESULTS AND CONCLUSION

In the following theorem, we express in equality form in terms of the number of blocks n of G.

Theorem 1.
For any connected (p, q) graph G, $\gamma_{sslb}(G) = n$, where n is the number of blocks in G.
Proof.
Let $B = \{B_1, B_2, \ldots\ldots, B_n\}$ be the set of blocks of G and $E = \{e_1, e_2, \ldots., e_q\}$ be the set of edges of G. Now $B' = \{b_1, b_2, \ldots.b_n\}$ be the set of vertices corresponding to the blocks $Bi \in B$, $1 \le i \le n$ of G and $E' = \{u_1, u_2, \ldots\ldots\ldots\ldots u_q\}$ be the set of vertices corresponding to the edges of E. Now $V[L_b(G)] = \{B'\} \cup \{E'\}$ and each $b_i \in B'$ is a cut vertex of $L_b(G)$. Since each $bi \in B'$ $1 \le i \le n$ is adjacent to at least one element $u_i \in E'$, $1 \le i \le q$, then $\langle V[Lb(G)] - B'\rangle$ is totally disconnected with at least two vertices. Hence B' is a γ_{sslb}- set of G. Clearly $|B'| = n$ gives $\gamma_{sslb}(G) = n$.
We have the following corollary from theorem 1.
Corollary 1. For any nontrivial (p, q) tree T, $\gamma_{sslb}(T) = q$.
Proof.
For any nontrivial tree T with q edges, the number of blocks n = q, From, Theorem 1 $\gamma_{sslb}(T) = q$.
Now we establish the relation between strong split domination number of a tree and strong split line domination number of a tree.
Theorem 2.
For any nontrivial tree T, $\gamma_{ss}(T) \le \gamma_{sslb}(T) - \delta(T)$.
Proof.
Let $E = \{e_1, e_2\ldots e_{p-1}\}$ and $S = \{v_1, v_2\ldots v_p\}$ be the set of edges and vertices of T respectively. Further $S_1 \subset S$ be the set of cut vertices of T. Suppose $S_2 \subseteq S_1$ such that $\forall v_i \in \langle S - S_2\rangle$ is an isolate with at least two vertices and $N[S_2] = V(T)$. Then S_2 is strong split dominating set of T. In $L_b(T)$, $E' = \{v'_1, v'_2, \ldots\ldots v'_{p-1}\}$ is the set of vertices corresponding to the elements of E. Since

each ei ∈ E $1 \le i \le p-1$ is a block of T, then $E'' = \{v''_1, v''_2, \ldots\ldots v''_{p-1}\}$ forms the set of vertices corresponding to the blocks of E. Since each $v'_i \in E'$ is a cutvertex of $L_b(T)$ and $V[L_b(T)] = E' \cup E''$, then $\forall\, v''_i \in \langle V[L_b(T)] - E''\rangle$ is an isolate and $|V[L_b(T) - E'']| \ge 2$. Hence E'' is a strong split dominating set of $L_b(T)$. Further $|S| < |E' \cup E''|$ which gives $|S_2| < |E'|$ and for any tree T there exists at least one vertex $v \in V(T)$ with deg $(v) = \delta(T)$. Then $|S_2| \le |E'| - \delta(T)$. Thus $\gamma_{ss}(T) \le \gamma_{sslb}(T) - \delta(T)$.

Theorem 3.

For any nontrivial tree T, $\gamma_{ssbc}(T) = \gamma_{sslb}(T)$.

Proof.

Since each edge of a tree is a block, then from Theorem 1 and Theorem A, the result follows.

In the following theorem we establish the equality of strong split semitotal block domination in terms of strong split semitotal block domination.

Theorem 4.

For any tree T, $\gamma_{sslb}(T) = \gamma_{sstb}(T) - 1$.

Proof.

From corollary 1 and Theorem B, the result follows.

Strong split middle domination can be expressed in terms of $\gamma_{sslb}(G)$ in the following theorem.

Theorem 5.

For any connected (p, q) graph G, $\gamma_{sslb}(G) \le \gamma_{ssm}(G)$. Equality holds for a tree.

Proof.

For any connected graph G with $E(G) = \{e_1, e_2, \ldots.., e_q\}$ be the set of edges and $B(G) = \{B_1, B_2, \ldots\ldots, B_n\}$ be the set of blocks of G. Let $B' = \{b_1, b_2 \ldots b_n\}$ be the set of vertices corresponding to the blocks of G and $E' = \{e'_1, e'_2, \ldots\ldots., e'_q\}$ be the set of vertices corresponding to the edges of G in $L_b(G)$. Then $V[L_b(G)] = E' \cup B'$. Since $\forall\, e'_i \in E'$, $1 \le i \le q$, deg $(e'_i) = 1$ and $\forall\, b_j \in B'$ $1 \le j \le n$, $N(b_j) = e'_i$, then $\langle V[Lb(G)] - B'\rangle$ is totally disconnected with $|V[L_b(G)] - B'| \ge 2$. Also $N[B'] = V[L_b(G)]$. Hence B' is a strong split dominating set of $L_b(G)$. So $|B'| = \gamma_{sslb}(G)$. Further the middle graph $M(G)$ of a graph G has $V[M(G)] = V(G) \cup E(G)$. From, Theorem C, $\gamma_{ssm}(G) = |E(G)|$. Also for any graph G, $|E(G)| \ge |B'|$. Thus $\gamma_{ssm}(G) \ge \gamma_{sslb}(G)$. For equality, suppose G is a tree. Then by corollary 1 and Theorem C, equality holds.

Theorem 6.

For any connected (p, q) graph G, $\beta_1[L_b(G)] = \gamma_{sslb}(G) = \alpha_0[L_b(G)]$.

Proof.

Let $B = \{B_1, B_2, \ldots\ldots., B_n\}$ be the set of blocks of G where each block b_i, $1 \le i \le n$ contains m_i edges. Then in $L_b(G)$, $|V[L_b(G)]| = |B| \cup \sum_{i=1}^{n} m_i$. Further for each block vertex b_i corresponding to the block B_i of G, there are m_i endedges in $L_b(G)$ adjacent to b_i. Thus there are n sets of endedges such that each set contains m_i , $1 \le i \le n$ endedges. Choosing one endedge from each of n sets gives maximal set say E_e of endedges which are independent such that $N[E_e] = E[L_b(G)]$. Clearly $|E_e| = \beta_1$. From Theorem 1, we have $\beta_1[L_b(G)] = \gamma_{sslb}(G)$. Also from Theorem D, $\gamma_{sslb}(G) = \alpha_0[L_b(G)]$. Hence $\beta_1[L_b(G)] = \gamma_{sslb}(G) = \alpha_0[L_b(G)]$.

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