THE EFFECT OF SUCTION/INJECTION ON VISCOUS FLUID FLOW, AND HEAT TRANSFER OVER A PERMEABLE STRETCHING SHEET IN PRESCENCE OF PARTIAL SLIP

[1]NOORJAHAN
Department of Mathematics, Dravidian University, Kuppam-517425, Andhra Pradesh, India

***ABSTRACT:** In this study, the effect of suction/injection on viscous fluid flow, and heat transfer over a permeable stretching sheet in prescence of partial slip is considered for investigation.The nonlinear ordinary differential equations of the corresponding flow field momentum,temperature equations are derived by employing the similarity transformation technique.The dimensionless nonlinear ordinary differential equations have been composed of momentum and temperature at the sheet,which have been solved analytically.The effects of various flow and heat transfer parameters have been analysed, and shown with the aid of graphs.*

***KEYWORDS:** Viscous fluid;Dimensionless nonlinear ordinary differential equations;Suction/Injection;Stretching Sheet.*

I Introduction

Analysis of viscous and visco-elastic fluids over a stretching surface has received much attention because of their extensive applications in the field of metallurgy and chemical engineering. For example, in the extrusion of polymer sheet from a dye or in the drawing of plastic films. Such investigations of magneto hydrodynamic (MHD) flows are very important industrially and have applications in different areas such as petroleum production and metallurgical process. Partial velocity slip may occur on the stretching boundary when the field is particulate such as emulsions, suspensions, foams and polymer solutions.

In certain cases, partial slip between the fluid and the moving surface may occur. Examples include situations when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. Also the extrudate may be rough or porous where the no slip condition is considered. In these cases the proper boundary condition is well described by Navier's condition, where the amount of relative slip is proportional to local shear stress.

Study of heat transfer in a laminar boundary layer flow over a moving stretching surface has gained considerable practical relevance in the field of electrochemistry and polymer processing (Gorla [1].The important studies of these transport processes have so for been devoted to flows induced by surface moving with constant velocity. Pioneering work was carried out by Sakiadis [2] and that was extended by Crane [3]. Due to the increasing application of non Newtonian fluids in industry.

Researchers Z. Abbas,etal[4] and Cortell[5] studied the heat transfer problem associated with the non Newtonian boundary layer-flow past a stretching sheet. Crane [3] studied the steady two dimensional flow caused by a stretching sheet whose velocity U varies linearly with the distance x from a fixed point on the sheet i.e., U = cx. This problem has later been extended also to fluids obeying non-Newtonian constitutive equations. The viscous slip flow due to a two dimensional stretching surface was studied by Wang [6].

While Chiam [7] and Anderson and Dandapat [8] considered the motion of Micropolar and power-law fluids, the Rivlin-Erickson fluid studied by Siddappa and Khapate [9]and Walter's liquid B' being considered by siddappa and Abel [10],both exhibit normal-stress difference in simple shear flows. Rajagopal et al [11] analyzed the effects of visco-elasticity on the flow of second order fluid with gradually fading memory and arrived at the same governing boundary layer equation as that in Abel etal [12].

A common feature of all these analyses is the assumption that the flow field obeys the conventional no-slip condition at the sheet that is the velocity component $u(x,y)$ parallel with the sheet becomes equal to the sheet velocity $U = cx$ at the sheet. In certain situations, however, the assumption of no-slip does no longer apply and should be replaced by a partial slip boundary condition

$$u(x,y) = L\frac{\partial u}{\partial y} \qquad (1)$$

which relates the fluid velocity u to the shear rate $\frac{\partial u}{\partial y}$ at the boundary. Here L is the slip length, and y denotes the coordinate perpendicular to the surface. This slip-flow condition was first introduced by C-L.M.H Navier more than a century ago and has more recently been used in studies of fluid flow past permeable walls , slotted plates,rough and coated surfaces and gas and liquid flow in micro devices . The no-slip boundary condition is known as the central tenets of the Navier-Stokes theory. But there are situations wherein such condition is not appropriate .Especially; no slip condition is inadequate for most non-Newtonian fluids. For example polymer melts often exhibit macroscopic wall slip and that in general is governed by a non-linear and monotone relation between the slip velocity and traction. The fluids exhibiting boundary slip find applications in technology such as in the polishing of artificial heart valves.
transfer. Miksis et al. [13], have studied Slip over rough and coated surfaces. Ariel and Asghar [14], have studied the flow of an elasticoviscous fluid past a stretching sheet with partial slip.

Motivated by aforementioned all these works, we contemplate to study the effects slip and no-slip condition. The focal point is to consider viscous flow past a stretching sheet in the presence of partial slip for which heat and mass transfer solutions will be derived and discussed.

II Problem formulation and Mathematical solution

Let us consider the laminar flow of viscous incompressible fluid past a flat and impressible elastic sheet. By applying two equal and opposite forces along the x-axis the sheet is stretched with a speed $u_w(x)$ proportional to the distance from the origin x=0. The resulting motion of the otherwise quiescent fluid is caused by the moving sheet, and the flow is governed by the constant property Navier-stokes equations for steady two-dimensional flow. The viscous fluid is only partially adhering to the stretching sheet, and the fluid motion is thus subjected to the slip-flow condition and the condition far away from the stretching sheet which are expressed as

$$u(x,y)-u_w(x)=L\frac{\partial u}{\partial y} \quad \text{at} \quad y=0 \qquad \text{and } u\rightarrow 0 \text{ as } y\rightarrow\infty \qquad (2)$$

It is noteworthy that in the present problem the fluid is dragged by the moving sheet. The Navier Stokes equations of motion for steady viscous incompressible fluid are given by

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \qquad (3)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 y}{\partial y^2}\right) \qquad (4)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (5)$$

Where υ is the kinematic viscosity, ρ is the density, p is the fluid pressure.

$$\text{Let } u=cxf'(\eta), \qquad v=-\left(\frac{c\mu}{\rho}\right)^{\frac{1}{2}}f(\eta), \qquad \eta=\left(\frac{c\rho}{\mu}\right)^{\frac{1}{2}}y \qquad (6)$$

Introducing similarity transformation (6), equations (2), (3), (4) and (5) takes the form,

$$(f')^2-ff''=f''', \qquad (7)$$

$$ff'=-f''+\frac{1}{2}g', \qquad (8)$$

$$f(0)=0,$$

$$f'(0)=1+\gamma f''(0), \qquad (9)$$

$$g(0)=0,$$

and $f'\rightarrow 0 \quad as \quad \eta\rightarrow\infty$

Here prime denotes differentiation with respect to η. The solution of (7) and (8) subjected to boundary conditions (9) can be found in the form

$$f(\eta)=\beta\left[1-\exp(-\beta\eta)\right], \qquad (10)$$

$$g(\eta)=f^2+2f'-2\beta^2, \qquad (11)$$

The streamline $\varphi=\varphi_0$ is given by

$$y=\frac{-\left(\frac{v}{c}\right)^{\frac{1}{2}}\ln\left[1-\beta\varphi_0(cv)^{-\frac{1}{2}}x^{-1}\right]}{\beta} \qquad (12)$$

The shear stress coefficient is given by

$$c_f=\frac{\mu\frac{du}{dy}/_{y=0}}{\frac{pu_w{}^2}{2}}=\frac{2f''(0)}{\mathrm{Re}_x^{\frac{1}{2}}}=-2\beta\,\mathrm{Re}_x^{-\frac{1}{2}} \qquad (13)$$

Where the local Reynolds number is defined as $\mathrm{Re}_x=U_w\frac{x}{v}$ .The velocity component v does not contribute to shear stress at the sheet as that using the boundary layer assumption.

The new dimensionless parameter β is the positive root of:

$$\gamma\beta^3+\beta^2-1=0 \qquad (14)$$

This seems from the partial slip condition (9). Here, with γ =0 (i.e., no-slip) β becomes equal to unity, and the solution derived earlier by Crane [1] is recovered.

For $\gamma > 0$ the only real and positive root of (14) is

$$\beta = \frac{1}{3\gamma}\left[2\cos\left(\varphi/3\right)-1\right] \text{ for } \gamma < \frac{2}{3\sqrt{3}} \quad (15)$$

and

$$\beta = \frac{1}{3\gamma}\left[\frac{A}{2}+\frac{2}{A}-1\right] \quad for \quad \gamma \geq \frac{2}{3\sqrt{3}}, \quad (16)$$

Where

$$\cos\varphi = \frac{27}{2}\gamma^2 - 1 \quad (17)$$

and

$$A = \left[108\gamma^2 - 8 + 12\sqrt{3}\sqrt{27\gamma^2 - 4}\gamma\right]^{1/3} \quad (18)$$

Define φ and A in terms of the slip factor γ. For the special case $\gamma = \frac{2}{3\sqrt{3}}$ both (15) A and (16) gives $\beta = \frac{\sqrt{3}}{2}$. Moreover, for infinitely high values of γ, (16) shows that β tends to $\gamma^{1/3}$.

III Heat transfer analysis

The governing boundary layer heat transport equation in the presence of viscous dissipation and non-uniform internal heat source/sink for the two-dimensional flow problem under consideration is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \quad (19)$$

Where ρ is the density, Cp is the specific heat at constant pressure.

The solution of equation (19) is obtained using two different types of heating processes namely,

(i) Constant surface Temperature (CST)

(ii) Prescribed surface Temperature (PST) conditions as described below.

Constant Surface Temperature (CST)

The boundary conditions in case of CST is given by

$$\begin{aligned} T &= T_w \quad at \quad y = 0 \\ T &\to T_\infty \quad as \quad y \to \infty \end{aligned} \quad (20)$$

Where T_w is the temperature of the sheet and T_∞ is the temperature of the fluid far away from the sheet.

Defining the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (21)$$

Where $T - T_\infty = (T_w - T_\infty)\theta(\eta)$ *and* $T_w - T_\infty$ is a constant

Using equation (21), equation (19) can be written in the form

$$\theta''(\eta) + Prf(\eta)\theta'(\eta) = 0 \quad (22)$$

Where $Pr = \frac{\mu Cp}{k}$ is the Prandtl number, consequently the boundary conditions in (20) take the form

$$\begin{aligned} \theta(\eta) &= 1 \quad at \quad \eta = 0 \\ \theta(\eta) &\to 0 \quad as \quad \eta \to \infty \end{aligned} \quad (23)$$

Introducing the new independent variable

$$\xi = -\Pr e^{-\beta\eta} \quad (24)$$

Substituting in (22) we obtain

$$\xi\frac{d^2\theta}{d\xi^2} + (1 - \Pr - \xi)\frac{d\theta}{d\xi} = 0 \quad (25)$$

The corresponding boundary conditions are

$$\begin{aligned} \theta(\xi) &= 1 \quad at \quad \xi = -\Pr \\ \theta(\xi) &\to 0 \quad as \quad \xi \to 0 \end{aligned} \quad (26)$$

The solution of (25) subject to the boundary conditions (26) is obtained in terms of ξ and η as

$$\theta(\xi)=\frac{\xi^{\Pr}M(\Pr,\Pr+1,\xi)}{(-\Pr)^{\Pr}M(\Pr,\Pr+1,-\Pr)},\qquad (27)$$

$$\theta(\eta)=\frac{e^{-\beta\eta\Pr}M\left(\Pr,\Pr+1,-\Pr e^{-\beta\eta}\right)}{M(\Pr,\Pr+1,-\Pr)}.$$

The heat transfer rate, characterized by the Nusselt number, at the sheet is given by

$$Nu_x=\frac{-k\dfrac{\partial T}{\partial y}}{k(T_w-T_\infty)}x=-\left(\frac{cx^2}{\nu}\right)^{\frac{1}{2}}\theta'(0)=\mathrm{Re}_x^{\frac{1}{2}}\theta'(0) \qquad (28)$$

$$\theta'(0)=\frac{\dfrac{\Pr-1}{\Pr+1}\Pr\beta M(\Pr,\Pr+2,-\Pr)}{M(\Pr-1,\Pr+1,-\Pr)}-\Pr\beta \qquad (29)$$

Prescribed Surface Temperature (PST):

The boundary conditions in case of PST are

$$T=T_w=T_\infty+A\left(\frac{x}{l}\right)\quad at\quad y\to 0 \qquad (30)$$

$$T\to T_\infty\quad as\quad y\to\infty,$$

Using 5.3.30) with $T_w-T_\infty=A\left(\frac{x}{l}\right)$, equation (19) can be written in the form

$$\theta''(\eta)+\Pr f(\eta)\theta'(\eta)-\Pr f'(\eta)\theta(\eta)=0 \qquad (31)$$

The constants appearing in the above equation have their usual definition as given in case of CST. The boundary conditions (30) take the form

$$\theta(\eta)=1\quad at\quad \eta=0 \qquad (32)$$

$$\theta(\eta)\to 0\quad as\quad \eta\to\infty$$

Using the new independent variable defined in equation (24) we get

$$\xi\frac{d^2\theta}{d\xi^2}+[1-\Pr-\xi]\frac{d\theta}{d\xi}+\theta=0 \qquad (33)$$

The corresponding boundary conditions (5.3.31) take the form

$$\theta(\xi)=1\quad at\quad \xi=-\Pr \qquad (34)$$

$$\theta(\xi)\to 0\quad as\quad \xi\to 0$$

The solution of (33) subjected to the boundary conditions (34) is obtained in the form

$$\theta(\xi)=\frac{\xi^{\Pr}M(\Pr-1,\Pr+1,\xi)}{(-\Pr)^{\Pr}M(\Pr-1,\Pr+1,-\Pr)} \qquad (35)$$

and in terms of η is

$$\theta(\eta)=\frac{e^{-\beta\eta\Pr}M\left(\Pr-1,\Pr+1,-\Pr e^{-\beta\eta}\right)}{M(\Pr-1,\Pr+1,-\Pr)} \qquad (36)$$

IV Results and Discussion

An analysis has been made to study the behavior of a viscous incompressible fluid taking into consideration of partial slip condition. Analytical solution is obtained for the flow problem. Heat transfer are also addressed in the present investigation. Analytical solution for the heat transport equations are sought in terms of hypergeomeric Kummer's functions. The effect of governing parameters like, slip parameter γ, Prandtl number *Pr*, are shown graphically from fig.2 to fig.6. Before discussing the results of the present investigation we mention the following:

Fig.3 and 4 represents the effect of slip parameter γ on the heat transfer in CST and PST cases respectively. The increasing values of slip parameter γ results in increase of temperature of fluid. Fig.5 and 6 represents the effect of Prandtl number *Pr* on the heat transfer in CST and PST cases respectively. From these plots it is evident that large values of Prandtl number results in decrease in temperature of the flow field.

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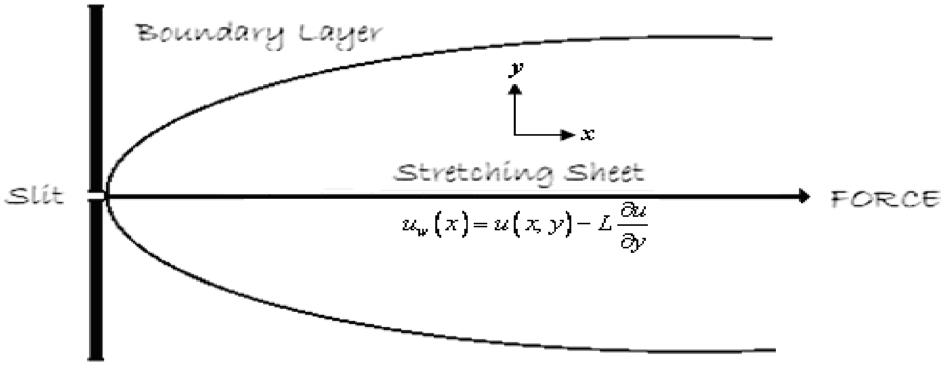

Fig.1. Schematic diagram of stretching sheet.

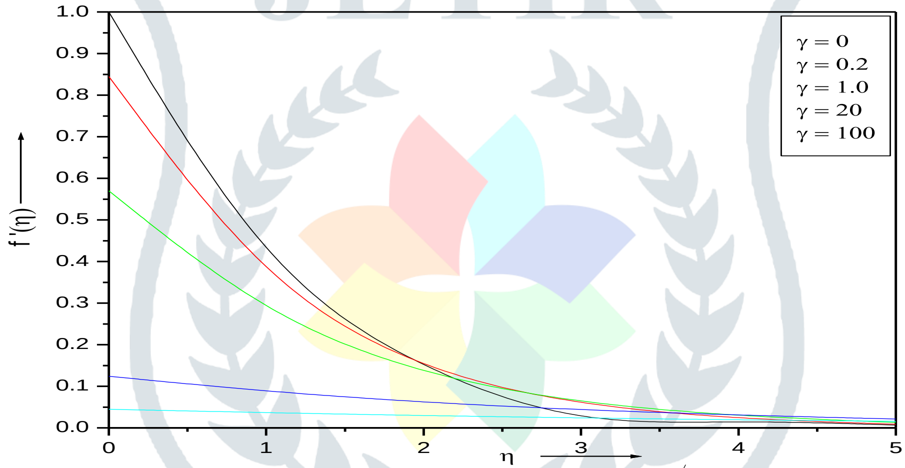

Fig.2. Dimensionalless velocity profiles f′(η) for different values of the slip factor γ

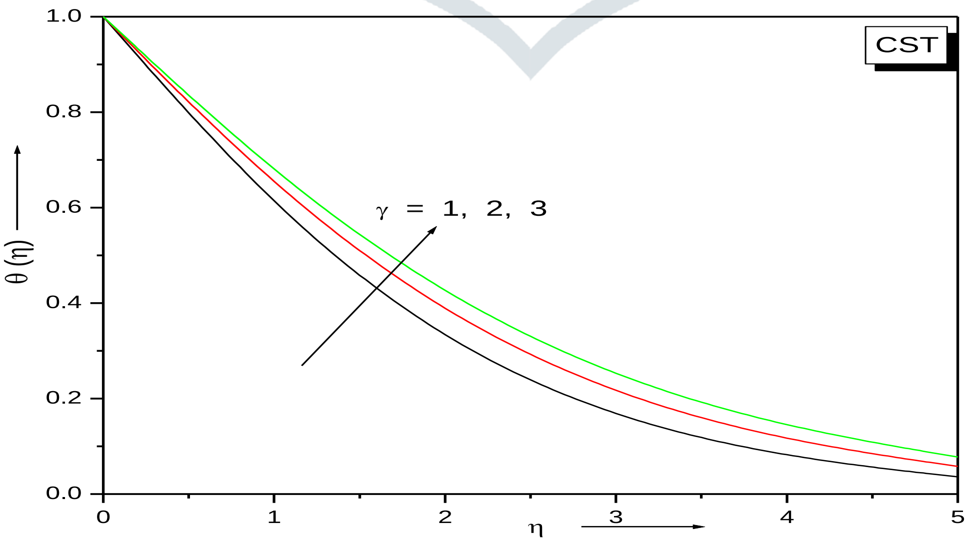

Fig.3. The effect of temperature profile θ (η) for different γ

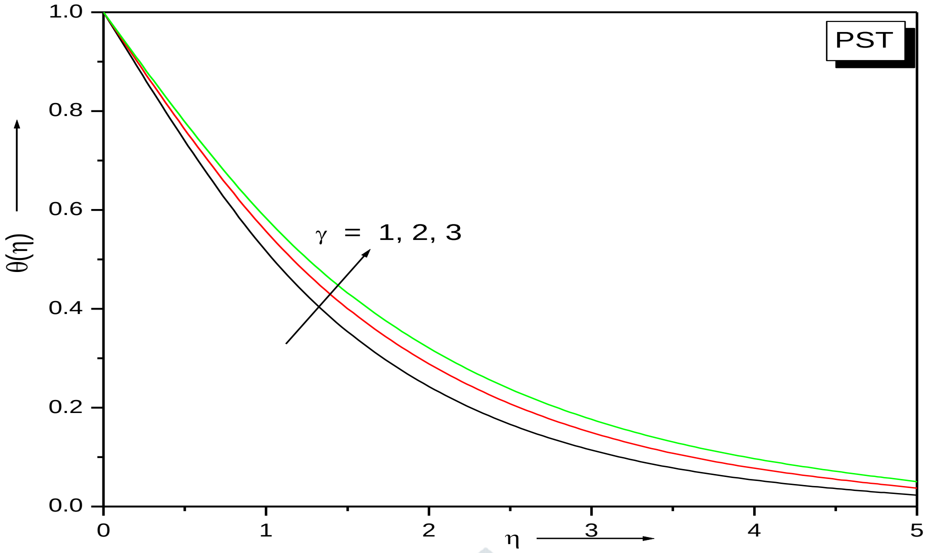

Fig.4. The effect of temperature profile θ (η) for different values of γ

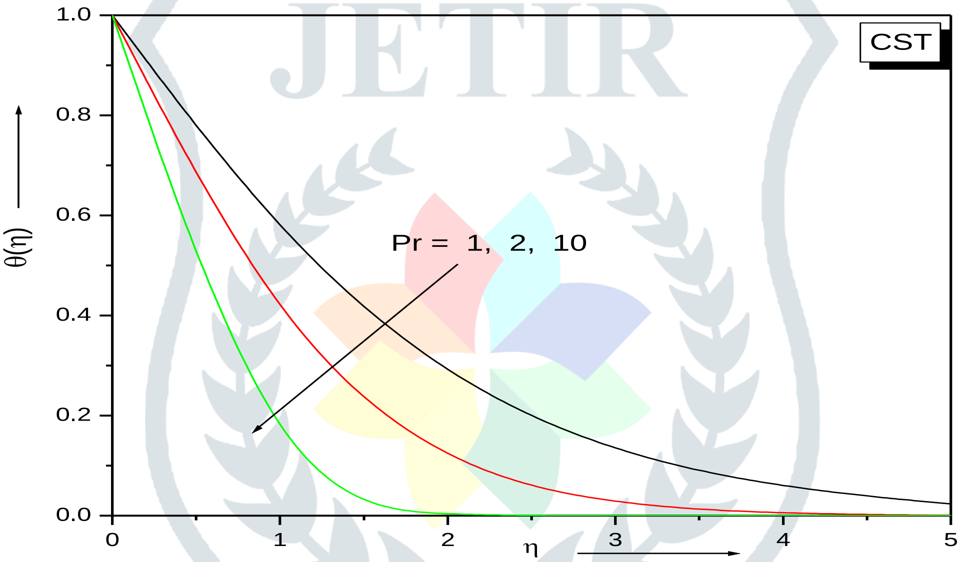

Fig.5. The effect of temperature profile θ (η) for different Pr

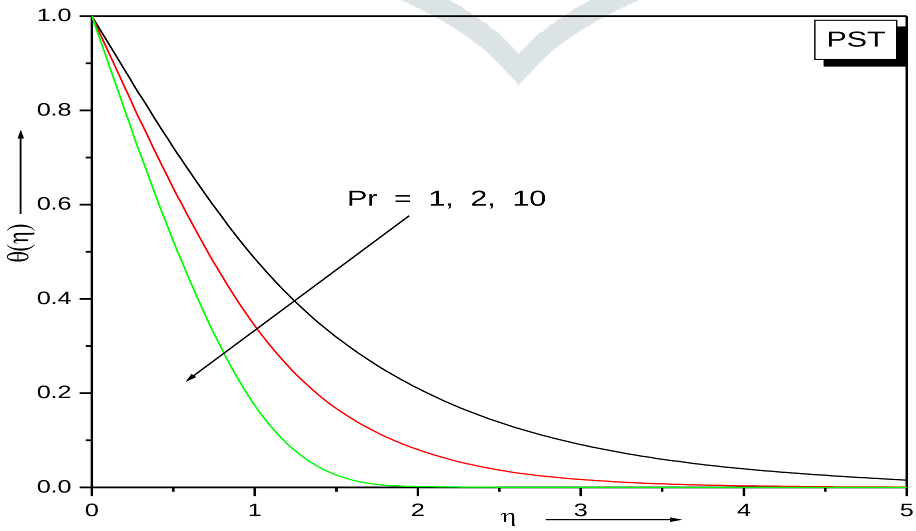

Fig.6. The effect of temperature profile θ (η) for different values of Pr