FUZZY SET THEORETIC APPROACH TO STOCHASTIC GAME

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ABSTRACT

In this paper stochastic game with fuzzy approach are defined as hierarchical non cooperative games. Here we have approved that for any strategic alternatives of one of the participants there exists only a fuzzy set of strategic alternatives.

Key words:- stochastic game, fuzzy approach, strategy.

1. Definition of Stochastic game :

Firstly we define a stochastic game as follows:

There are two players I and II. The play proceeds by steps from position to position. Let there be two polish spaces A & B. It has following five objects:

S - S is a non empty Borel subset of a polish space, the set of states of the system,

A (s) - A (s) is a non empty subset of a polish space, the set of actions available to player I at State. S

B (s) - B (s) is a non-empty subset of a polish space, the set of actions available to player II at state s.

q - q associates Borel measurably with each triple (s,a,b) $\in$ SxAxB a probability measure on the Borel subsets of S.

r - r the rewards function, is a bounded measurable function on SxAxB.

We shall assume throughout that A(s) $\subseteq$ A & B (s) $\subseteq$ B for all s $\in$ S.

Initially player I and II observe the current state s of the system and choose actions a(s) $\in$ A(s) and b(s) $\in$ B(s) respectively, the choice of the actions is made with full knowledge of the history of the system as it has evolved to the present.

2. Fuzzy Games:-

Game theory has an importance in the field of decision theory. A game is determined by informations decisions and goals. As far As real Situations are concerned it is very common to deal with elements defined in an approximate way. Human ideas and decisions are fuzzy. Human thinks during facing a new problem about the recognition and classification of the problem. What kind of actions have been taken in the past for the same or similar problems to the present one and what kinds of results have been obtained (remembrance and re-evaluation of the past experience). What kinds of environmental changes have happened and so on. The experience can be defined as the association of set of actions and set of results where a relation from former set to later is changeable with the environment. A fuzzy decision D may be defined as an intersection of fuzzy goals G and fuzzy constraints C i.e.

$$D(x) = C(x) \cap G(x) ,$$

where fuzzy goals and fuzzy constraints are defined precisely as fuzzy subsets i.e. as membership functions G and C. Fuzzy decisions are made one after another. During each step of the fuzzy decision making procedure (FDMP) having limited resources and subjective information, different criteria can exist. Now fuzzy games are defined as follows.

Fuzzy games are defined as hierarchical no cooperative games of n + 1 participants (FDMP and n mutually independent objects Fj) provided the information about objects, the value of the particular goals and the parameter evaluation are of fuzzy nature.

If X, U and V_i be the finite set of states (V_i = 1, 2, 3, ... n) and **f :** $X \times Y \times V_i \rightarrow X$ be the function then the following particular case describes the relations between the $FDMP\text{-}F_0$ and the objects F_1 :

$$X_{t+1} = f(x_t, u_t; v_{it}) : t = 0,\ldots\ldots\ldots,N;$$

$$x \in X : u_t \in U, v_{it} \in V_i ;$$

$$U_{ot} = u(X^{\circ}, X^1,\ldots\ldots, X^4, e) ;$$

$$V_i = v(x, x^i) ;$$

where, X^0 are the coordinates of FDMP F_0.

X^i are the coordinates of object F_i.

U_{ot} is the strategy of FDMP F_0.

V_{it} is the strategy of object F_i.

At each time moment t the system input is constrained, the constraints B_t and C_t are the fuzzy patterns in set of states U and V_i correspondingly and are defined by their membership functions. The initial state and the final time moment are fixed. Evaluation of the solution data is done by means of the quantitative scales $q_i \in$ C (length, cost) or on the quantity

evaluation plan (q,p) $\in$ C (better-worse, Fast-slow). Built on such bases, membership surfaces represented by the equipotential lines define the properties of states and the class of decisions. In the solving process a multistep procedure is realized by $F_0(u_k) = F_0(u_{k1}), F_0(u_{k2}), F_0(u_{kn}) = 0$; where F_0 is the membership function of the rational decision of the FDMP F_0 at K-step in the situation of the fuzzy elements presented in the situation data, criteria, constraints and the evaluation system is done according to some definite rule.

3. Fuzzy approach to stochastic games

Game theory has an importance in the field of decision theory. Many real problems can be modeled as games. As far as real situations are concerned, it is very common to deal with elements defined in an approximate way. These problems are modeled classically without taking into account such lacks of accuracy, which leads to less suitability of the model for the proposed problem. In such cases, the theoretical support provided by the fuzzy subsets can become a useful tool to model these problems properly.

(c) For any player must be given a method for deciding if a given exchange of information is preferable to another.

4. Stochastic game with fuzzy classical concept.

A two person game consists of the following data

(a) two players denoted by 1 and 2 respectively.

(b) for any k, a set $\sum_k = \{\sigma_1^{(k)}, \ldots\ldots\ldots, \sigma_k^{(k)}\}$ is given where $\sum_k$ is called the set of pure strategies of K ;

(c) for any pair ($\sigma_{i_1}^{(1)}$, $\sigma_{i_2}^{(1)}$) form $\sum_1$ x $\sum_2$, there is a unique real number F_k($\sigma_{i_1}^{(1)}$, $\sigma_{i_2}^{(1)}$) called the gain of K.

In such a game, a possible individual choice of the player K consists of a mixed strategy of k, where we understand by mixed strategy an n-vector p = (p_1,……….,p_n) with non-negative components and with $\sum_{i=1,}^{n} p_i = 1$.

An exchange of information in the given two-person game consists of the following facts: player 1 chooses and mixed strategy $\xi^1 \in S_1$, and player chooses the mixed strategy $\xi^1 \in S_2$, is the given information from each of the two players; player 1 receives his gain $\phi_1(\xi^1, \xi^2)$ and player 2 receives his gain $\phi_2(\xi^1, \xi^2)$ where :

$$\phi_1(\xi^1, \xi^2) = \sum_{i=1}^{n} \sum_{j=1}^{n} \xi_i^k F_k(\sigma_i^{(1)}, \sigma_i^{(2)}) \xi_j^2 \qquad (1)$$

with $\xi^k = (\xi_i^k, \ldots\ldots\ldots, \xi_{mk}^k)$, k = 1, 2- this is the received information of the two players. It is clear that any possible exchange of information in such a game can be completely described by a pair $(\xi^1, \xi^2) \in$ $S_1 x S_2$. Let two possible exchanges of information (ξ_0^1, ξ^2) and (ξ^1, ξ^2) be given. Here we say that (ξ_0^1, ξ^2) is preferable to (ξ^1, ξ^2) from the point of view of the player k, and we denote

$(\xi^1, \xi^2) > (\xi_0^1, \xi^2)$ iff.

$$\phi_k(\xi^1, \xi^2) > \phi_k(\xi_0^1, \xi^2) \qquad (2)$$

Hence a classical two person game is also a game according to the general definition.

However, we can must made an important critical remark on the classical two person game: all the elements $\xi^k \in S_k$ are equally possible choices of the player k. There exist many social, political or economic phenomena which are games in our enlarged sense, and which can be studied satisfactorily with the restriction, but not every game (i.e, in the extended sense) has this property.

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