Effects of Single wall carbon nano tube and variable viscosity on blood flow through an artery with overlapping stenosis

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Abstract:

This paper is concerned with the analysis ofsingle wall carbon nanotube on blood flow through an overlapping stenosis. The nature of blood through tube is considered as nano viscous fluid. The mild stenosis approximation and corresponding boundary conditions are usedto obtain analytic expressions for axial velocity, temperature distribution, wall shear stress and resistance to flow. The variation in different flow parameters are studied through graphs.

***Keywords*: ** Nanofluid viscosity, Thermal conductivity, Heat capacitance, Overlapping stenosis.

Introduction:

Blood flow is an eminent topic to the researchers due to its huge applications in arterial mechanics. Mainly blood flow in blood vesselsis a crucial part to researchers because many arterial deceases are major cause fordeath in the present world like heart diseases and stroke. These arterial diseases affect the blood vessels of heart, kidneys, legsetc. That is the main reason why researchers have interest in examining the flow behavior in the blood vessels. One of such disease is arterial stenosis, is an irregular condition in blood vessels with vascular disease called as stenosis. In recent past, huge experimental and theoretical studies [1-5] are available to analyze the blood flow characteristics in narrowed tubes.

In the recent time a new wing of fluid dynamics has emanated, namely nanofluid.Nanofluids have huge applications in various fields like biology, engineering and medical science. It is discovered by Choi [6] with suspension of nanoparticles in base fluid. The suspended nanoparticles in base fluid alone are not enough to increase the thermal conductivity due to the shape and size of the nanoparticles. Murshed et al. studied that the carbon nanotube provides better thermal conductivity due to the physical and chemical properties[7]. N.S. Nadeem and Ijaz examined single wall carbon nanotube analysis on blood flow through multiple stenosis by considering viscous nanofluid[8].More studies related to this is discussed in the references[9-11].

Motivated from above studies available on blood flows in the narrowed tube, the intention of present paper is to discuss the effects of single wall carbon nanotube on blood flow through a stenosed artery with overlapping stenosis. The exact solutions are obtained for the governing equations of the given model. The conclusions are made for the present analysis based on the graphs.

FORMULATION OF THE PROBLEM:

Consider the steady and incompressible blood flow through an axially symmetrical uniformtube of length is L and radius R_0 through an overlapping stenosis with variable viscosity. The cylindricalpolar coordinate system (r, θ, z) is considered. Here the z axis is taken along the axis of the artery, while θ and r are taken along the circumferential and radial directions respectively.

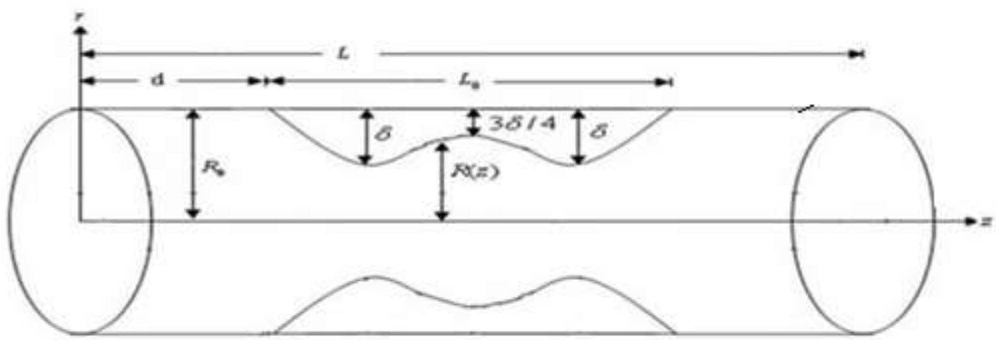


Fig1: Geometry of the Stenosed artery

The geometry of the arterial wall with overlapping stenosis is given as[5]

$$h = \frac{R(z)}{R_0} = 1 - \frac{3\delta}{2R_0 L_0^4}\left[11(z-d)L_0^3 - 47(z-d)^2 L_0^2 + 72(z-d)^3 L_0 - 36(z-d)^4\right], d \le z \le d + L_0,$$

$$= 1, \quad otherwise. \quad (1)$$

Here R is the tube radius in the presence of stenosis, d indicates stenosis location and L_0 it's length, δ is the maximum height of the stenosis located at $z = d + \frac{L_0}{6}, z = d + \frac{5L_0}{6}$. The critical height is taken as $\frac{3\delta}{4}$ at $z = d + L_0/2$, from the origin.

The governing equations for conservation of mass, momentum and temperature for variable viscous nanofluid can be written as,

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\pi} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (2)$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{r}} + \bar{\omega}\frac{\partial \bar{\omega}}{\partial \bar{z}}\right) = -\frac{\partial \bar{p}}{\partial \bar{r}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(2\bar{r}\mu_{nf}\frac{\partial \bar{u}}{\partial \bar{r}}\right) + \frac{\partial}{\partial \bar{z}}\left(\mu_{nf}\left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}}\right)\right) - 2\mu_{nf}\left(\frac{\bar{u}}{(\bar{r})^2}\right) \quad (3)$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{r}} + \bar{\omega}\frac{\partial \bar{\omega}}{\partial \bar{z}}\right) = -\frac{\partial \bar{p}}{\partial \bar{r}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\mu_{nf}\left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}}\right)\right) + \frac{\partial}{\partial \bar{z}}\left(2\mu_{nf}\frac{\partial \bar{w}}{\partial \bar{z}}\right) - g(\rho\gamma)_{nf}(\bar{T} - \bar{T}_0) \quad (4)$$

$$\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{\omega}\frac{\partial \bar{T}}{\partial \bar{z}}\right) = \frac{K_{\eta f}}{(\rho c_p)_{\eta f}}\left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{\omega}\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) + \frac{Q_0}{(\rho c_p)_{\eta f}} \quad (5)$$

In these equations $\bar{u}$and $\bar{w}$are components of velocity, $\bar{T}$is the temperature of fluid, Q_0isthe constant heat absorption or heat generation.[11]For the proposed nanofluid model $\mu_{\eta f}$is thevariable nanofluid viscosity,[10]$K_{\eta f}$is the thermal conductivity, $\rho_{nf}$is the density, $\gamma_{nf}$is the thermalexpansion coefficient and $(\rho cp)_{nf}$is the heat capacitance with the thermo physical properties givenas Ref[9],

$$\mu_{nf} = \frac{\mu_0 e^{-\alpha\theta}}{(1-\varphi)^{2.5}}\,, \qquad \mu_{nf} = \frac{K_{\eta f}}{(\rho c_p)_{\eta f}}\,, \qquad \rho_{nf} = (1-\varphi)_{\rho f} + \varphi_{\rho SWCNT}\,,$$

$$(\rho c_p)_{nf} = (1-\varphi)_{(\rho c_p)f} + \varphi_{(\rho c_p)SWCNT}\,, \quad (\rho\gamma)_{nf} = (1-\varphi)_{(\rho\gamma)f} + \varphi_{(\rho\gamma)SWCNT}\,,$$

$$\frac{K_{\eta f}}{K_f} = \frac{(1-\varphi)+2\varphi\frac{K_{SWCNT}}{K_{SWCNT}-K_f}\ln\frac{K_{SWCNT}+K_f}{2K_f}}{(1-\varphi)+2\varphi\frac{K_f}{K_{SWCNT}-K_f}\ln\frac{K_{SWCNT}+K_f}{2K_f}}$$

For the base fluid $\rho_f$is density, $\mu_f$is viscosity, $\gamma_f$is thermal expansion oefficient, $(\rho cp)_f$isheat capacitance and $k_f$is thermal conductivity, while for single carbon nanotubes $\rho_{SWCNT}$is density,$\gamma_{SWCNT}$is thermal expansion coefficient, $(\rho c_p)_{SWCNT}$is heat capacitance, $k_{SWCNT}$is thermalconductivity and φ is the volume fraction.

Non dimensional variables are defined as,

$$r = \frac{\bar{r}}{e_0}\,,\ w = \frac{\bar{w}}{u_0}\,,\ u = \frac{L\bar{u}}{u_0\bar{\delta}}\,,\ p = \frac{e_0^2\bar{p}}{u_0 S_l \mu_0}\,,\ \beta = \frac{Q_0 e_0^2}{T_0 K_f}\,,\ R_{en} = \frac{u_0 e_0 \rho_f}{\mu_0}\,,\ G_r = \frac{g T_0 \gamma_f \rho_f e_0^2}{u_0 \mu_0}\,,$$

$$\theta = \frac{T-T_0}{T_0}\,,\ z = \frac{\bar{z}}{z_0}\,.$$

Making use of the non-dimensional variables in eqs. (2)- (5) and applying the additional condition$\epsilon = \frac{R_0}{L_0} = o(1)$ for the case of mild stenosis$\left(\frac{\delta}{R_0} \ll 1\right)$, the non-dimensional governing equations after dropping the dashes can be written as

$$\frac{\partial p}{\partial r} = 0 \quad (6)$$

$$\frac{dp}{dz} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\mu_{nf}}{\mu_0}\left(r\frac{\partial w}{\partial r}\right)\right) + \frac{(\rho\gamma)_{nf}}{(\rho\gamma)_f} G_r \theta \qquad (7)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) + \beta\left(\frac{k_{nf}}{k_f}\right) = 0 \qquad (8)$$

Where β and G_r are the Hartmann number, heat absorption parameter and Grashof number respectively.

The non dimensional boundary conditions are

$$\frac{\partial w}{\partial r} = 0, \frac{\partial\theta}{\partial r} = 0 \quad \text{at } r = 0 \qquad (9)$$

$$w = 0, \theta = 0 \quad \text{at } r = h \qquad (10)$$

The nanofluid viscosity is defined as

$$\frac{\mu_{nf}}{\mu_0} = \frac{1}{e^{\alpha\theta}(1-\varphi)^{2.5}} \text{ and } e^{\alpha\theta} = 1 + \alpha\theta, \alpha \ll 1$$

SOLUTION OF THE PROBLEM:

The solutions of Eqs(7-8). under the given boundary conditions are

$$\theta = \frac{-\beta}{4}\left(\frac{k_{nf}}{k_f}\right)\left(r^2 - h^2\right) \tag{11}$$

$$w = \frac{\partial p}{\partial z}(1-\varphi)^{2.5}\left[(1+\alpha a_2)\left(\frac{r^2-h^2}{4}\right) - \alpha a_1\left(\frac{r^4-h^4}{8}\right)\right] - A\left[(a_1 - 3\alpha a_1 a_2)\left(\frac{r^4-h^4}{16}\right) - \alpha a_1^2\left(\frac{r^6-h^6}{24}\right) + \alpha a_2^2\left(\frac{r^2-h^2}{4}\right)\right]$$

(12)

Where $A = \frac{(\rho\gamma)_{nf}}{(\rho\gamma)_f} G_r (1-\varphi)^{2.5}$, $a_1 = \frac{\beta}{4}\frac{k_{nf}}{k_f}$,$a_2 = \frac{\beta h^2}{4}\frac{k_{nf}}{k_f}$

The flux q is calculated as by$q = \int_0^h 2ru\, dr.$ (13)

After integrating Eq.(13), we obtain as

$$\frac{dP}{dz} = \frac{q - A\left[\frac{(a_1 - 3\alpha a_1 a_2)h^6}{48} - \frac{\alpha a_1^2 h^8}{64} + \frac{\alpha h^4}{1\ldots}\right.}{(1-\varphi)^{2.5}\left(\frac{-(1+\alpha a_2)h^4}{4} + \frac{\alpha a_1 h^6}{24}\right.} \tag{14}$$

The pressure drop across the stenosis between z = and z =is given as

$$\Delta p = -\int_0^l \frac{dp}{dz} \tag{15}$$

The resistance to the flow is given by

$$\lambda = \tag{16}$$

The pressure drop in the absence of stenosis (h =) is obtained from eq. (14)as

$$\Delta p_N = \int_0^l \frac{q - A\left[\frac{(a_1)}{48} - \frac{\alpha a_1^2}{64}\right]}{(1-\varphi)^{2.5}\left(\frac{-(1+\alpha a_1)}{4} + \frac{\alpha a_1}{24}\right)}, \tag{17}$$

The wall shear stress is calculated as

(18)

$$S_{rz} = -\frac{h}{2}$$

RESULTS AND DISCUSSION:

In this section the results are analyzed through the graphs and the effects of various parameters on velocity, temperature, resistance to flow and wall shear stress are studied. The graphs are plotted for pure blood and SWCNT ($\varphi = 0.01,0.02$) by taking the constant values as $\beta = 2, \alpha = 0.1, L = 1, \delta = 0.02$

Temperature Profile:

The temperature profile for different values of stenosis height, heat absorption parameter is plotted in Figs.2-3. It is observed that with an increase in stenosis height and heat absorption parameter the temperature increases. Further the temperature is more for pure blood when compared to SWCNT.The temperature is more at the centre of the tube and it gradually decreases from the centre to the arterial wall. It is seen that when we increase the copperquantity inbase fluid then the arteries become more flexible and the temperature declines gradually.

Velocity Profile:

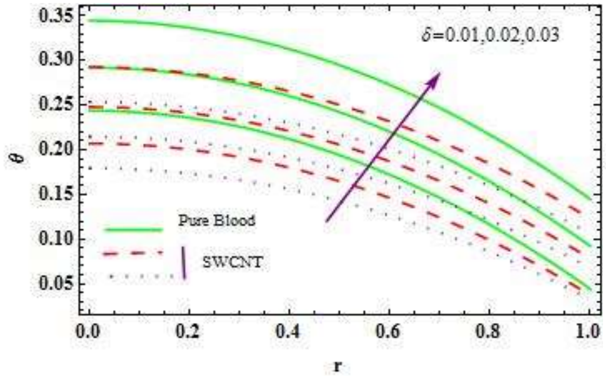
The variation in velocity for different values of stenosis height, heat absorption parameter, Grashof number is given in Figs.4-6. It is noticed that the velocity increases with the increase in values of stenosis height, heat absorption parameter, Grashof number. The velocity is maximum at the centre of the tube and minimum at the wall. The velocity is low for SWCNT when it is compared to pure blood.

Resistance to the flow:

In Figs.7-8, we observed that the resistance increases with the stenosis height, fluid viscosity constant but decreases with heat absorption parameter. The wall shear stress for different values of stenosis height is shown in Fig-9. It is seen that the wall shear stress is increases with stenosis height and it gives higher results for SWCNT as compared to pure blood.

Stream lines:

From Fig.10, it is observed that the size of the trapping bolus increases with the height of stenosis.



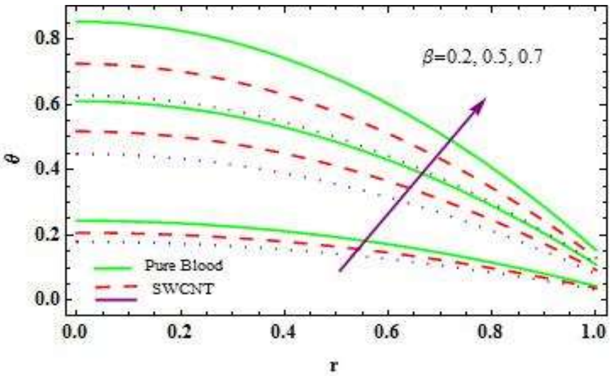
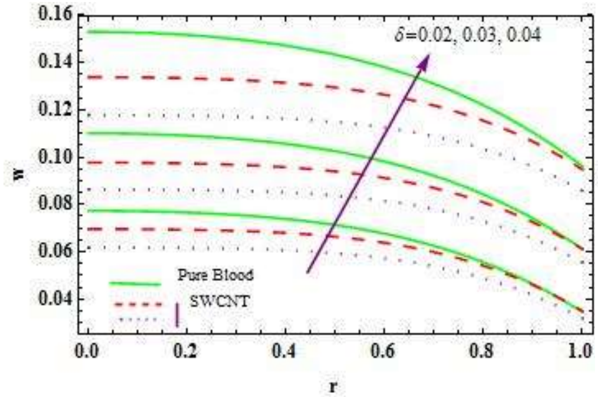


Fig 2: Change in temperature for different values of δ

Fig 3: Change in temperature for different values of β



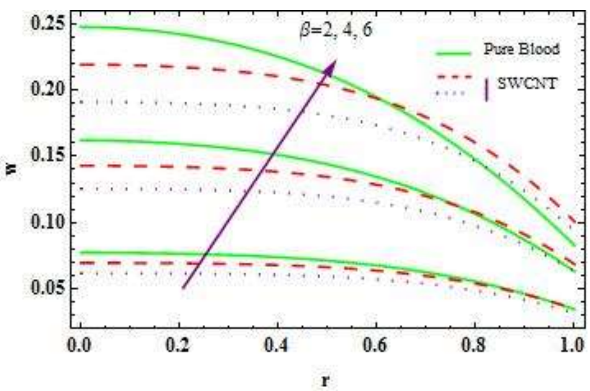
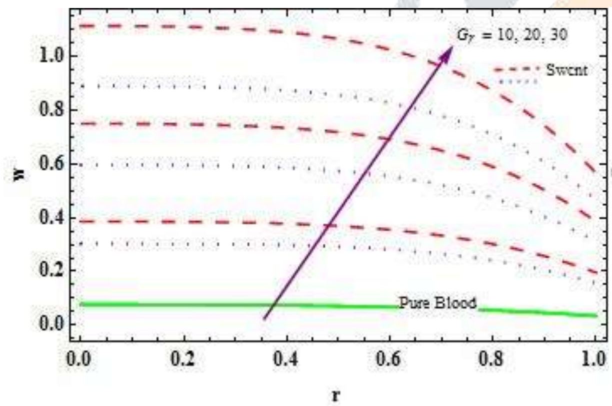


Fig 4: Change in velocity for different values of δ

Fig 5: Change in velocity for different β



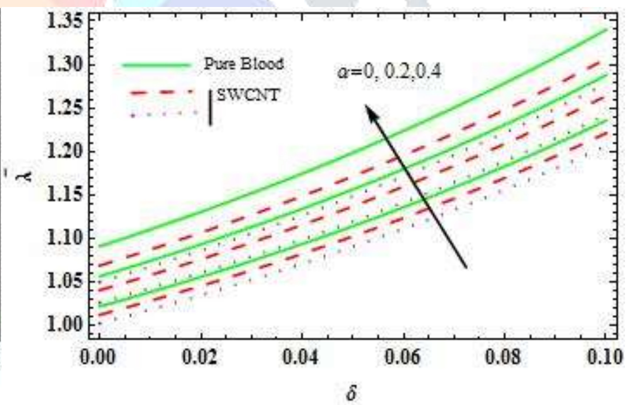
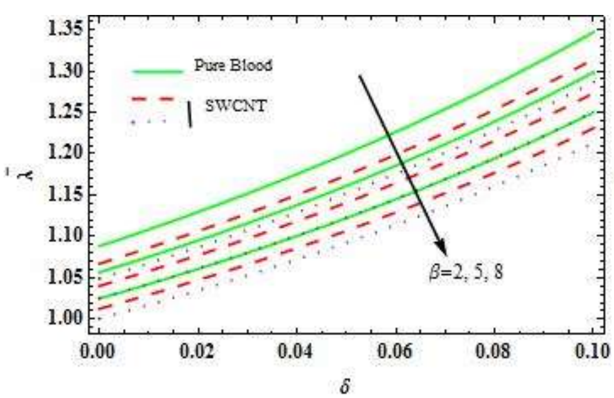


Fig 6: Change in velocity for different values of G_r

Fig 7: Variation in flow resistance for different α



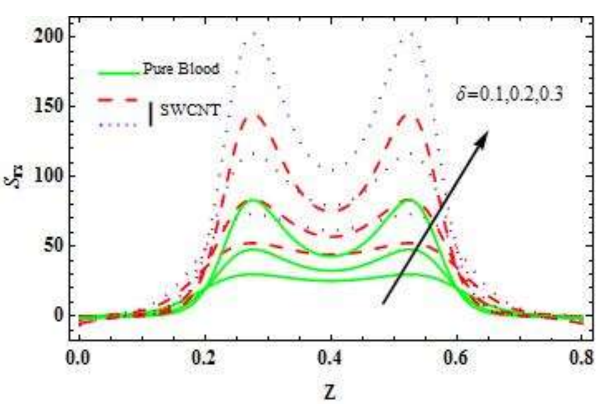


Fig 8: Variation in flow resistance for different β

Fig 9: Change in wall shear stress for different values of δ

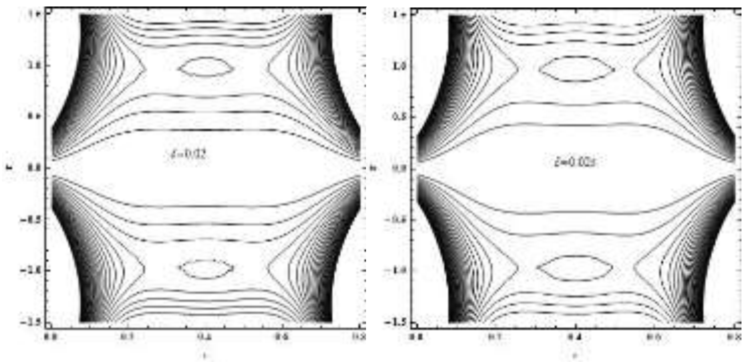


Fig 10: Stream lines for different values of stenosis height δ

Conclusions:

SWCNT characteristics on blood flow through an overlapping stenosed artery have been investigated. Based on the Mathematical calculations the following conclusions are made that:

• Temperature of the fluid increases with the height of stenosis and heat source parameter.

• The velocity increases with height of stenosis, heat source parameter, Grashof number.

• The resistance to flow decreases with heat source parameter and increases with fluid viscosity parameter.

• The trapping bolus phenomenon shows that the bolus increases with the height of the stenosis.

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