TWO UNIT COLD STANDBY SYSTEM WITH OVERHAULING AND INSPECTION

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ABSTRACT

The present paper deal with the analysis of two unit cold standby system with overhauling and inspection. In this system one unit is operative and other is as cold standby. The cold standby unit comes into operation properly at the time of failure of an operative unit if it is good otherwise we send it for overhauling and then it comes into operation properly. After repair, the unit is inspected to decide whether the repair is perfect or not. If the repair is not perfect then it is sent for post repair. Various reliability characteristics of the system model under study are obtained by using regenerative point technique.

Key words : Mean time to system failure, Availability, Markov Renewal Process. Profit analysis.

INTRODUCTION

Many authers [1, 4, 6, 9, 10] Working in the field of reliability have analysed several reliability models in which they assumed that the cold standby unit can not fail and at the time of failure of an operative unit it becomes operative. But in real practical situation we can observe some engineering systems in which the cold standby unit do not able to operate satisfactorily at the time of need or in other words we can say that the cold standby is in the working position but not operating properly due to reason that it is kept as standby position for a long period of time. So for operating it satisfactorily we need its overhauling.

Keeping the above view in mind we in the present study develop an engineering system which consists of two units with configuration that one is operative and other as cold standby. The cold standby unit comes into operations properly at the time of failure of an operative unit if it is good otherwise we send it for overhauling and then it comes into operation properly.

Using regenerative point technique in Markov Renewal Process the following measures of the system effectiveness are obtained.

(i) Steady state transition probabilities
(ii) Mean sojourn time
(iii) Mean time to system failure
(iv) Pointwise and steady state availability of the system
(v) Expected busy period of the repairman in time interval (o, t]
(vi) Expected number of visits by the repairman in (o, t]
(vii) Profit analysis of the system.

MODEL DESCRIPTION AND ASSUMPTIONS

(i) Consider a two unit system with one unit operative and the other as cold standby.

(ii) Upon failure of operative unit, the cold standby unit comes into operation satisfactorily if the standby unit is good at the time of need otherwise it is sent for overhauling.

(iii) After repair, the unit is inspected to decide whether the repair is perfect or not. If the repair of a unit is found to be perfect then the repaired unit becomes operative or cold standby other wise, it goes for post repair. The Probability of having perfect repair is fixed.

(iv) Failure rate of an operative unit is constant, while the distribution of time for overhauling, repair, inspection and post repair are general.

(v) A single repair facility is available for overhauling, repair, inspection and post repair.

(vi) Service discipline is FCFS.

NOTATIONS AND STATES OF THE SYSTEM

α	Constant failure rate of an operative unit.
f(.), F(.)	P.d.f. and c.d.f. of time to complete repair.
g(.), G(.)	P.d.f and c.d.f. of the time to complete overhauling.
h(.), H(.)	P.d.f and c.d.f. of the time to complete inspection.
k(.), K(.)	P.d.f and c.d.f. of the time to complete post repair.
p	Probability that the standby unit in good at the time of need.
q	Probability that the standby unit is not good at the time of need.

m_1, m_2, m_3, m_4 Mean time for repair, overhauling, inspection and post repair.

a Probability that the repair is perfect after inspection.

b. Probability that the repair is imperfect after inspection.

N_O Unit in normal mode and operative.

N_S Unit in normal mode and cold standby.

N_{Oh} Unit in normal mode under overhauling.

F_{wr} Failed unit waiting for repair.

F_r Failed unit under repair

F_R Repair of failed unit is continued from earlier state.

F_I Failed unit under inspection after repair.

F_{IC} Inspection is continued from earlier state.

F_P Failed unit under post repair.

F_{PC} Post repair is continued from earlier state.

Considering the above notations the possible states of the system are:

Up States

S_O : (N_O, N_s) S_1 : (F_r, N_O) S_2 : (F_{wr}, N_{oh})

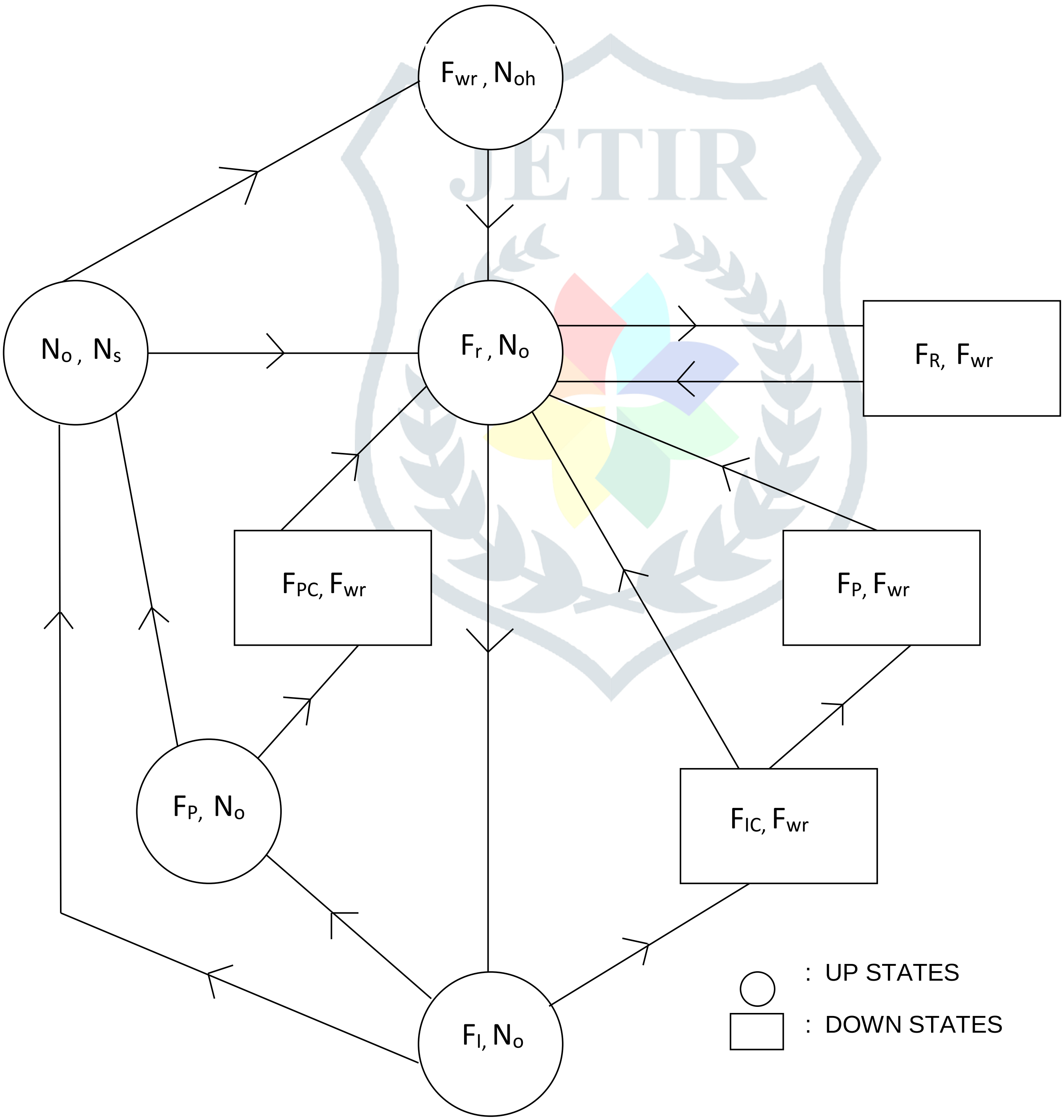
S_3 : (F_I, N_O) S_4 : (F_P, N_O)

Down States

S_5 : $(F_{R,} F_{wr})$, S_6 : $(F_{IC,} F_{wr})$

S_7 : $(F_{PC,} F_{wr})$, S_8 : $(F_{P,} F_{wr})$

The states S_0, S_1, S_2, S_3, S_4 and S_8 are regenerative while the states S_5, S_6, and S_7, are non-regenerative. The possible transitions between these states are shown in following figure.



Transition Probability and Sojourn Times

The non zero elements of the transition probability, P = (P_{ij}) are given as

$$\mathrm{P}_{01} = \mathrm{p}, \ \mathrm{P}_{02} = \mathrm{q}, \ P_{11}^{(5)} = 1 - f^*(\alpha) = P_{15}, \ P_{13} = f^*(\alpha),$$

$$P_{21} = P_{51} = P_{71} = P_{81} = 1, \ P_{30} = ah^*(\alpha), \quad P_{31}^{(6)} = a\,[1 - h^*(\alpha)],$$

$$P_{34} = bh^*(\alpha), \quad P_{38}^{(6)} = b[1 - h^*(\alpha)], \ P_{36} = 1 - h^*(\alpha), \ P_{40} = k^*(\alpha),$$

$$P_{41}^{(7)} = 1 - k^*(\alpha) = P_{47}, \ P_{61} = a, \ P_{68} = b$$

The above probabilities satisfies the following relations:

$$P_{01} + P_{02} = 1, \ P_{13} + P_{15} = 1 = P_{11}^{(5)} + P_{13}$$

$$P_{30} + P_{34} + P_{36} = 1 = P_{30} + P_{31}^{(6)} + P_{34} + P_{38}^{(6)}$$

$$P_{40} + P_{47} = 1 = P_{40} + P_{41}^{(7)}, \ P_{61} + P_{68} = 1$$

Also, the mean rojourn times are

$$\mu_0 = \frac{1}{\alpha}, \quad \mu_1 = \frac{[1 - f^*(\alpha)]}{\alpha}$$

$$\mu_2 = \int t dG(t) = m_2, \quad \mu_3 = \frac{[1 - h^*(\alpha)]}{\alpha}$$

$$\mu_4 = \frac{[1 - k^*(\alpha)]}{\alpha}, \quad \mu_5 = \int t dF(t) = m_1$$

$$\mu_6 = \int t dH(t) = m_3, \quad \mu_7 = \int t dK(t) = m_4 = \mu_8$$

And conditional mean sojourn times are

$$m_{01} = \frac{p}{\alpha} \ , \ m_{02} = \frac{q}{\alpha} \ , \ m_{13} = \int te^{-\alpha t}\, dF(t)$$

$$m_{15} = \alpha \int te^{-\alpha t}\,[1 - F\,(t)]dt \ , \ m_{21} = \int tdG(t)$$

$$m_{30} = a \int te^{-\alpha t}\, dH(t)\, , \quad m_{34} = b \int te^{-\alpha t}\, dH(t)$$

$$m_{36} = \alpha \int te^{-\alpha t}\,[1 - H\,(t)]dt \ , \qquad m_{40} = \int te^{-\alpha t} dK(t)$$

$$m_{47} = \alpha \int te^{-\alpha t}\,[1 - K(t)]dt \ , \qquad m_{11}^{(5)} = \int t(1 - e^{-\alpha t})dF(t)$$

$$m_{31}^{(6)} = a \int t\,(1 - e^{-\alpha t})dH(t) \ , \qquad m_{38}^{(6)} = b \int t(1 - e^{-\alpha t})dH(t)$$

$$m_{41}^{(7)} = \int t\,(1 - e^{-\alpha t})dK(t)$$

It can be easily verified that

$$m_{01} + m_{02} = \mu_0 \ , \ m_{13} + m_{15} = \mu_1$$

$$m_{11}^{(5)} + m_{13} = m_1 \ , \qquad m_{21} = m_2$$

$$m_{30} + m_{34} + m_{36} = \mu_3 \quad , \qquad m_{30} + m_{31}^{(6)} + m_{34} + m_{38}^{(6)} = m_3$$

$$m_{40} + m_{47} = \mu_4 \quad , \qquad m_{40} + m_{41}^{(7)} = m_4$$

MEAN TIME TO SYSTEM FAILURE

To find MTSF, we consider that the failed states Sj (j = 5, 6, 7, 8) are absorbing. By using simple probabilistic arguments , we have

$$\pi_0(t) = Q_{01}(t)\,\$\,\pi_1(t) + Q_{02}(t)\,\$\,\pi_2(t)$$

$$\pi_1(t) = Q_{13}(t)\,\$\,\pi_3(t) + Q_{15}(t)$$

$$\pi_2(t) = Q_{21}(t)\,\$\,\pi_1(t)$$

$$\pi_3(t) = Q_{30}(t)\,\$\,\pi_0(t) + Q_{34}(t)\,\$\,\pi_4(t) + Q_{36}(t)$$

$$\pi_4(t) = Q_{40}(t)\,\$\,\pi_0(t) + Q_{47}(t)$$

(1 – 5)

Taking Laplace-Stieltjes transform of the relation (1 – 5) and solving for $\tilde{\pi}_0(s)$ and omitting the argument 's' for brevity, we have

$$MTSF = E\,[T] = \frac{d\tilde{\pi}_0(s)}{ds}\Big|_{s=o} = \frac{N_1}{D_1} \qquad (6)$$

Where

$$N_1 = \mu_0 + \mu_1 + m_2\,p_{02} + \mu_3 P_{13} + \mu_4\,P_{13}\,P_{34}$$

And

$$D_1 = 1 - (P_{13}\,P_{30} + P_{13}\,P_{34}\,P_{40})$$

AVAILABILITY ANALYSIS

As defined, $M_i(t)$ is the probability that the system starting in up state $S_i \;\epsilon\; E$ is up at time t without passing through any regenrative state. Thus, we have

$$M_0(t) = e^{-\alpha t}, \qquad M_1(t) = e^{-\alpha t}\bar{F}\,(t), \quad M_2\,(t) = \bar{G}(t)$$

$$M_3(t) = e^{-\alpha t}\bar{H}(t) \quad , \quad M_4(t) = e^{-\alpha t}\bar{K}\,(t)$$

Using the arguments of theory of regenerative process, the set of recursive relations of $A_i(t)$ are

$$A_0(t) = \quad M_0(t) + q_{01}(t)© A_1(t) + q_{02}(t)© A_2\,(t)$$

$$A_1(t) = \quad M_1(t) + q_{11}^{(5)}(t) © A_1(t) + q_{13}(t)© A_3\,(t)$$

$$A_2(t) = \quad M_2(t) + q_{21}(t)© A_1(t)$$

$$A_3(t) = \quad M_3(t) + q_{30}(t)© A_0(t) + q_{31}^{(6)}(t)© A_1\,(t) + q_{34}(t)© A_4(t)$$

$$+ q_{38}^{(6)}\,(t)©A_8(t)$$

$$A_4(t) = \quad M_4(t) + q_{40}(t)© A_0(t) + q_{41}^{(7)}(t)© A_1\,(t)$$

$$A_8(t) = \quad q_{81}(t) © A_1(t)$$

(7 – 13)

Solving $A_0^*(s)$ after taking the Laplace tansform of the above relation (7 – 13).

By omitting the argument 's' for brevity, we have

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)} \tag{14}$$

The steady state availability, when the system starts from S_i , is obtained as.

$$A_0 = \lim_{s\to 0} s\, A_0^*(s) = \frac{N_2(0)}{D_2(0)} = \frac{N_2}{D_2} \tag{15}$$

Where

$$N_2 = (\mu_0 + P_{02}\, m_2)(P_{13}\, P_{30} + P_{13}\, P_{34}\, P_{40}) + \mu_1 + P_{13}\,(\mu_3 + P_{34}\,\mu_4)$$

And

$$D_2 = (\mu_0 + P_{02}\, m_2)(P_{13}\, P_{30} + P_{13}\, P_{34}\, P_{40}) + m_1 + P_{13}\,(m_3 + P_{34}\, m_4)$$

BUSY PERIOD ANALYSIS

$B_i(t)$ be the probability that the repaiman is busy at epoch t starting from $S_i \in E$.

Using the elementary probabilistic arguments, we have

$$B_0(t) = q_{01}(t)©B_i(t) + q_{02}(t)©B_2(t)$$

$$B_i(t) = W_1(t) + q_{11}^{(5)}(t)©B_1(t) + q_{13}(t)©B_3(t)$$

$$B_2(t) = W_2(t) + q_{21}(t)©B_1(t)$$

$$B_3(t) = W_3(t) + q_{30}(t)©B_0(t) + q_{31}^{(6)}(t)©B_1(t) + q_{34}(t)©B_4(t) + q_{38}^{(6)}(t)©B_8(t)$$

$$B_4(t) = W_4(t) + q_{40}(t)©B_0(t) + q_{41}^{(7)}(t)©B_1(t)$$

$$B_8(t) = W_8(t) + q_{81}(t)©B_1(t)$$

(16 – 21)

Where

$W_1(t) = e^{-\alpha t}\bar{F}(t)$, $W_2(t) + \bar{G}(t)$, $W_3(t) = e^{-\alpha t}\bar{H}(t)$

$W_4(t) = e^{-\alpha t}\bar{K}(t)$, $W_8(t) = \bar{K}(t)$

Taking the Laplace transform of the relation (16 – 21) and then solving for $B_0^*(s)$. Omithing the orguments 's' for brevity, we get

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)} \tag{72}$$

The steady busy period, when the system starts from S_i, is obtained as

$$B_0 = \lim_{s \to 0} s\, B_0^*(s) = \frac{N_3}{D_2} \tag{23}$$

Where

$N_3 = \mu_1 + P_{02}\, P_{13}(P_{30} + P_{34}\, P_{40})m_2 + P_{13}\,(\mu_3 + P_{34}\,\mu_4 + P_{38}^{(6)}\, m_4)$

And D_2 is same as defiened in availability.

EXPECTED NUMBER OF VISITS BY THE PEPAIRMAN

$V_i(t)$ be the expected number of visits by the repairman in (0 , t], given that the system initially starts from regenerative state S_i. By using the probabilistic arguments, we get the following recursive relations :

$V_0(t) = Q_{01}(t)\$ [1 + V_1(t)] + Q_{02}(t)\$ [1 + V_2(t)]$

$V_1(t) = Q_{11}^{(5)}(t)\$ V_1(t) + Q_{13}(t)\$ V_3(t)$

$V_2(t) = Q_{21}(t) \$ V_1(t)$

$V_3(t) = Q_{30}(t) \$ V_0(t) + Q_{31}^{(6)}(t) \$ V_1(t) + Q_{34}(t) \$ V_4(t) + Q_{38}^{(6)}(t) \$ V_8(t)$

$V_4(t) = Q_{40}(t) \$ V_0(t) + Q_{41}^{(7)}(t) \$ V_1(t)$

$V_8(t) = Q_{81}(t) \$ V_1(t)$

(24 – 29)

Taking Laplace – Stieltjes transform of (26 – 29) and solving for $\tilde{V}_0(s)$ by omitting the argument 's' for brevity, we have

$$\tilde{V}_0(s) = \frac{N_4(S)}{D_2(S)} \tag{30}$$

Under steady state, the number of visits per unit time is given by

$$V_0 = \underset{t \to 0}{Lim} \left[\frac{V_0(t)}{t}\right] = \underset{s \to 0}{Lim}\, s.\, \tilde{V}_0(s) = \frac{N_4}{D_2} \tag{31}$$

Where

$$N_4 = P_{13}(P_{30} + P_{34} P_{40})$$

And $D_2$is same as in availability.

PROFIT ANALYSIS

The Profit obtained to the system model in steady state can be obtained as.

$$P = C_0 A_0 - C_1 B_0 - C_2 V_0 \tag{32}$$

Where

C_0 = Revenue per unit up time of the system.

C_1 = Cost per unit time for which the repairman is busy

C_2 = Cost per visit by the repairman.

CONCLUSION

In the previous study it was assumed that the cold standby unit will work properly at the failure of an operative unit. But in some engineering systems the standby unit does not work properly at the time of need. In the present study, the concept of overhauling the standby unit for satisfactory working is considered to improve the effectiveness of the system. The optimum results are obtained which are shown in equations (6), (15), (23), (31) and (32). The behaviour of the mean time to system failure and frofit can also be studied from the equation (6) and (32) with respect to the system failure rate.

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