LAPLACE-DECOMPOSITION METHOD TO STUDY SOLITARY WAVE SOLUTIONS OF COUPLED NON LINEAR PARTIAL DIFFERENTIAL EQUATION

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ABSTRACT

In this paper, exact and numerical solutions are obtained for coupled Schrödinger–Korteweg–de Vries (Sch–KdV) equation by the well known Laplace decomposition method. The Adomian decomposition method (ADM) is an analytical method to solve linear and nonlinear equations and gives the solution a series form. we combine Laplace transform and ADM and present a new approach for solving coupled Schrödinger–Korteweg–de Vries (Sch–KdV) equation which is an imaginary equation, with initial condition The method does not need linearization, weak nonlinearity assumptions or perturbation theory.We compare the numerical solutions with corresponding analytical solutions.

***Key words*: Coupled Schrödinger–KdV equation; Laplace decomposition method; Adomian decomposition method; solitary wave solution; Numerical solution.**

1. INTRODUCTION:

Systems of partial differential equations have attracted much attention in a variety of applied sciences. The general ideas and the essential features of these systems are of wide applicability. These systems were formally derived to describe wave propagation [1−5], to control the shallow water waves [1−5], and to examine the chemical reaction-diffusion model of Brusselator [4−6]. The method of characteristics, the Riemann invariants, and Adomian method [6] were the commonly used methods.

In this work, we will use Laplace decomposition method introduced by Khuri [7, 8]. Agadjanov [9] solved Duffing equation with the help of this method. This numerical technique basically illustrates how the Laplace transform may be used to approximate the solutions of the nonlinear partial differential equations by manipulating the decomposition method. Elgasery [10], applied Laplace decomposition method for the solution of Falkner-Skan equation. Here Laplace- Adomian decomposition is implemented to systems of partial differential equations [11, 12]. The modification of Laplace decomposition method introduced by Hussain and Majid Khan [13]

2. LAPLACE-DECOMPOSITION METHOD :

In this section, we outline the main steps of our method to solve imaginary equations by using the LDM. We consider the non linear partial differential equations written in an operator form

$$\begin{aligned} L_t u + R_1(u,v) + N_1(u,v) &= f_1 \\ L_t v + R_2(u,v) + N_2(u,v) &= f_2 \end{aligned} \qquad (1)$$

With initial data

$$\begin{aligned} u(x,0) &= g_1(x) \\ v(x,0) &= g_2(x) \end{aligned} \qquad (2)$$

Where L_t is considered a first-order partial differential operator $L_t = \frac{\partial}{\partial t}$, R_1, R_2 and N_1, N_2 are linear and nonlinear operators, respectively, and f_1 and f_2 are source terms. The method consists of first applying the Laplace transform to both sides' of equations in system (1) and then by using initial conditions (2), we have:

$$\begin{cases} L(L_t u) + L(R_1(u,v)) + L(N_1(u,v)) = L(f_1) \\ L(L_t(v)) + L(R_2(u,v)) + L(N_2(u,v)) = L(f_2) \end{cases} \tag{3}$$

Using the differentiation property of Laplace transform, we get

$$\begin{cases} L(u) = \frac{g_1(x)}{p} + \frac{L(f_1)}{p} - \frac{1}{p}\left[L(R_1(u,v)) + L(N_1(u,v))\right] \\ L(v) = \frac{g_2(x)}{p} + \frac{L(f_2)}{p} - \frac{1}{p}\left[L(R_2(u,v)) + L(N_2(u,v))\right] \end{cases} \tag{4}$$

The LADM defines the solutions $u(x, t)$ and $v(x, t)$ by the infinite series

$$u(x,t) = \sum_{n=0}^{\infty} u_n, \qquad v(x,t) = \sum_{n=0}^{\infty} v_n \tag{5}$$

The nonlinear terms N_1 N_2 are usually represented by an infinite series of the so-called Adomian polynomials [14]

$$N_1(x,t) = \sum_{n=0}^{\infty} A_n \quad ; \quad N_2(x,t) = \sum_{n=0}^{\infty} B_n \tag{6}$$

The Adomian polynomials can be generated for all forms of nonlinearity. They are determined by the following relations:

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N_1 \sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0} \qquad B_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N_2 \sum_{i=0}^{\infty} \lambda^i v_i\right)\right]_{\lambda=0} \tag{7}$$

Substituting (5) and (6) into (4), gives

$$\begin{cases} L\left(\sum_{n=0}^{\infty} u_n\right) = \frac{g_1(x)}{p} + \frac{L(f_1)}{p} - \frac{1}{p}\left[L\left(R_1\left[\left(\sum_{n=0}^{\infty} u_n\right), \left(\sum_{n=0}^{\infty} v_n\right)\right]\right) + L\left(\sum_{n=0}^{\infty} A_n\right)\right] \\ L\left(\sum_{n=0}^{\infty} v_n\right) = \frac{g_2(x)}{p} + \frac{L(f_2)}{p} - \frac{1}{p}\left[L\left(R_2\left[\left(\sum_{n=0}^{\infty} u_n\right), \left(\sum_{n=0}^{\infty} v_n\right)\right]\right) + L\left(\sum_{n=0}^{\infty} B_n\right)\right] \end{cases} \tag{8}$$

Applying the linearity of the Laplace transform, we define the following recursively formula

$$\begin{cases} L(u_0) = \frac{g_1(x)}{p} + \frac{L(f_1)}{p} \\ L(v_0) = \frac{g_2(x)}{p} + \frac{L(f_2)}{p} \end{cases} \tag{9}$$

$$\begin{cases} L(u_1) = -\frac{1}{p} L\left[\left(R_1(u_{0,} v_0)\right)\right] - \frac{1}{p} L[A_0] \\ L(v_1) = -\frac{1}{p} L\left[\left(R_2(u_{0,} v_0)\right)\right] - \frac{1}{p} L[B_0] \end{cases} \tag{10}$$

In general, for $k \geq 1$, the recursive relations are given by

$$\begin{cases} L(u_{k+1}) = -\frac{1}{p} L\left[R_1(u_{k,} v_k)\right] - \frac{1}{p} L[A_k] \\ L(v_{k+1}) = -\frac{1}{p} L\left[R_2(u_{k,} v_k)\right] - \frac{1}{p} L[B_k] \end{cases} \tag{11}$$

Applying the inverse Laplace transform, we can evaluate u_k and v_k $(k \geq 0)$. In some cases the exact solution in the closed form may also be obtained.

3. APPLICATION:

At the classical level, a set of coupled nonlinear wave equations describing the interaction between high-frequency Langmuir waves and low-frequency ion-acoustic waves were firstly derived by Zakharov [15]. We can consider the Schrödinger–KdV (Sch–KdV) equation as a model for the interaction of long and short nonlinear waves, which is following.

$$iE_t = E_{xx} + E\eta$$
$$\eta_t = -6E\eta_x - \eta_{xxx} + \left(|E|^2\right)_x \qquad (12)$$

With initial conditions

$$E(x,0) = 2\sqrt{2}k^2\left(1 - 3\tanh^2(kx)\right)e^{i\alpha x}$$
$$\eta(x,0) = \frac{8k^2 - \alpha}{3} - 6k^2\tanh^2(kx)$$

Where α, k, are arbitrary constant.

Calculation of exact and numerical solutions of above equation, in particular, travelling wave solutions, play an important role in wave-wave interaction and soliton theory [1, 16].

Taking Laplace−Adomian decomposition method Eqs.(16) Then, by using the differentiation property of Laplace transform ,initial conditions and Applying the inverse Laplace transform,

$$E(x,t) = E(x,0) - iL^{-1}\left[\frac{1}{p^2}L\left(E_{nxx} + \sum_{n=0}^{\infty} A_n(\eta, E)\right)\right]$$
$$\eta(x,t) = \eta(x,0) + L^{-1}\left[\frac{1}{p^2}L\left(\sum_{n=0}^{\infty} B_n(E) - 6\sum_{n=0}^{\infty} C_n(\eta, E) - \eta_{nxxx}\right)\right]$$

The LADM defines the solutions series $E(x,t)$ $\eta(x,t)$

$$E(x,t) = \sum_{n=0}^{\infty} E_n \ , \ \eta(x,t) = \sum_{n=0}^{\infty} \eta_n$$

Where

$\sum_{n=0}^{\infty} A_n(\eta, E) = \eta E$ $\sum_{n=0}^{\infty} B_n(E) = \left(|E|^2\right)_x$ $\sum_{n=0}^{\infty} C_n(\eta, E) = E\eta_x$ is Adomian polynomials that represent nonlinear terms then we get

$$\sum_{n=0}^{\infty} E_n(x,t) = E(x,o) - iL^{-1}\left[\frac{1}{p}L\left(E_{nxx} + \sum_{n=0}^{\infty} A_n(\eta, E)\right)\right]$$
$$\sum_{n=0}^{\infty} \eta_n(x,t) = \eta(x,o) + L^{-1}\left[\frac{1}{p}L\left(\sum_{n=0}^{\infty} B_n(E) - 6\sum_{n=0}^{\infty} C_n(\eta, E) - \eta_{nxxx}\right)\right]$$

The recursive relation is given below

$$E(x,0) = E_0 = 2\sqrt{2}k^2\left(1 - 3\tanh^2(kx)\right)e^{i\alpha x}$$

$$E_1(x,t) = -iL^{-1}\left[\frac{1}{p}L\left(E_{0xx} + \sum_{n=0}^{\infty} A_0(\eta, E)\right)\right]$$

$$E_{n+1}(x,t) = -iL^{-1}\left[\frac{1}{p}L\left(E_{nxx} + \sum_{n=0}^{\infty} A_n(\eta, E)\right)\right]$$

And

$$\eta(x,o)=\eta_0=\frac{8k^2-\alpha}{3}-6k^2\tanh^2(kx) \qquad \eta_1(x,t)=L^{-1}\left[\frac{1}{p}L\left(\sum_{n=0}^{\infty}B_0(E)-6\sum_{n=0}^{\infty}C_0(\eta,E)-\eta_{0xxx}\right)\right]$$

$$\eta_{n+1}(x,t)=L^{-1}\left[\frac{1}{p}L\left(\sum_{n=0}^{\infty}B_n(E)-6\sum_{n=0}^{\infty}C_n(\eta,E)-\eta_{nxxx}\right)\right] \qquad n\geq 1$$

By this recursive relation we can find other components of the solution $E_1(x,t)=-iL^{-1}\left[\frac{1}{p}L(E_{0xx}+\eta_0E_0)\right]$

$$E_1(x,t)=\frac{2\sqrt{2}k^2t}{3}\left[(3\alpha^2+\alpha+10k^2)(1-3\tanh(kx))\cos(\alpha x)+36k\alpha\,\mathrm{sec}h^2(kx)\tanh(kx)\cos(\alpha x)\right]$$
$$+i\frac{2\sqrt{2}k^2t}{3}\left[(3\alpha^2+\alpha+10k^2)(1-3\tanh(kx))\sin(\alpha x)-36k\alpha\,\mathrm{sec}h^2(kx)\tanh(kx)\cos(\alpha x)\right]$$

$$\eta_1(x,t)=L^{-1}\left[\frac{1}{p}L\left(\left(|E_0|^2\right)_x-6E_o\eta_{ox}-\eta_{0xxx}\right)\right]$$

$$\eta_1(x,t)=-48k^5\mathrm{t\,sech}^2(\mathrm{kx})(4-9\tanh^2(\mathrm{kx}))+144\sqrt{2}\,\mathrm{k}^5\mathrm{t\,sech}^2(\mathrm{kx})\tanh(\mathrm{kx})(1-3\tanh^2(\mathrm{kx}))\cos(\alpha x)$$
$$+144\mathrm{i}\sqrt{2}\,\mathrm{k}^5\mathrm{t}\ \mathrm{sech}^2(\mathrm{kx})\tanh(\mathrm{kx})(1-3\tanh^2(\mathrm{kx}))\sin(\alpha x)$$

$$E_2(x,t)=-iL^{-1}\left[\frac{1}{p}L(E_{1xx}+\eta_1E_0+\eta_0E_1)\right]$$

$$E_2(x,t)=-\frac{\sqrt{2}k^2t^2}{3}\Big[6k^2(15\alpha^2+\alpha+10k^2)\mathrm{sec}h^2(kx)(1-3\tanh^2(kx))$$
$$+(3\alpha^2+\alpha+10k^2)(1-3\tanh^2(kx))\left(\frac{3\alpha^2+\alpha-8k^2}{3}-6k^2 3\tanh^2(kx)\right)\Big]e^{\alpha ix}$$
$$+48\sqrt{2}k^7t^2i\,\mathrm{sec}h^2(kx)\tanh(kx)(4-9\tanh^2(kx))(1-3\tanh^2(kx))e^{\alpha ix}$$
$$-288k^7t^2i\,\mathrm{sec}h^2(kx)\tanh(kx)(1-3\tanh^2(kx))^2e^{2\alpha ix}$$
$$-\frac{\sqrt{2}k^2t^2}{3}i\left[216k^3\alpha\,\mathrm{sec}h^2(kx)\tanh^3(kx)-24\alpha k(3\alpha^2+\alpha+13k^2)\mathrm{sec}h^2(kx)\tanh(kx)\right]e^{\alpha ix}$$

$$\eta_2(x,t)=\eta_1(x,t)-6tk^4\,\mathrm{sec}h^5(kx)\{k4t\,\mathrm{sec}h3(kx(1208-1191\cosh(2kx)+120\cosh(4kx)$$
$$-\cosh(6kx))+24e^{2i\alpha x}(i\alpha\cosh(kx)+2k\sinh(kx))\}$$

The other components of the decomposition series can be determined in a similar way, we can obtain the expression of E(x, t) which is in a Taylor series, and then the closed form solutions yield as follows

:

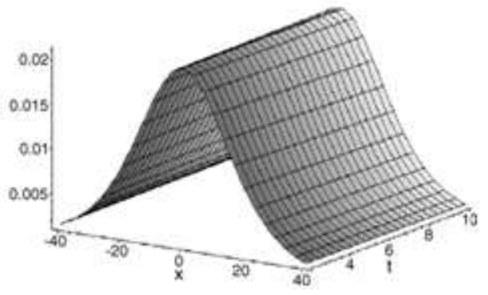
$$E(x,t)=2\sqrt{2}k^2(1-3\tanh^2(k(x+2\alpha t)))\exp\left(i\left[\alpha x+\frac{1}{3}(3\alpha^2+\alpha-10k)t\right]\right)$$

$$\eta(x,t)=\frac{8k^2-\alpha}{3}-6k^2\tanh^2(k(x+2\alpha t))$$

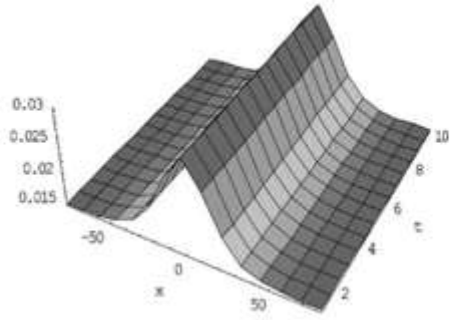
4. NUMERICAL DESCRIPTION OF THE SOLUTION:

The Laplace Decomposition Method is used for finding the exact and approximate traveling-waves solutions of the Sch–KdV equation. Both the exact and approximate solutions obtained for n = 2 by using LDM are plotted in Fig. 1. It is evident that when compute more terms for the decomposition series the numerical results are getting much more closer to the corresponding analytical solutions.

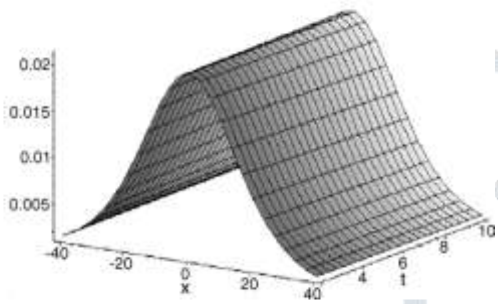
The behaviour of the two solutions obtained by Laplace Decomposition Method with the exact solutions for different values of time are plotted in Figure1



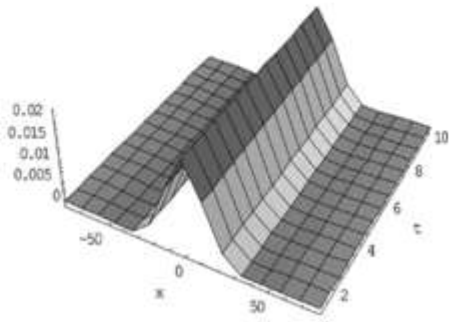
(a)



(b)



(c)



(d)

Fig.1. The plots of results for solution of Sch-KdV equations with a fixed values of $\alpha = 0.05$, $k = 0.05$ and for different values of time (a) Analytical solutions for $E(x,t)$ (b) Numerical results for $E_2(x,t)$ by means of LDM (c) Analytical solutions for $\eta(x,t)$ (d) Numerical results for $\eta_2(x,t)$ by means of LDM

5. CONCLUSIONS:

The Laplace decomposition method is a powerful method which has provided an efficient potential for the solution of physical applications modeled by nonlinear differential equations. The algorithm can be used without any need to complex calculations except for simple and elementary operations.

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