From Cones to Computations: A Journey Through Conic Sections with wxMaxima

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Abstract : *This abstract discusses the integration of wxMaxima, an open-source software, for analyzing conic sections—curves formed from the intersection of a plane with a double-napped cone. It highlights the significance of conic sections in various fields, including engineering and astronomy, while providing definitions and mathematical descriptions of the four main conic types: circles, ellipses, parabolas, and hyperbolas. The abstract also details various commands within wxMaxima that enable users to graphically represent these conics and perform related calculations, thereby illustrating their application in calculus and other mathematical contexts.*

Index Terms **–conic section, parabola, hyperbola, ellipse, kill(all), print.**

I. INTRODUCTION

Maxima is a powerful open-source software that combines analytical, graphical, and numerical features, We make use of wxMaxima, which is incredibly user-friendly. There are several uses for conic sections in engineering, architecture, science, and astronomy. For example, satellites and planets frequently have elliptical orbits, and telescopes and satellite dishes employ parabolic mirrors to focus signals or light. The focus of this page is on conic sections, which are utilized in calculus both manually and with the open-source program Maxima. The maximum program command and many kinds of graphing conics by hand are explained in the article. Conic sections are curves created when a plane and a double-napped cone intersect. The angle at which the plane intersects the cone determines the curve's characteristics. The four main forms of conic sections are hyperbolas, parabolas, ellipses, and circles.

II. OBJECTVES

The main objective is to find the commands for finding the conic sections using wxMaxima.

III. METHODOLOGY

1. Definitions

1.1 Formed when the plane cuts the cone perpendicular to its axis. All points on a circle are equidistant from a fixed point called the centre.

1.2 Created when the plane intersects the cone at an angle, but does not pass through the base. An ellipse is the set of all points where the sum of the distances to two fixed points (foci) is constant.

1.3 Parabola: Resulting when the plane is parallel to a generator of the cone. A parabola is the set of all points equidistant from a fixed point called as focus and a fixed line called as directrix.

1.4 Hyperbola: Formed when the plane intersects both nappes of the cone at an angle. A hyperbola consists of two separate curves, each being the set of all points where the absolute difference of the distances to two fixed points (foci) is constant.

These conic sections have been studied for several years and have provided a rich source of interesting and beautiful results in Euclidean geometry. The general equation of conic is $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$. The type of conic section represented by this equation depends on discriminant B^2-4AC.

2. Different Menus for Graphic User Inference

About Interface:File, Edit, View, Cell, Maxima, Equations, Algebra, Calculus, Simplify, List, Plot, Numeric, Help. Commands for performing calculations, defining and evaluating functions, and generating plots are written in the input line of the wxMaxima interface.

To assign a value to a variable, utilize a colon (:). When defining a function, employ a colon followed by an equal sign (: =). Maxima expressions conclude with a semi-colon (;). If the semi-colon is omitted in the input line, wxMaxima will automatically add it. In addition to the semi-colon, there is a character used to suppress output, which is the dollar sign ($). By using the dollar sign ($) at the end of a Maxima statement, the output of that command is suppressed. Nonetheless, the command is executed directly in memory. The individual statements cannot be separated by a colon (:) or a dollar sign ($), but rather by a comma (,). To launch wxMaxima, simply type any expression on the blank screen and press Shift+Enter. To display the results of each computation, conclude the input line with a semicolon (;) and then press Shift+Enter. Arithmetic Operations: + for Addition, - for Subtraction, * for Multiplication, / for Division, and ^ for Power. Each time you input an opening parenthesis in the command line, a closing parenthesis will be automatically appended. The percentage operator (%) signifies the most recent output. A polar curve is represented by the equation, radius = R(θ). The function string (x) returns the string representation of x, which can be a number, function, or equation. The command string (%) provides the string representation of the output from the previous line. The polar ( ) function is used in wxdraw2d and follows the format polar (R (θ), θ, l1, l2), where l1 and l2 represent the lower and upper limits for the angle θ, typically set to 0 and 2pi. The command proportional_axes=xy is utilized to ensure equal scaling on both the x-axis and y-axis. The nticks = k parameter, applicable for k greater than 100, can be employed in wxdraw2d to enhance the smoothness of a curve's appearance. If this parameter is not specified, a small default value is assumed, resulting in a curve that appears less smooth. The dimensions are defined as [n, n].The ratsimp(f(x)) command cancels out any shared factors in the numerator and denominator of a rational function f(x). The num(f(x)) command extracts the numerator from a rational function f(x). The denom(f(x)) command retrieves the denominator of a rational function f(x). The divide(f(x), g(x)) command performs the division of f(x) by g(x), returning both the quotient and the remainder. The wxdraw2d() command creates a graph within the same window. It can be utilized to plot a curve where y is explicitly defined as a function of x, to illustrate a vertical asymptote (as a parametric curve), and to indicate critical points. Using wxdraw2d with the implicit option allows for the drawing of a curve defined by f(x, y) = constant. To begin, assign the equation to a variable, labeled as a, and then use implicit (a, x, lower_limit_for_x, upper_limit_for_x, y, lower_limit_for_y, upper_limit_for_y) in wxdraw2d. These can all be incorporated in a single wxdraw2d command to create multiple level curves within the same graph. For creating 3D surfaces, the wxdraw3d() command can be utilized, but it results in a static figure in the same window. By employing plot3d(), a figure can be generated in a separate graphic window. You can manipulate the view from different angles by holding down the left mouse button and moving. It is important to note that implicit options are unavailable for 3D surfaces; z must be explicitly defined as a function of x and y using the solve() command.

3.Various kinds Conic Section and Related Maxima Commands for Conic Section Illustration

3.1 Cricle**:**As an example consider $x^2 + y^2 = 4$ which is a circle with centre (0,0) and radius 2.Commands for sketching this conic is as follows.

kill(all)$a:1$b:0$c:1$d:0$e:0$f:-9$ eq:a*x^2+b*x*y+c*y^2+d*x+e*y+f=0$

wxdraw2d(grid=true,color=green,xaxis =true,yaxis=true, proportional_axes=xy,

implicit(eq ,x,-10,10,y,-10 ,10))$

3.2 Parabola: For example, $y^2 = 12x$ is a parabola with focus (3,0), vertex (0,0) and directrix $x = -3$. The following commands are used to sketch this conic.

kill(all)$ g:-3;a:0$b:0$c:1$d:-12$e:0$f:0$ eq:a*x^2+b*x*y+c*y^2+d*x+e*y+f=0$wxdraw2d(grid=true,color=red,implicit(eq,x,-10,10,y,-10,10),color=blue,parametric(g,t,t, 10,10),proportional_axes=xy)$. Also,$x^2 = 12y$ is a parabola with focus (0,3),vertex (0,0) and directrix $y = -3$. The commands corresponding to these are as follows.

kill(all)$g:-3;a:1$b:0$c:0$d:0$e:12$f:0$ eq:a*x^2+b*x*y+c*y^2+d*x+e*y+f=0$

wxdraw2d(grid =true,color =red,implicit(eq ,x,-10,10,y,-10 ,10),color=blue, parametric(t, g, t, -10,10),proportional_axes=xy)$

3.3 Hyperbola: For instance the hyperbola $x^2 - y^2 = 1$ has major axis along x-axis with foci $(\pm\sqrt{2}, 0)$, vertices on the x-axis are $(\pm 1,0)$ and asymptotes are $x= \pm y$.The following are the commands for this conic.

```
kill(all)$z1:x;z2:-x;a:1$b:0$c:-1$d:0$e:0$f:-1$  eq:a*x^2+b*x*y+c*y^2+d*x+e*y+f=0$
wxdraw2d(grid =true,color =red, implicit(eq ,x,-10,10,y,-10 ,10),color =green, explicit ((z1), x, -10,10) ,color =green, explicit ((z2), x, -10,10))$
```

3.4 Ellipse**:** Consider the ellipse $3x^2 + 2y^2 = 6$**.** Standard form this ellipse is $\frac{x^2}{2} + \frac{y^2}{3} = 1$. Major axis along y-axis and minor axis along x-axis, foci are $(0, \pm 1)$,vertices on the major axis are $(0, \pm\sqrt{3})$.Vertices on the minor axis are $(\pm\sqrt{2}\,, 0)$. The following are the commands that go with these.

```
kill(all)$ a:3$b:0$c:2$d:0$e:0$f:-6$  eq:a*x^2+b*x*y+c*y^2+d*x+e*y+f=0$
wxdraw2d(grid =true,color =red,implicit(eq ,x,-5,5,y,-5 ,5),proportional_axes=xy)$
```

4.Recognizing the conic.

The equation $Ax^2 + Bxy+Cy^2+Dx+Ey+F=0$ is known as the generic quadratic equation, in which A, B, C, D, E, and F are constants.The discriminant of the equation is the value B^2-4AC. If $B^2-4AC = 0$, then the quadratic curve is a parabola,if $B^2- 4AC < 0$, an ellipse,a hyperbola if $B^2 - 4AC > 0$. As an example, if $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ is the quadratic equation, then $B^2 - 4AC = 0$**.** Therefore, quadratic curve is parabola.

IV. CONCLUSION

This article promotes critical and creative thinking while providing readers with the fundamental concepts needed for research in conic section using wxMaxima. Also, helps to evaluate different conic section using discriminant test.

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