THE UNION ∪ AND INTERSECTION ∩ OF TWO FUZZY

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Abstract : A powerful tool of economic analysis is utility theory. However the indispensable fuzzy aspects of a human mind in the area of utility and performance remains to exploratory. The primary purpose of this paper is to investigate certain properties of a fuzzy utility function. Unlike the traditional approach utility indicators will be expressed by a fuzzy set in lie *a* of real numbers. This paper aim to study the union ∪ and intersection ∩ of two fuzzy sets.

Keywords : fuzzy commodity bundles, commodity space.

INTRODUCTION

A powerful tool of economic analysis is utility theory. However the indispensable fuzzy aspects of a human mind in the area of utility and performance remains to exploratory. The primary purpose of this paper is to investigate certain properties of a fuzzy utility function. Unlike the traditional approach utility indicators will be expressed by a fuzzy set in lie *a* of real numbers. This paper aim to study the union ∪ and intersection ∩ of two fuzzy sets.

Fuzzy sets : to begin with, let x be a universe of discourse, whose generic elements set A in X is characterized by a membership function/$\mu_A(x)$ which is a associated with each point in X, a real number in the interval [0, 1] with the value of $\mu_A(x)$ at x representing the grade of membership of x in A. the closer the value of $\mu_A(x)$ to unity, the high the grade of membership of x in A. When A is an ordinary set, its membership can take on only two values of 0 and 1 with $\mu_A(x) = 1$ or 0 according as x does not belong to A. $\mu_A(x)$ is referred to as characteristic function of the set A.

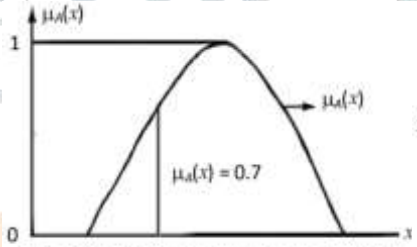

Fig. 1 : A one dimensional fuzzy set and its membership function.

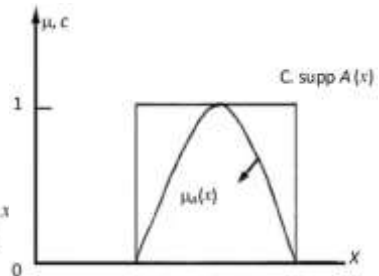

Fig. 3 : (support of a fuzzy set)

A fuzzy set A is completely characterized by the set of pairs

$A = \{x, \mu_A(x); x \in X\}$

A more convenient notation is as follows : when X is a finite set,

Two fuzzy set A and B are equal iff

$\mu_A(x) = \mu_B(x)$

$(x_1, \ldots, x_n)$, a fuzzy set of X is expressed as

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \ldots + \frac{\mu_A(x_n)}{x_n} = \sum_{i=1}^{n} \frac{\mu_A(x_i)}{x_i}$$

Where '+' and '∑' represent the symbol only. The support of a fuzzy set A is an ordinary subset of X :

Supp $A = \{x \in X, \mu_A(x) > 0\}$

A is said to be normalized if $\exists\, x \in X, \mu_A(x) = 1$

A fuzzy set is empty iff its membership function is identically zero on X.

Two fuzzy sets A and B are equal iff

$\mu_A(x) = \mu_B(x)\ \forall\ x$ in X.

A is contained in B iff

$\mu_A(x) \le \mu_B(x)$

Symbolically $A \subset B \leftrightarrow \mu_A \le \mu_B$.

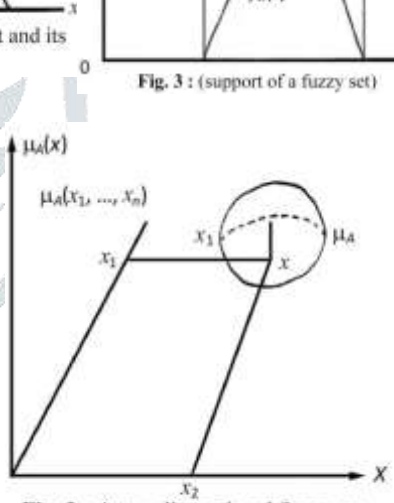

Fig. 2 : A two dimensional fuzzy set and its membership function

The union ⋃ and intersection ⋂ of two fuzzy sets : Let A and B are two fuzzy sets.

Then the union ⋃ and intersection ⋂ of two fuzzy sets A and B are defined as

$$\forall x \in X, \mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x)$$

$$\forall x \in X, \mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x)$$

where $\mu_{A \cup B}$ and $\mu_{A \cap B}$ are the membership functions of $A \cup B$ and $A \cap B$, respectively. The union and intersection are two fuzzy sets are in Fig. 4.

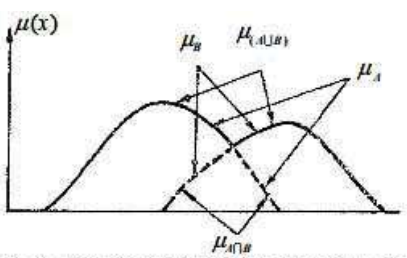

Fig. 4. : The union and intersection of two fuzzy sets

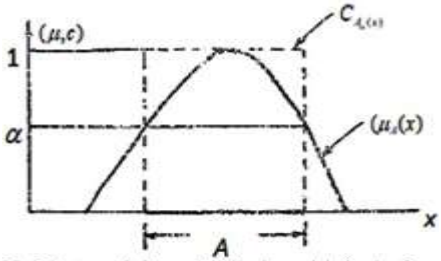

Fig. 5 : An α-cut of a fuzzy set A and its characteristic function $C_{A(x)}$

***α*-cut :** Let A be a Fuzzy set A then α-cut of a fuzzy set A is an ordinary set of such elements with membership grade greater than or equal to a threshold α, $0 \le \alpha \le 1$.

$$A_\alpha = \{x \in X, \mu_A(x) \ge \alpha\}$$

Fuzzy relation : Let X_1, ..., X_n be n universes. An n-ary fuzzy relation R in $X_1 \times \ldots \times$

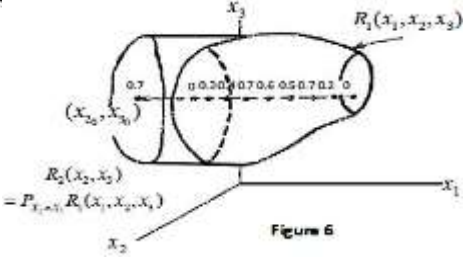

Figure 6

X_n is a fuzzy set on $X_1 \times \ldots \times X_n$.

Projection : The projection of a fuzzy relation R on $X_{i_1} \times ... \times X_{i_k}$, where $(i_1, ..., i_k)$ is a subsequence of $(1, 2, ..., n)$, is a relation of $X_{i_1} \times ... \times X_{i_k}$ defined by

$$P_{X_{i_1} \times ... \times X_{i_k}} R = X_{i_1} \times \overset{\sum}{\ldots} \times X_{i_k} \; X_{j_1} \overset{\sup}{\ldots} X_{j_l} \; \mu_R(x_1, ..., x_n)/(x_{i_1} \ldots x_{i_k})$$

where $j_1, ..., j_P$ is the subsequence complementary to $(i_1, \ldots, i_k)$ in $(1, ..., n)$.

Fuzzy relation and its projection are in the Fig. 6.

Cylindrical extension : Expansion of a one-dimensional fuzzy set into a three-dimensional fuzzy set through cylindrical extension are in Fig. 7.

If A is a fuzzy set in $X_{i_1} \times ... \times X_{i_k}$, then its cylindrical extension in $X_1 \times \ldots \times X_n$ is a fuzzy set A in $X_1 \times ... \times X_n$ defined by

$$\bar{A} = \sum_{X_1 \times \ldots \times X_n} \mu_A(x_{i_1}, ..., x_{i_k})/(x_1, ..., x_n), k < n.$$

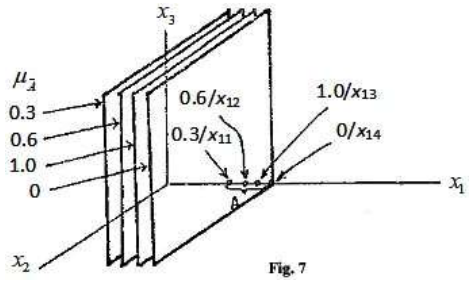

Fig. 7

Figures 7 and 8 give two examples of cylindrical extensions. In Fig. 7 a 1-dimensional fuzzy set

$$A = 0.3/x_{11} + 0.6/x_{12} + 1.0/x_{13} + 0/x_{14}$$

becomes a 3-dimensional fuzzy set $\bar{A}$ via a cylindrical extension operation. We note that $\bar{A}$ is a fuzzy set in $X_1 \times X_2 \times X_3$, $\mu_{\bar{A}}(x_1, x_2, x_3)$ is independent of x_2 and x_3. In Fig. 8, a 2-dimensional fuzzy set A in $X_1 \times X_2$ is extended into a 3-dimensional fuzzy set $\bar{A}$ in $X_1 \times X_2 \times X_3$, with its $\mu_{\bar{A}}(x_1, x_2, x_3)$ independent of X_3.

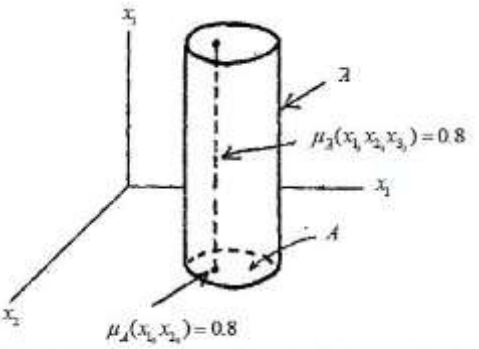

Fig. 8 : Expansion of a two-dimensional fuzzy set into three dimensional fuzzy set through cylindrical extension.

Composition of fuzzy relation : Let R be a fuzzy relation in $X \times Y$ and S be a fuzzy relation in $Y \times Z$, the composition of R and S denoted RoS is defined as

$$RoS = P_{X \times Z}(\bar{R} \cap \bar{S})$$
$$\mu_{RoS}(x, z) = V[\mu_R(x, y) \vee \mu_S(y, z)]$$
$$\max_y [\min(\mu_R(x, y) \vee \mu_S(y, z)]$$

Where $\bar{R}$ and $\bar{S}$ are cylindrical extension in $X \times Y \times Z$. Fig. 9 demonstrates this operation with a simplified example.

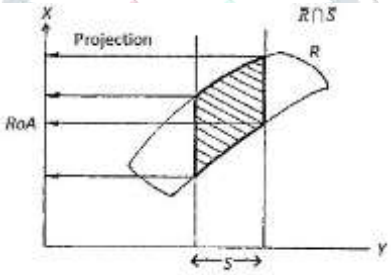

Fig. 9 : Composition of fuzzy relations R and S

Convex fuzzy set : A fuzzy set A is convex iff all of its α-cut sets are convex. An equivalent definition of convexity is : A is convex iff

$$\forall x_1 \in X, \forall x_2 \in X, \forall \lambda \in [0,1],$$
$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

We note that this definition does not necessarily imply that μ_A is a convex function of x. Convex fuzzy set and its convex α-cut are in the fig. 10(a) and non concave fuzzy set and its non convex α-cut.

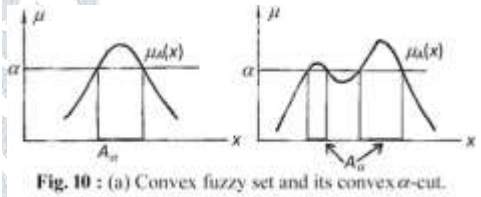

Fig. 10 : (a) Convex fuzzy set and its convex α-cut. (b) Non-convex fuzzy set and its non-convex α-cut.

Fuzzy number : A fuzzy number is a convex normalized fuzzy set A defined on the real line E such that

(a) $\exists x \in E \; \mu_A(x_0) = 1$

(b) μ_A is piecewise continuous.

Fuzzy variable : A fuzzy variable is a variable which takes on fuzzy numbers as its values.

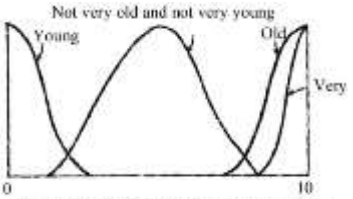

Fig. 11 : Age described by fuzzy numbers

If E is restricted to a real interval, one can express adjectives in natural language as fuzzy sets in this interval. For example, assume the universe of discourse is [0, 100], one can express old, young, very old, not very old and not very young, etc., as fuzzy numbers in [0, 100] interval as depicted in Fig. 11.

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