

Heat transfer in a Visco-elastic fluid flow in presence of internal heat generation / absorption

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Abstract-This Paper considers the study of convective heat transfer in a viscoelastic fluid flow in a porous media over a stretching sheet in presence of internal heat generation / absorption and entropy generation. Similarity transformation is used to transform the boundary layer equations of momentum and heat transfer, which are partial differential equations into ordinary differential equations. The proposed problem has been solved numerically using Runge- Kutta fourth order method with shooting technique. The effects of Grashof number, Prandtl number, heat source/sink parameter and entropy generation number on heat transfer have been investigated and represented with the aid of graphs.

Keywords: Entropy Analysis, Stretching Surface, Prandtl Number, Eckert Number, Grashof Number.

1. INTRODUCTION

Boundary layer flow through porous media has many engineering and geophysical applications such as geothermal reservoirs, drying porous solids, thermal insulation enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors and underground energy transport.

Vajravelu [6] independently extended the above work to the case of suction and blowing. Vajravelu and Hadjinicolaou [7] studied convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. Ali and Al-Yousef [9] have investigated laminar mixed convection boundary layers induced by a linearly stretching permeable surface. Sravanan and Kandaswamy [11] investigated hydromagnetic stability of convective flow of variable viscosity fluids generated by internal heat sources. Merkin and Mahmood [5] on the study of convective flow through porous media reveals that the work is not carried out for visco-elastic fluid of the type Walters' liquid B model. Jagadish V Tawade [13] has studied the convective heat transfer in a visco-elastic fluid flow over a stretching sheet through a porous media with internal heat generation / absorption and entropy generation. Abel et al [14] discussed about the Heat transfer in a Walter's liquid B fluid over an impermeable stretching sheet with non-uniform heat source/ sink and elastic deformation. Khan et al [15] evaluated about Unsteady Free Convection Flow in a Walters'-B Fluid and Heat Transfer Analysis.

The objective of this paper is to consider steady convective heat mass transfer in a porous medium over a stretching surface with internal heat generation or absorption in the flow of visco-elastic fluid. In contrast to the work of Abel and Veena [1998], the present work considers the effect of free convection current with temperature dependent internal heat generation or absorption for two general cases namely;

- (i) Prescribed Surface Temperature (PST case)
- (ii) Prescribed Heat Flux (PHF case)

The presence of buoyancy force makes the momentum and heat transfer equations to coupled and highly non-linear form of partial differential equations. These equations are reduced to ordinary differential equations by using similarity transformations. To deal with the coupling and non-linearity, a numerical shooting technique with Runge-Kutta Fourth order integration scheme has been used.

2. MATHEMATICAL FORMULATION

Consider the laminar free convective flow of an incompressible visco-elastic fluid (Walters' liquid B [1]) through porous media past a stretching surface coinciding with the plane $y = 0$. Two equal and opposite forces are applied along the x-axis so that the surface is stretched with the origin fixed. This flow satisfies the rheological equation of state derived by Beard and Walters [1].

The basic boundary layer equations for the steady flow and heat transfer in usual notations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} - k_0 \left\{ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right\} - \frac{\gamma}{k'} u + g\beta(T - T_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p} (T - T_\infty) \quad (3)$$

Here, k_0 is the visco-elastic parameter, k' , g , β , Q denote the permeability parameter of porous medium, gravitational acceleration, volumetric coefficient of thermal expansion, volumetric rate of heat generation respectively and the other symbols have their usual meanings. The last term in the momentum equation takes into account the buoyancy force.

In deriving these equations it is tacitly assumed that in addition to the usual boundary layer approximations, the contribution due to the normal stress is of the same order of magnitude as that of due to shear stresses. Hence both γ and k are of $O(\delta^2)$, δ being the boundary layer thickness. Note that $k > 0$.

The thermal boundary conditions depend on the type of heating process being considered. We consider two different heating processes namely,

- (i) Prescribed surface temperature (PST)
- (ii) Prescribed heat flux (PHF).

The prescribed surface temperature and prescribed heat flux are considered to be a function of x only (Vajravelu [6]). The appropriate boundary conditions for the flow with heat transfer are

$$u = bx, v = 0, \left\{ \begin{array}{l} T = T_w = T_\infty + A\left(\frac{x}{l}\right) \quad (PST) \\ -KT_y = q_w = D\left(\frac{x}{l}\right) \quad (PHF) \end{array} \right\} \quad \text{at } y = 0 \tag{4}$$

$$u \rightarrow 0, \quad u_y \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty$$

Where l is the characteristic length, b the stretching rate and A, D are constants. T_y represents the derivative of T with respect to y. T the temperature in the boundary layer, q_w is the heat flux at the wall, T_w the wall temperature and T_∞ the temperature at infinity.

2.1. NON-DIMENSIONALISATION

In order to reduce the number of independent variables and dimensionless forms of equations (1)-(4), we define the new variables

$$u = bx f_\eta(\eta), \quad v = -\sqrt{b\gamma} f(\eta), \quad \text{where } \eta = \sqrt{\frac{b}{\gamma}} y \tag{5}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \text{where } \left\{ \begin{array}{l} T_w - T_\infty = A\left(\frac{x}{l}\right) \quad (PST \text{ Case}) \\ T_w - T_\infty = D\left(\frac{x}{l}\right) \quad (PHF \text{ Case}) \end{array} \right\} \tag{5}$$

with these transformations equation of continuity (1) is identically satisfied and equations (2) and (3) reduce to the following forms

$$f_\eta^2(\eta) - f(\eta) f_{\eta\eta}(\eta) = f_{\eta\eta\eta}(\eta) - k_1 \{ 2f_\eta(\eta) f_{\eta\eta\eta}(\eta) - f(\eta) f_{\eta\eta\eta\eta}(\eta) - f_{\eta\eta}^2(\eta) \} - k_2 f_\eta(\eta) + Gr \theta \tag{6}$$

$$\theta_{\eta\eta}(\eta) + Pr f \theta_\eta(\eta) + Pr(-f_\eta(\eta) + \alpha)\theta = 0 \tag{7}$$

Where $k_1 = \frac{k_0 b}{\gamma}$ - Visco-elastic parameter, $Gr = \frac{g\beta(T_w - T_\infty)}{b^2 x}$ - Free convection parameter, $k_2 = \frac{\gamma}{k b}$ - Permeability parameter, $Pr = \frac{\mu C_p}{k}$ -

Prandtl number, $\alpha = \frac{Q}{b\rho C_p}$ - Heat source / sink parameter.

The corresponding boundary conditions are

(i) PST CASE:

$$\begin{array}{llll} f_\eta(\eta) = 1, & \theta(\eta) = 1, & f(\eta) = 0 & \text{at } \eta = 0 \\ f_\eta(\eta) \rightarrow 0, & f_{\eta\eta}(\eta) \rightarrow 0, & \theta(\eta) \rightarrow 0 & \text{as } \eta \rightarrow \infty \end{array} \tag{8}$$

(ii) PHF CASE:

$$\begin{array}{llll} f_\eta(\eta) = 1, & \theta_\eta(\eta) = -1, & f(\eta) = 0 & \text{at } \eta = 0 \\ f_\eta(\eta) \rightarrow 0, & f_{\eta\eta}(\eta) \rightarrow 0, & \theta(\eta) \rightarrow 0 & \text{as } \eta \rightarrow \infty \end{array} \tag{9}$$

2.2. PHYSICAL QUANTITIES

Our interest lies in investigation of the flow behaviour and heat transfer characteristics by analyzing the non-dimensional local shear stress (τ_w) and Nusselt number (Nu). These non-dimensional parameters are defined as

$$\tau_w = \frac{\tau^*}{\mu b x \sqrt{b/\gamma}} = f_{\eta\eta}(0), \quad \text{where } \tau^* = -\mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \tag{10}$$

$$Nu = \frac{-h}{T_w - T_\infty} T_y = \theta_\eta(0) \tag{11}$$

3. NUMERICAL SOLUTION

Momentum and the thermal boundary equations being non-linear and coupled, therefore exact solutions do not seem feasible for complete set of equations (6), (7), (8) & (9) so solution must be obtained numerically. In order to solve them, we employ most efficient shooting technique with fourth order Runge-Kutta integration scheme, which is described in Abel et al [10].

Selection of an appropriate finite value of η_∞ is most important aspect in this method. To select η_∞ , we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $f_{\eta\eta}(0)$ and $\theta_k(0)$ ($\theta_k = \theta_\eta$ in PST case and $\theta_k = \theta$ in PHF case). The solution process is repeated with another larger (or smaller, as the case may be) value of η_∞ . The values of $f_{\eta\eta}(0)$ and $\theta_k(0)$ compared to their respective previous values, if they agreed to about six significant digits, the last value of η_∞ used was considered

the appropriate value for that particular set of parameters; otherwise the procedure was repeated until further changes in η_∞ did not lead to any more change in the values of $f_{\eta\eta}(0)$ and $\theta_k(0)$. The initial step size employed was $h = 0.01$. The convergence criterion was largely depends on fairly good guesses of the initial conditions in the shooting technique (T. C. Chaim [3]).

Equations (6) and (7) are solved numerically by the method of superposition. Here the coupled boundary value problem of fourth order in f and second order in θ respectively, has been reduced to a system of six simultaneous coupled ordinary differential equations as follows

$$f(\eta) = f_1(\eta);$$

$$\theta(\eta) = \theta_1(\eta)$$

$$\frac{df_1(\eta)}{d\eta} = f_2(\eta);$$

$$\frac{df_2(\eta)}{d\eta} = f_3(\eta);$$

$$\frac{df_3(\eta)}{d\eta} = f_4(\eta) \tag{12}$$

$$\frac{df_4(\eta)}{d\eta} = \frac{1}{k_1 f(\eta)} \left[f_2^2(\eta) - f_1(\eta)f_3(\eta) - f_4(\eta) + 2k_1 f_2(\eta)f_4(\eta) - k_1 f_3^2(\eta) + k_2 f_2(\eta) - Gr\theta_1(\eta) \right]$$

$$\frac{d\theta_1(\eta)}{d\eta} = \theta_2(\eta)$$

$$\frac{d\theta_2(\eta)}{d\eta} = -Pr f_1(\eta)\theta_2(\eta) + Pr(f_2(\eta) - \alpha)\theta_1(\eta)$$

In order to solve above system we need to have six initial conditions, whilst, we have only two initial conditions on f , one initial condition on θ . The third initial condition on f is obtained by applying the initial conditions of (8) and (9) in equation (6). This is obtained as,

$$f_4(0) = f_{\eta\eta\eta}(0) = \frac{1 + k_2 - Gr - k_1 f_3^2(0)}{1 - 2k_1} \quad \text{-PST Case} \tag{13}$$

$$f_4(0) = f_{\eta\eta\eta}(0) = \frac{1 + k_2 - Gr\theta_1(0) - k_1 f_3^2(0)}{1 - 2k_1} \quad \text{-PHF Case} \tag{14}$$

Since $f_3(0)$ and $\theta_k(0)$ ($k=1$ for PHF case and $k=2$ for PST case) are not prescribed, we start with initial approximation as $f_3(0) = \alpha_0$ and $\theta_k(0) = \beta_0$. Let α and β be the correct initial values of $f_3(0)$ and $\theta_k(0)$. Now integrate the system (12) using fourth order Runge-Kutta method and denote the values of f_3 and θ_k at $\eta = \eta_\infty$ by $f_3(\alpha_0, \beta_0, \eta_\infty)$ and $\theta_k(\alpha_0, \beta_0, \eta_\infty)$ respectively. As f_3 and θ_k at $\eta = \eta_\infty$ are clearly functions of α and β , they are expanded into a Taylor series retaining only the linear terms. They are as follows,

$$f_3(\alpha, \beta, \eta_\infty) - f_3(\alpha_0, \beta_0, \eta_\infty) = (\alpha - \alpha_0) \frac{\partial f_3}{\partial \alpha}(\alpha_0, \beta_0, \eta_\infty) + (\beta - \beta_0) \frac{\partial f_3}{\partial \beta}(\alpha_0, \beta_0, \eta_\infty) \tag{15}$$

$$\theta_k(\alpha, \beta, \eta_\infty) - \theta_k(\alpha_0, \beta_0, \eta_\infty) = (\alpha - \alpha_0) \frac{\partial \theta_k}{\partial \alpha}(\alpha_0, \beta_0, \eta_\infty) + (\beta - \beta_0) \frac{\partial \theta_k}{\partial \beta}(\alpha_0, \beta_0, \eta_\infty) \tag{16}$$

To solve the above equations for $\delta\alpha_0 = (\alpha - \alpha_0)$ and $\delta\beta_0 = (\beta - \beta_0)$ the difference quotients for the derivative as

$$\frac{\partial f_3}{\partial \alpha}(\alpha_0, \beta_0, \eta_\infty) \approx \frac{f_3(\alpha_0 + \Delta\alpha_0, \beta_0, \eta_\infty) - f_3(\alpha_0, \beta_0, \eta_\infty)}{\Delta\alpha_0} \tag{17}$$

$$\frac{\partial \theta_k}{\partial \beta}(\alpha_0, \beta_0, \eta_\infty) \approx \frac{\theta_k(\alpha_0, \beta_0 + \Delta\beta_0, \eta_\infty) - \theta_k(\alpha_0, \beta_0, \eta_\infty)}{\Delta\beta_0} \tag{18}$$

Where $\Delta\alpha_0$ and $\Delta\beta_0$ are arbitrarily chosen small increments of α_0 and β_0 respectively. Using fourth order Runge-Kutta integration scheme, we solve the above equation (12) making the use of α_0 and β_0 and other four known initial conditions. Hence all the values of f_3 and θ_k are computed at $\eta = \eta_\infty$. Now solving the system (15)-(16) we obtain new estimates $\alpha_1 = \alpha_0 + \delta\alpha_0$ and $\beta_1 = \beta_0 + \delta\beta_0$. The entire process is repeated starting with $f_1(0), f_2(0), f_3(0) = \alpha_1, f_4(0), \theta_{1+k}(0), \theta_{2-k}(0) = \beta_1$ ($k=0$ for PST case and $k=1$ for PHF case) as initial conditions. Iteration of the whole outlined procedure is continued with the latest estimates of α and β until L.H.S. of (15) & (16) become infinitesimally small. Finally we obtain,

$$\begin{aligned}\alpha_n &= \alpha_{n-1} + \Delta\alpha_{n-1} \\ \beta_n &= \beta_{n-1} + \Delta\beta_{n-1}\end{aligned}\quad (19)$$

as the desired most approximate initial values of $f_3(0)$ and $\theta_k(0)$ ($k=2$ for PST case and $k=1$ for PHF case). With this now all the six initial conditions become known and so we solve the system of (12) by fourth order Runge-Kutta integration scheme and get profiles of $f_1, f_2, f_3, f_4, \theta_1, \theta_2$ for a particular set of physical parameters. The method developed as above is analogous to the modified Newton's method of finding roots of equations in several variables.

4. RESULTS AND DISCUSSION

In order to test the accuracy of our method we have compared our results with those of Crane [2], McLeod and Rajagopal [4], Abel and Veena [8] in absence of buoyancy force with internal heat generation or absorption. The results are found to be in good agreement. Numerical results are shown graphically in Figures (2)–(10). Non-dimensional horizontal velocity profiles are depicted in figures (2)–(5). Results for prescribed surface temperature (PST) are drawn in figures (6)–(7) and for prescribed power law heat flux (PHF) are drawn in figures (8)–(9). Results for different values of Prandtl number for both PST and PHF cases are shown graphically in figure (10). Table.1. represents the numerical values of coefficient of Skin friction and Nusselt number for different physical parameters.

Figure 2 (a) & 2 (b) gives the graphical representation of horizontal velocity profile $f_\eta(\eta)$ for visco-elastic parameter $k_1=0.1$ and $k_1=0.2$ respectively. It is noticed that velocity decreases in the boundary layer with the increase of distance from the boundary. Comparison of these two graphs reveals that the effect of visco-elastic parameter is to decrease the horizontal velocity profile. This is because of the fact that the introduction of tensile stress due to visco-elasticity causes transverse contraction of the boundary layer. The effects of variation of porosity parameter and the free convection parameter Gr are also shown in these figures. We notice that the velocity profile $f_\eta(\eta)$ increases as the free convection parameter increases. Physically $Gr > 0$ means the heating of the fluid or cooling of the surface, $Gr < 0$ means the cooling of the fluid or heating of the surface and $Gr=0$ corresponds to the absence of free convection current. It is observed that the effect of porous parameter k_2 is to decrease the horizontal velocity profile. This is due to the increase of porous parameter k_2 leading to the enhanced deceleration of the flow. This behaviour is even true in the absence of heat source / sink parameter (α) shown in the figure (3). Figures (4) and (5) provide the information about horizontal velocity profile $f_\eta(\eta)$ for $k_1=0.1$ and $k_1=0.2$ when the surface heat flux is prescribed. Effects of all physical parameters are noticed to be similar, with reduced magnitude as that in PST case.

In figures (5) & (6), temperature profile is plotted for the same magnitude of values of the physical parameters as those considered in figures (2)–(5). It is noticed from these figures that the temperature distribution is unchanged (unit value) at the wall with the change of physical parameters in PST case. However, it changes with the change of physical parameters when the wall is maintained with prescribed heat flux. The non-dimensional temperature distribution asymptotically tends to zero in the free stream in all cases. This result is similar to the result obtained by Mohamed Ali and Fahd Al-Yousef [9]. Figure (6) is plotted for the effects of Grashof number and porosity parameter on temperature distribution. It is observed that the increase of Grashof number leads to decrease in the temperature profile and this behaviour is even true in the presence of porous medium. This is consistent with the fact that increase of porous parameter introduces some additional stress, which is responsible for thickening of the boundary layer. Comparative study of figure 6 (a) with the figure 6 (b) reveals that the increase of visco-elastic parameter k_1 leads to increase the temperature profile $\theta(\eta)$. Physically, this means thickening of thermal boundary layer due to increase of non-Newtonian visco-elastic normal stress. Figure 8 (a) and 8 (b) gives temperature profile $\theta(\eta)$ versus η for $k_1=0.1$ and $k_1=0.2$ respectively when the wall has been prescribed with surface heat flux. The effects of all the physical parameters on the temperature profile $\theta(\eta)$ are similar to those of as seen in PST case.

The presence of heat source ($\alpha > 0$) or heat sink ($\alpha < 0$) in the boundary layer on temperature profile $\theta(\eta)$ is presented in figures (7) & (9) for PST and PHF cases respectively. The presence of heat source ($\alpha > 0$) in the boundary layer generates the energy, which causes the temperature of the fluid to increase. On the other hand, the presence of heat absorption ($\alpha < 0$) effects has the tendency to reduce the fluid temperature, which results in decreasing the fluid velocity. This behavior is even true in the presence of Grashof number. From these figures it is clearly observed that both the hydrodynamic (velocity) and thermal (temperature) boundary layers decrease as the heat absorption effects increase. These results were compared with those results of Chamkha et al [12], which are found to be in good agreement. Figure 10 (a) and 10 (b) shows graphical representation of temperature profile $\theta(\eta)$ in PST and PHF cases respectively for different values of Prandtl number and heat source/ sink parameter. It is noticed that the effect of Prandtl number is to decrease the temperature profile in both PST and PHF cases. Physically it means that the thermal boundary layer thickness decreases with increase of the values of Prandtl number. This behaviour is even true in the presence of heat source ($\alpha > 0$).

The values of coefficient of Skin friction $f_{\eta\eta}(0)$, Nusselt number Nu (dimensionless rate of heat transfer) for various values of Gr, α , k_1 , k_2 , are recorded in Table.1 for both PST and PHF cases. Analysis of the table shows that magnitude of surface velocity gradient is found to increase with the non-Newtonian parameter k_2 in the absence of buoyancy effect. The effect of free convection parameter is to increase the coefficient of skin friction. From the table we also observe that the effect of heat absorption parameter ($\alpha < 0$) is to decrease the Nusselt number in both cases and is even true in presence of porous medium.

5. CONCLUSION

The governing equations for a steady, laminar free convective flow of an incompressible visco-elastic fluid over a continuously moving stretching surface embedded in a porous medium in presence of heat generation or absorption was formulated. The resulting partial differential equations were transformed into ordinary differential equations by using similarity transformations. Numerical results of the transformed boundary layer equation have been obtained. In this paper, we have investigated the influences of permeable parameter, porosity parameter, Grashof number, Prandtl number, heat source / sink parameter on the dimensionless velocity, temperature distribution, Skin friction coefficient and the rate of heat transfer coefficient on horizontal velocity profile as well as temperature distribution. The important results of our investigation are

- Horizontal velocity profile decreases with the increase of distance from the boundary
- The effect of visco-elastic parameter is to decrease the horizontal velocity profile in the boundary layer in presence / absence of porous medium.
- The increase of porosity parameter leads to the enhanced deceleration of the flow and hence the velocity decreases.
- The effect of increase in Grashof number leads to increase in the horizontal velocity profile but decrease the temperature in the boundary layer.
- The effect of increase in Grashof number leads to increase in the horizontal velocity profile but decrease the temperature in the boundary layer.
- In the presence of heat source ($\alpha > 0$) in the boundary layer, generates the energy, which causes the temperature of the fluid to increase. On the other hand in the presence of heat absorption ($\alpha < 0$) effects caused reductions in the fluid temperature, which resulted in decrease in the fluid velocity.
- The effect of Prandtl number is to decrease the thermal boundary layer thickness in presence / absence of porosity parameter
- Skin friction coefficient increased due to increase in the Grashof number.
- Nusselt number decreased as the heat source/sink parameter increased.

Table 1: Values of skin friction parameter and heat transfer co-efficient for various values of k_1, k_2, α and gr when $pr=1.0$.

k_1	k_2	α	Gr	Skin Friction Parameter $f_{\eta}(0)$		Heat Transfer Co-efficient	
				PST	PHF	PST $\theta_{\eta}(0)$	PHF $\theta(0)$
0.1	0.0	0.0	0.5	-0.7924	-0.8134	-1.0514	0.9541
			0.0	-1.0651	-1.0646	-0.9877	1.0032
			-0.5	-1.4212	-1.3492	-0.5837	1.0415
	0.5		-1.2812	-1.2775	-0.9299	1.0492	
	0.0		-1.5285	-1.5285	-0.6749	1.1331	
	-0.5		-1.8085	-1.9440	-0.4877	1.4246	
	1.0	-0.5	0.5	-0.7929	-0.8785	-1.2872	0.7823
			0.0	-1.0648	-1.0646	-1.2552	0.7966
			-0.5	-1.3339	-1.2670	-1.1940	0.8144
	0.5		-1.3135	-1.3511	-1.2189	0.8233	
	0.0		-1.5285	-1.5285	-1.1906	0.8397	
	-0.5		-1.7560	-1.7204	-1.1497	0.8599	
0.2	0.0	0.0	0.5	-0.8001	-0.8345	-1.0338	0.9635
			0.0	-1.1167	-1.1133	-0.9564	1.0239
			-0.5	-1.4801	-1.5200	-0.5143	1.1848
	0.5		-1.3310	-1.3212	-0.9117	1.0664	
	0.0		-1.5812	-1.5812	-0.6329	1.1656	
	-0.5		-1.8926	-1.9508	-0.3445	1.3960	
	1.0	-0.5	0.5	-0.8242	-0.9032	-1.2776	0.7866
			0.0	-1.1126	-1.1126	-1.2435	0.8041
			-0.5	-1.3830	-1.3351	-1.2004	0.8269
	0.5		-1.3587	-1.3964	-1.2073	0.8313	
	0.0		-1.5812	-1.5812	-1.1761	0.8500	
	-0.5		-1.8070	-1.7749	-1.1317	0.8713	
	0.0	0.1	0.5	-0.7752	-0.8122	-0.9587	1.0142
			0.0	-1.1167	-1.1126	-0.9085	1.1005
			-0.2	-1.2413	-1.2758	-0.5596	1.2139
	0.5		-1.3114	-1.2955	-0.7896	1.1440	
	0.0		-1.5812	-1.5812	-0.7533	1.3197	
	-0.2		-1.6923	-1.7361	-0.3637	1.4414	

Figures:

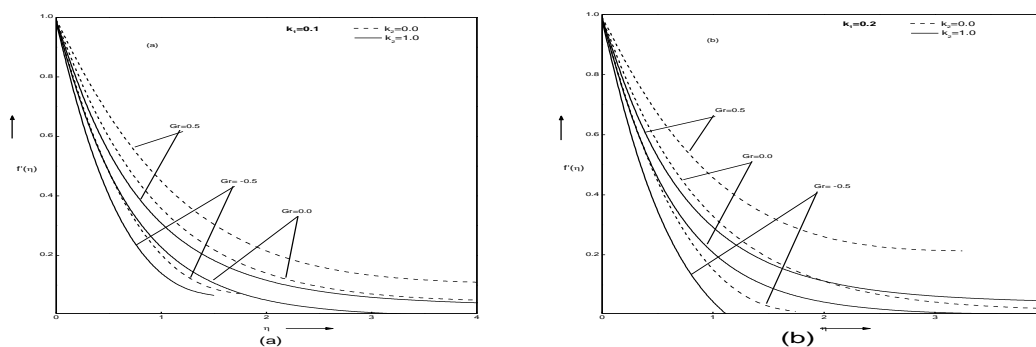


Fig.2: Variation of $f_{\eta}(\eta)$ vs. η for different values of Grashof number Gr and Porosity parameter k_2 , when $Pr=1.0, \alpha=0.0$ in PST case.

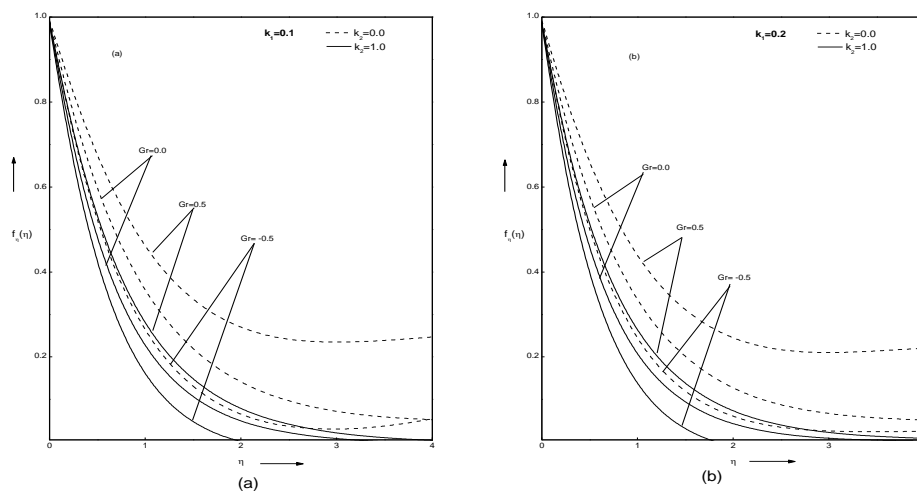


Fig.3 Variation of $f_{\eta}(\eta)$ vs. η for different values of Grashof number Gr and porosity parameter k_2 , when $Pr=1.0$, $\alpha=-0.5$ in PST case.

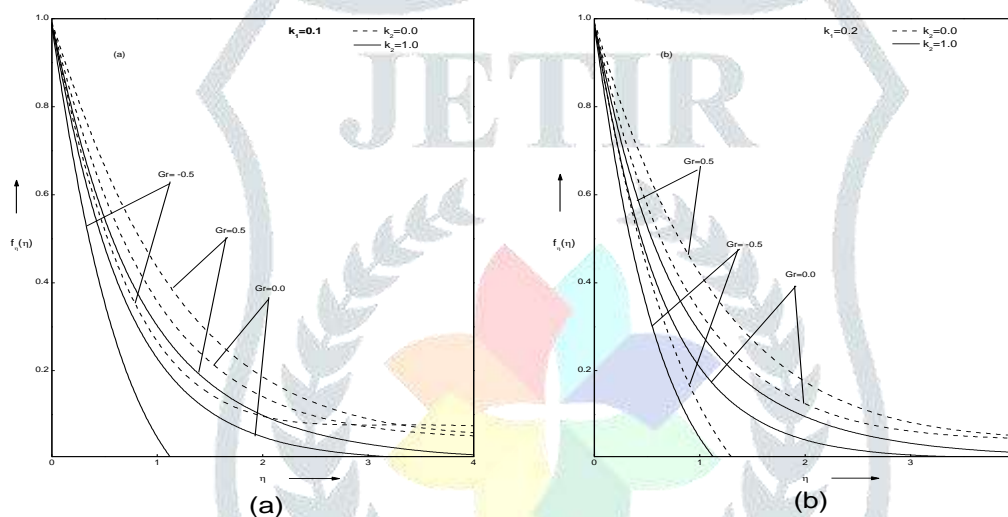


Fig.4 Variation of $f_{\eta}(\eta)$ vs. η for different values of Grashof number Gr and Porosity parameter k_2 , when $Pr=1.0$, $\alpha=0.0$ in PHF case.

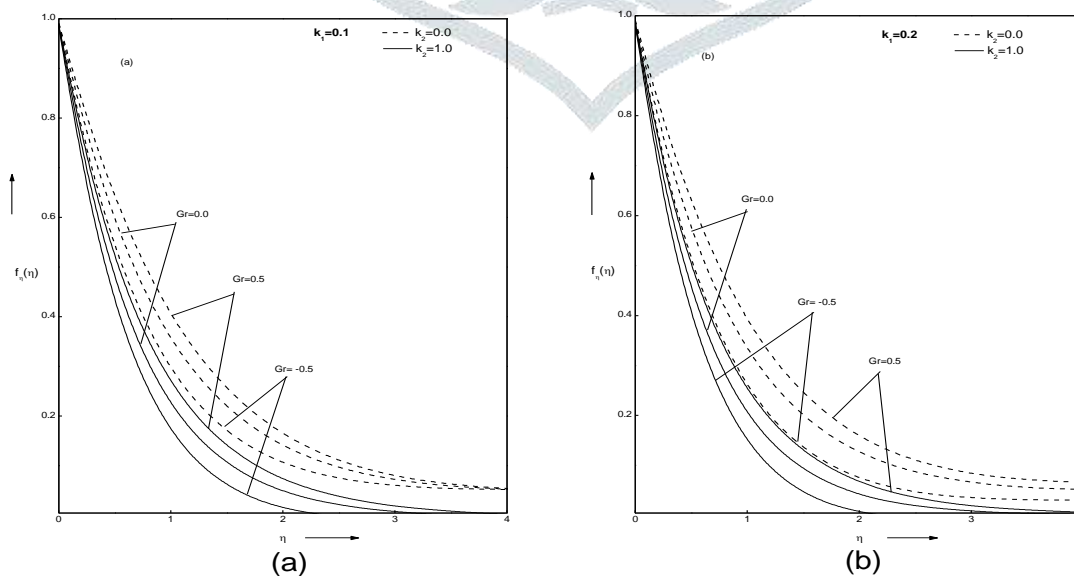


Fig.5 Variation of $f_{\eta}(\eta)$ vs. η for different values of Grashof number Gr and porosity number k_2 , when $Pr=1.0$, $\alpha=-0.5$ in PHF Case.

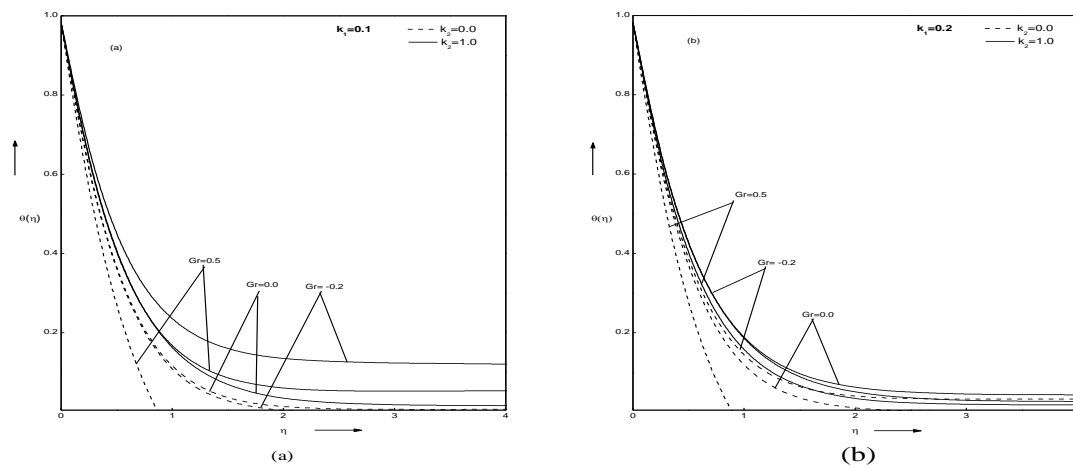


Fig.6 Variation of $\theta(\eta)$ vs. η for different values of Grashof number Gr and porosity parameter k_2 , when $Pr=3.0$, $\alpha=0.0$ in PST case.

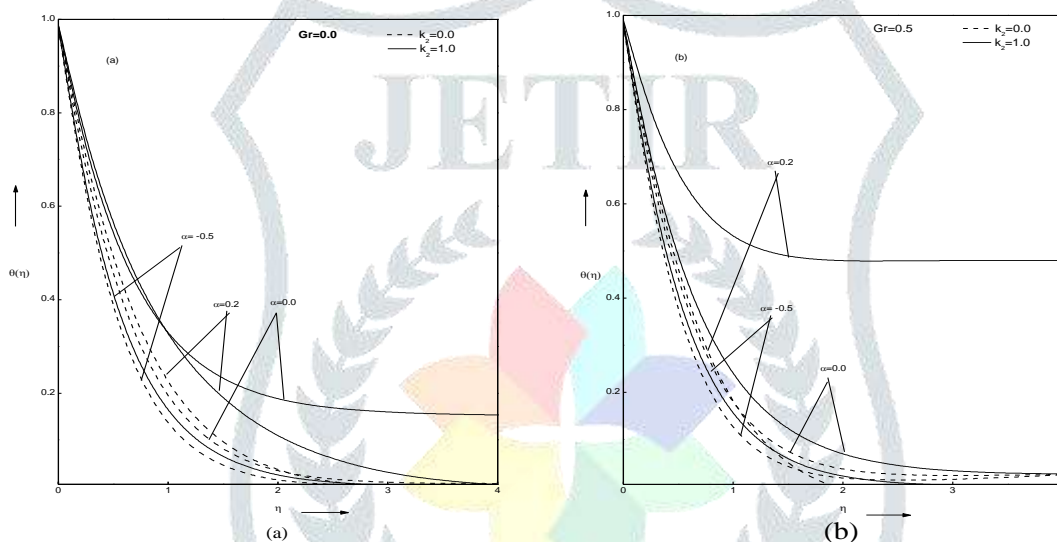


Fig.7 Variation of $\theta(\eta)$ vs. η for different values of heat source / sink parameter α and grashof number Gr when $k_1=0.1$, $Pr=2.0$ in PST case.

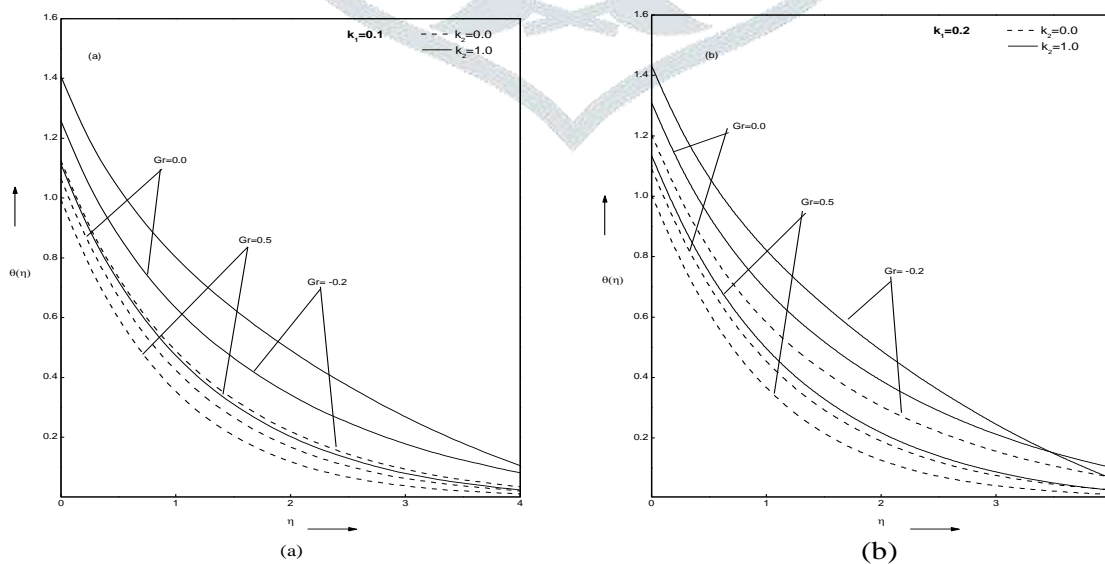


Fig.8 Variation of $\theta(\eta)$ vs. η for different values of Grashof number Gr and porosity parameter k_2 when $Pr=1.0$, $\alpha=0.1$ in PHF Case.

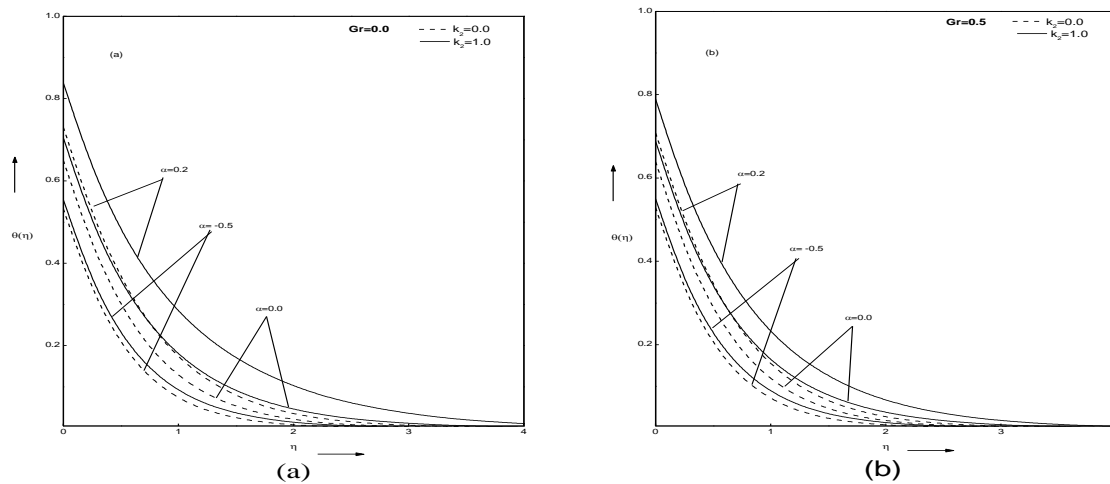


Fig.9 Variation of $\theta(\eta)$ vs. η for different values of heat source / sink parameter α and Grashof number Gr when $Pr=2.0, k_1=0.1$ in PHF case

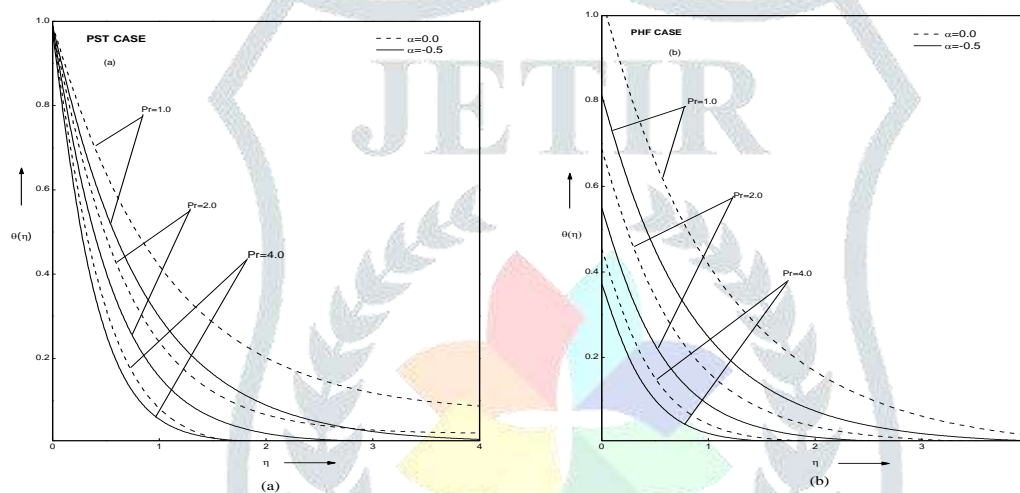


Fig.10 Variation of $\theta(\eta)$ vs. η for different values of Prandtl number (Pr) and heat source / sink parameter α , when $k_1=0.1, k_2=1.0, Gr=0.5$.

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