Joule Heating and Melting Heat Transfer Effects on Electrically Conducting Dissipative Magnetic Nanofluid

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Abstract— Joule heating and melting heat transfer over a stretching sheet with non-uniform heat source/sink of a Casson fluid is analysed. Numerical solutions of highly non-linear momentum equations after subjecting to the similarity transformation are computed. The fluid flow and fluid temperature is examined for Magnetic field, melting parameter, Casson Parameter and few other parameters. The Variation of skin fraction and Nusselt numbers is also examined. The results obtained are found to be justified.

Index Terms— Heat transfer, Nanofluid, Jjoule ,Fluid temparature.

I. Introduction

Casson fluid is a type of non-Newtonian fluid which exhibits yield stress. It is well known that Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear, i.e., if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move. The examples of Casson fluid are of the type are as follows: jelly, tomato sauce, honey, soup, concentrated fruit juices, etc. Human blood can also be treated as Casson fluid. Due to the presence of several substances like, protein, fibrinogen, and globulin in aqueous base plasma, human red blood cells can form a chainlike structure, known as aggregates or rouleaux. If the rouleaux behave like a plastic solid, then there exists a yield stress that can be identified with the constant yield stress in Casson's fluid (Fung [02]). Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear (Dash et al. [03]). Nadeem et al. [04] proposed a work on MHD Casson fluid flow in two lateral directions past a porous linear stretching sheet and results are presented and discussed with respect to variation in Casson flow parameter as well as other fluid flow parameter. Pramanik [05] analysed the boundary layer flow of Casson fluid accompanied by heat transfer towards an exponentially stretching surface in presence of thermal radiation.

This Piece of this investigation is arranged to analyse the melting effect of a Casson fluid on horizontally stretching sheet with non-uniform source/sink. The governing system of equations are comprehensively solved by using Runge-Kutta fourth order scheme. Melting heat transfer and Joule heating extended from the work of Hayat et al. [1]. The aim of this work is to provide an alternate numerical route for solving the problem of heat transfer with melting and Joule effect in presence non-uniform heat source/sink of a dissipative magnetic field. The non-linear coupled differential equations are subjected to the similarity solutions. The results are presented for the various dimensionless parameters on the velocity and temperature fields. And found that result admit the general behaviour of the fluid and well justified with the results of previously published works. To the best of our Knowledge, the present work has not been examined before.

II. Constricting the Problem

Consider an incompressible flow of a Casson nano fluid over a stretching surface located at $x=0$ and *y*-axis is taken normal to it and the flow is confined to $y \geq 0$. It is assumed that the velocity of the stretching sheet is $u_w(x) = ax$, where a is a positive constant. We have chosen $T_\infty > T_m$ where $T_m (= T_\infty - bx^2)$ the non-uniform temperature of the melting surface is and T_∞ is the ambient temperature. Also a uniform magnetic field of intensity B_0 acts in the *y*-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison with the applied magnetic field. We incorporate the Joule heating effects in the energy equations which govern such type of flow are written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{nf} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \mu_{nf} \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \sigma_{nf} B_o^2 u, \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho C_p)_{nf}} \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_{nf} B_0^2}{(\rho C_p)_{nf}} u^2 + q''' \tag{3}$$

The subjected boundary conditions are,

$$u = u_w = ax, \quad v = 0, \quad T = T_m \ at \quad y = 0 \tag{4}$$
$$u \to 0, \ T \to T_\infty \ as \quad y \to \infty$$
and
$$k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} = \rho_{nf}[l + c_s(T_m - T_0)]v(x,0) \tag{5}$$

Where u and v are the velocity components along the *x*- and *y*- directions respectively, σ_{nf} is the electrical conductivity of the nanofluid, $l(=l_0x^2)$ is the non-uniform latent heat of the fluid and c_s is the heat capacity of the solid surface. The boundary condition (5) shows that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required to raise the solid temperature $T_0(=T_m - cx^2)$ to its melting temperature T_m.

The space and temperature dependent heat generation/absorption (non-uniform heat source/sink) is given by,

$$q''' = \frac{k_f u_w(x)}{x\upsilon_f}\left[A^*(T_\infty - T_m)f' + B^*(T - T_m)\right] \tag{6}$$

The effective dynamic viscosity of nanofluid μ_{nf} given by

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{7}$$

Where ϕ is the nanoparticle volume fraction. The effective nanofluid density ρ_{nf}, heat capacity $(\rho C_p)_{nf}$ and thermal diffusivity α_{nf} are taken as follows:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{8}$$
$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \tag{9}$$
$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \tag{10}$$

The effective thermal conductivity and electrical conductivity of nanofluid k_{nf} and σ_{nf} written as,

$$k_{nf}^* = \left\{\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}\right\}, \qquad \sigma_{nf}^* = 1 + \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi}, \tag{11}$$

where, $k_{nf}^* = \frac{k_{nf}}{k_f}$, $\sigma_{nf}^* = \frac{\sigma_{nf}}{\sigma_f}, \sigma = \frac{\sigma_s}{\sigma_f}$

We now introduce the following similarity transformations,

$$u = axf'(\eta), \quad v = -(\upsilon_f a)^{0.5} f(\eta), \ \eta = (\frac{a}{\upsilon_f})^{0.5} y, \ \theta(\eta) = \frac{T - T_m}{T_\infty - T_m} \tag{12}$$

Substituting eqn. (11) into eqns. (1) - (3) yields the following non-dimensional ordinary differential equations.

$$A_1(1+\frac{1}{\beta})f''' - MA_1(1-\phi)^{2.5}\frac{\sigma_{nf}}{\sigma_f}f' - f'^2 + ff'' = 0 \tag{13}$$

$$\frac{1}{\Pr}\frac{k_{nf}}{k_f}(1-\phi)^{2.5}\theta'' + \frac{1}{\Pr}(1-\phi)^{2.5}(A^*f' + B^*\theta) + Ec(1+\frac{1}{\beta})f''^2 + EcM(1-\phi)^{2.5}\frac{\sigma_{nf}}{\sigma_f}f'^2$$
$$+\frac{1}{A_2}(f\theta' - 2\theta f' + 2f') = 0 \tag{14}$$

where prime indicates the differentiation with respect to η, M is the Hartman number, Pr is the Prandtl number, Ec is the Eckert number. Which are given by,

$$M = \frac{\sigma_f B_0^2}{a\rho_f}, \quad \Pr = \frac{\nu_f}{\alpha_f}, \quad Ec = \frac{\rho_f a^2}{b(\rho C_p)_f} \tag{15}$$

The relevant boundary conditions of eqn. (4) and (5) are,

$$f'(\eta) = 1, \ \Pr f(\eta) + \frac{k_{nf}}{k_f}\delta\theta'(\eta) = 0, \ \theta(\eta) = 0, at \qquad y = 0$$
$$f'(\eta) \to 0, \ \theta(\eta) \to 1, \qquad at \qquad y = \infty \tag{16}$$

Where δ is the dimensionless melting parameter

$$\delta = \frac{c_f(T_\infty - T_m)}{l + c_s(T_m - T_0)} = \frac{c_f b}{l_0 + c_s c} \tag{17}$$

Which is the combination of the Stefan numbers $\frac{c_f(T_\infty - T_m)}{l}$ and $\frac{c_s(T_m - T_0)}{l}$ for the liquid and solid phases, respectively. When $\phi = 0$ we obtain the governing equations for a viscous fluid. In absence of melting i.e., for δ =0 eq. (17) reduces to the classical equation,

$$A_1 = \frac{1}{(1-\phi)^{2.5}(1-\phi+\frac{\rho_s}{\rho_f}\phi)}, \quad A_2 = \frac{1}{(1-\phi)^{2.5}(1-\phi+\frac{(\rho C_p)_s}{(\rho C_p)_f}\phi)},$$
$$A_3 = 1-\phi+\frac{\rho_s}{\rho_f}\phi \tag{18}$$

Local skin friction coefficient C_f and the Nusselt number Nu_x are given by,

$$C_f = \frac{\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{q_w x}{k_{nf}(T_\infty - T_m)} \tag{19}$$

where the surface shear stress τ_w and wall heat flux q_w are given by,

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{20}$$

Using equations above equations we obtain,

$$C_f(\text{Re}_x)^{1/2} = \frac{1}{(1-\varphi)^{2.5}}(1+\frac{1}{\beta})f''(0), \ Nu_x(\text{Re}_x)^{-0.5} = -\frac{k_{nf}}{k_f}\theta'(0) \tag{21}$$

here $\text{Re}_x = x(a/\nu_f)$ is the local Reynolds number.

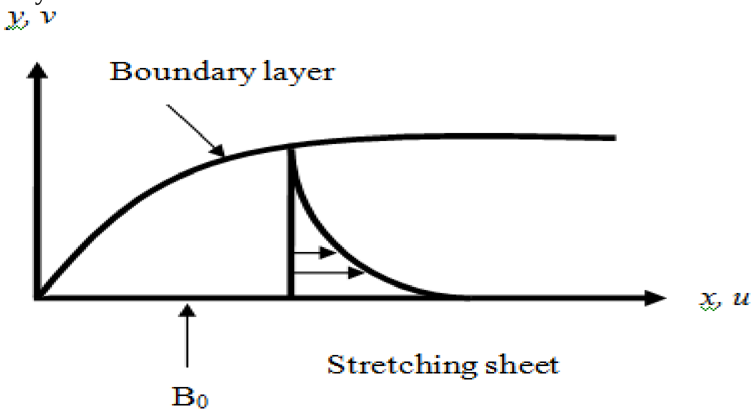


Figure 1. Sketch of Physical Flow

III. Discussions with Results

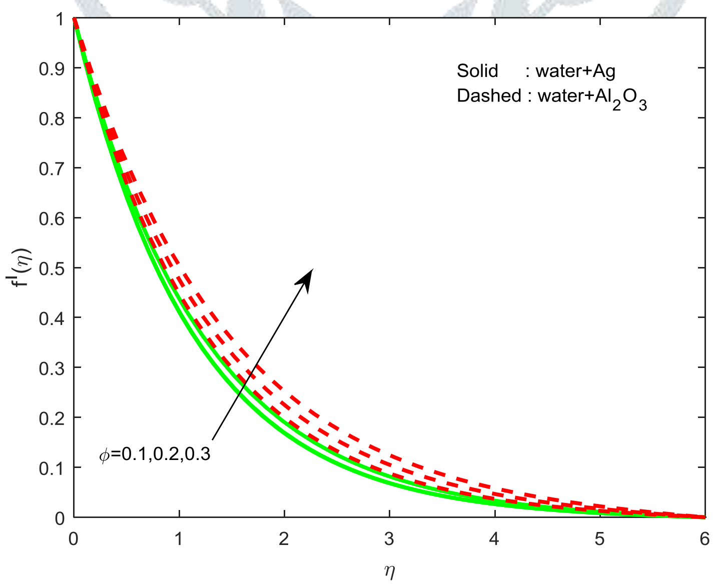
Under the influence of boundary conditions (16) the non-linear ordinary differential equations are solved using Runge-Kutta fourth order method with Shooting technique. Consider the parameters at fixed values of as $\delta = 0.5$, $Ec = 0.01$, $A^* = 0.1$, $B^* = 0.1$, $M = 1$, $\beta = 0.5$, $\phi = 0.1$. These values are fixed for the entire discussion except the changes in values are pictured in graphs and tables. The non-dimensional velocities $f'(\eta)$ and temperature $\theta(\eta)$ is observed for two nanofluids, obtained by mixing Silver nanoparticle and Aluminium oxide nanoparticle in base fluid as water, represented by continuous lines and dashed lines respectively. The study is concentrated on the influence of governing parameters like Magnetic effect (M), volume fraction (ϕ), Casson fluid parameter (β), melting parameter (δ), Eckert number (Ec) and non-uniform heat source/sink parameter A^* and B^*. Table 1 shows the thermo physical properties of the nano particle with base fluid.

Table 1. Thermo physical properties of the nano particles with base fluid

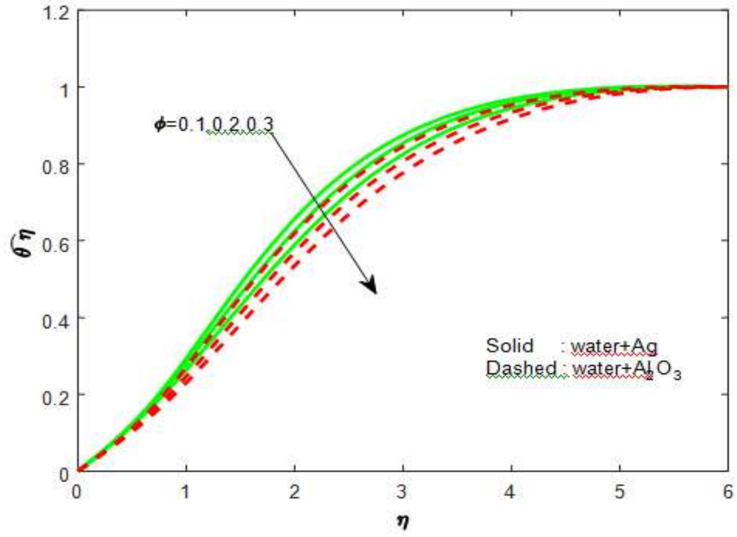
Physical properties	Fluid phase (water)	Ag	Al_2O_3
$c_p(J/KgK)$	4179	235	765
$\rho(Kg/m^3)$	997.1	10500	3970
$k(W/mK)$	0.613	429	40
$\sigma(S/m)$	0.005	6.30×10^7	3.5×10^7

Graph 1 and 2 illustrates the influence of volume fraction (ϕ) of the nanoparticles on the velocity and temperature profiles for both the Ag-water and Al_2O_3-water nanofluid. It is evident from the figures that increase in the volume fraction of the nanoparticles increases the strength of the flow. Because, the flow intensity increases with an increase in volume fraction which enhances the energy transport within fluid. Hence, the absolute value of stream function which represents the strength of the flow increases. It is observed that the thermal boundary layer of Ag-water combination is slightly more than the thermal conductivity of the nano Al_2O_3-water fluid. This is due to the fact that mixing of Al_2O_3-water declines the heat transfer due to poor conductivity than the Ag-metal. It is interesting to mention here that with a decreasing in the volume fraction of nanoparticle the thermal conductivity of the Al_2O_3-water nanofluid less. We can reduce the thermal conductivity of the Ag-water nanofluid.

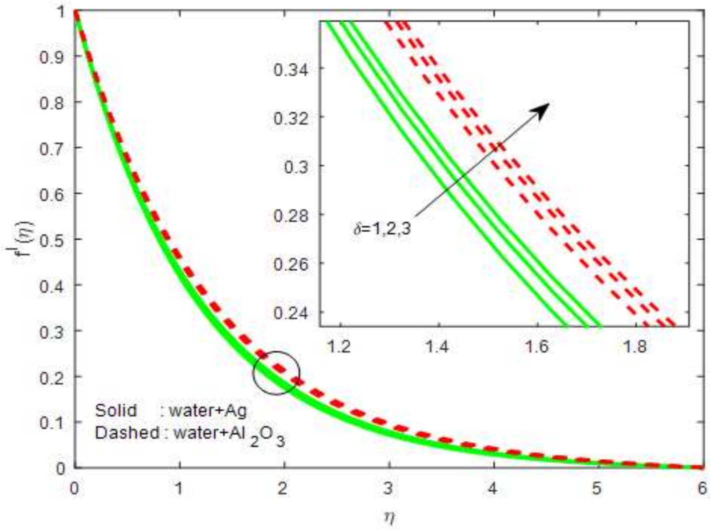
It has been observed that the thermal boundary thickness become thicker and thicker for both solutions. This is due to the fact that the temperature difference between ambient and melting surface increase which enhances the fluid flow and reduces the temperature of fluid with increasing δ for both solutions, is shown in graph no. 3. Graph no. 4 shows that the fluid flow profiles decreases with increase in melting parameter δ for the first solution consequently increases the thickness momentum boundary layer. Thus for the second solution the velocity is observed greater than the first fluid.



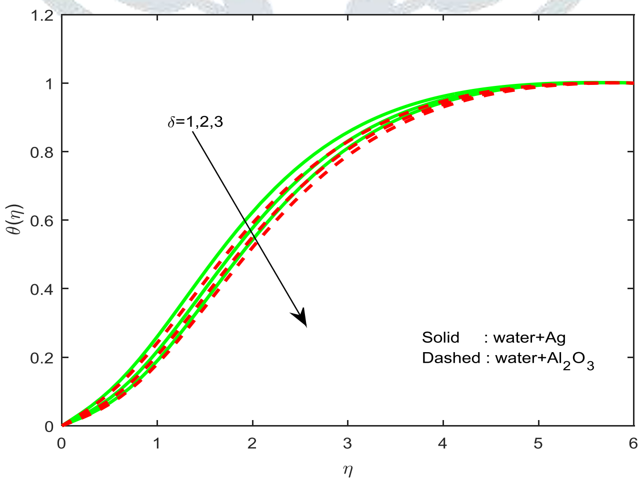
Graph 1: Fluid flow with respect to volume fraction (ϕ)



Graph 2: Fluid Temperature with respect to the Volume Fraction (ϕ)



Graph 3: Fluid Flow with respect to Melting Parameter (δ)



Graph 4: Fluid Temperature with respect to Melting Parameter (δ)

Table no. 2, shows the Variation of $f''(0)$ (skin friction) and $-\theta'(0)$ (Nusselt number) for a Ag-water nanofluid (Silver nanoparticle)

Table.2 . Variation of $f''(0)$ (skin friction) and $-\theta'(0)$ (Nusselt number) for a Ag-water nanofluid (Silver nanoparticle)

M	ϕ	β	δ	Ec	A^*	B^*	$f''(0)$	$-\theta'(0)$
1							-0.890211	-0.223487
2							-1.055380	-0.263575
3							-1.198403	-0.295932
	0.1						-0.890211	-0.223487
	0.2						-0.886640	-0.231448
	0.3						-0.827645	-0.226223
		0.3					-0.746925	-0.200900
		0.5					-0.890211	-0.223487
		0.7					-0.983804	-0.237649
			1				-0.873900	-0.179662
			2				-0.853062	-0.134196
			3				-0.839077	-0.109751
				0.2			-0.838117	-0.666939
				0.4			-0.796088	-1.047104
				0.6			-0.763446	-1.358437
					1		-0.787557	-1.127028
					2		-0.724079	-1.755558
					3		-0.679426	-2.239555
						0.1	-0.880604	-0.303176
						0.3	-0.844788	-0.608530
						0.5	-0.756860	-1.417524

Table no. 3, shows the Variation of $f''(0)$ (skin friction) and $-\theta'(0)$ (Nusselt number) for a Al_2O_3-water mixture nanofluid (Alumina Nanoparticle)

Table no. 3, Variation of $f''(0)$ (skin friction) and $-\theta'(0)$ (Nusselt number) for a Al_2O_3-water mixture nanofluid (Alumina Nanoparticle)

M	ϕ	β	δ	Ec	A^*	B^*	$f''(0)$	$-\theta'(0)$
1							-0.804446	-0.205618
2							-0.987186	-0.250562
3							-1.141413	-0.286106
	0.1						-0.804446	-0.205618
	0.2						-0.743305	-0.203378
	0.3						-0.685525	-0.198721
		0.3					-0.673638	-0.184359
		0.5					-0.804446	-0.205618
		0.7					-0.890421	-0.219359
			1				-0.793926	-0.167886
			2				-0.780107	-0.127368
			3				-0.770650	-0.105034
				0.2			-0.770445	-0.632843
				0.4			-0.742092	-1.004662
				0.6			-0.719442	-1.313042
					1		-0.733159	-1.125018
					2		-0.688684	-1.749894
					3		-0.656681	-2.229398
						0.1	-0.798992	-0.272878
						0.3	-0.779233	-0.520556
						0.5	-0.739554	-1.038695

Table 2 and 3 respectively display the influence of governing parameters on skin friction and local Nusselt number for Ag- water and Al_2O_3-water nanofluid. It is observed from the tables that the influence of Magnetic field parameter and Casson parameter declines the friction factor along with the heat transfer rate. However they are increasing with the increasing values of melting parameter. The Skin friction increases co-efficient and Nusselt number declines with the variation in the values viscous dissipation parameter. The same behaviour is observed for the other fluid also. Influence of A^* and B^* increases friction factor and declines the rate of heat transfer for both the fluids.

IV. CONCLUSION

The study presents a numerical solutios for the magneto-hydrodynamic flow of a Casson fluid over a stretching sheet in presence of the melting heat transfer and Joule heating. The effects of non-dimenssional governing parameters on velocity and temperature profiles of the flow are discussed with the help of graphs. The skin friction and Nusselt number are also calculated for variation of governing parameter and discussed. By the analysis the overall observation is listed out as:

- Fluid flow declines with rise in the magnetic field parameter but the reverse result is observed for temperature fields.
- The volume fraction of the nanoparticles increases the strength of the flow.
- By the variation of Casson parameter the rate of transport is considerably reduced and the rate of heat transfer is considerably enhances.
- The velocity is increased and the temperature is decreased with the variation of melting parameter δ
- The fluid flow decreases with Ec. The temperature is increasing initially and after some values of η the behaviour of graph is reversed .
- The temperature of the fluid increases by the influence of the non-uniform heat source/sink parameters
- The influence of the magnetic field declines the skin friction and Nusselt number.

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