CHARACTERIZATION OF SOME SPECIAL CLASSES OF GRAPHS

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ABSTRACT- Graph theory is an applied branch of mathematics which deals with problems with the help of graphs. One of the popular areas of research in graph theory is the study of the graphs that are obtained from given graphs. Concept of total graph comes under this category. Every graph has a total graph but all graphs are not total graphs of some graph. The total graph T(G) of G is the graph whose vertex set corresponds to the set of vertices and edges of G such that two vertices of T(G) are adjacent if and only if the corresponding elements of G are adjacent or incident. This paper gives formulas to find the regularity, the number of triangles and the number of edges of total graphs of certain standard graphs like path graph, complete graph cyclic graph.

KEYWORDS: Total graph, Regularity, Path graph, complete graph, cyclic graph.

I. INTRODUCTION

In the domain of mathematics and computer science, graph theory is the study of graphs that concerns with the relationship among edges and vertices. It is a popular subject having its applications in computer science, information technology, biosciences, mathematics, and linguistics to name a few. Without further ado, let us start with defining a graph. A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as vertices, and the links that connect the vertices are called edges. Formally, we consider a graph G (n, e), where n is the set of vertices and e is the set of edges, connecting the pairs of vertices. We consider Simple graphs; that is, finite undirected graphs with no loops or multiple edges.

Path is a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list. A cycle is a graph with an equal number of vertices and edges whose vertices can be placed around a circle so that two vertices are adjacent if and only if they appear consecutively along the circle. A simple graph G is said to be complete if every pair of adjacent vertices of G are adjacent in G. A graph is said to be regular if every vertex of graph has same degree. Line graph L(G) of graph G is obtained by taking the edges of G as the vertices of L(G) and two vertices are adjacent in L(G) if the corresponding edges have a common end vertices in G. The total graph T(G) of a graph G has a vertex for each edge and vertex of G and an edge in T(G) for every edge-edge, vertex-edge, and vertex-vertex adjacency in G. Total graphs are generalizations of Line Graph.

II. ON PROPERTIES OF TOTAL GRAPH OF PATH GRAPH (P_n)

Theorem 2.1

Let P_n be a path on n vertices $v_1, v_2, \ldots, v_n$ and T(P_n) be the total graph of P_n. Then number of triangles in T(P_n) is equal to n+e-2=n+(n-1)-2=2n-3

Proof:

Consider a path P_n on n vertices and its total graph T(P_n)

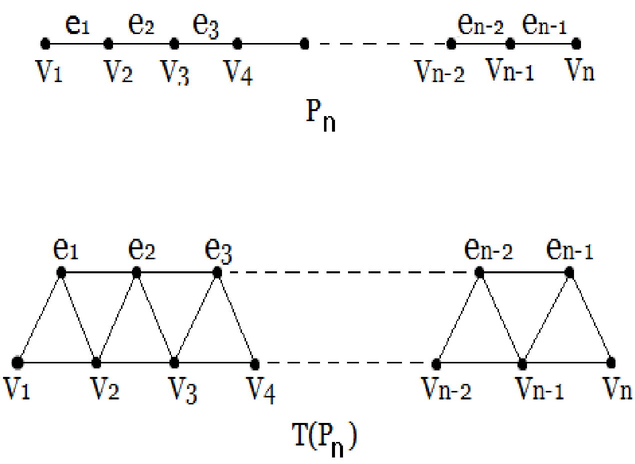

Figure A

We can observe that the triangles in T(P_n) are formed by either relation(incidence or adjacency) between two adjacent vertices and an edge between them (incident to both of them) or two adjacent edges and a vertex incident to both of them. In the first case we get a triangle $v_{n-1}e_{n-1}v_nv_{n-1}$ with $v_{n-1}v_n$ as base and in the second case we get a triangle $e_{n-1}\,v_{n-1}e_{n-2}e_{n-1}$ with $e_{n-1}e_{n-2}$ as base of triangle.

To prove that number of triangles in T(P_n)= n+e-2 it is enough to find the number of bases of triangles.

Number of triangles having $v_{n-1}v_n$ as base= length of the path v_1-v_n in T(P_n) =n-1 and number of triangles having $e_{n-1}e_{n-2}$ as base= length of the path e_1-e_{n-1} in T(P_n)= n-2

Then number of triangles in T(P_n) = n-1 + n-2 = 2n-3=n+e-2.

Example 2.1.1

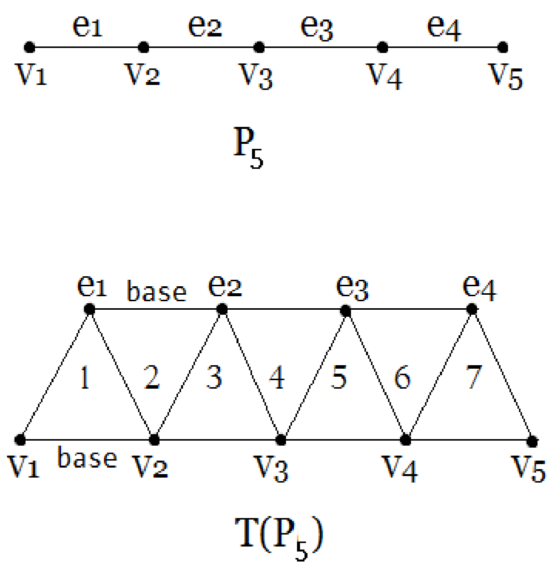

Figure 1

Given path P_5 , with n=5 and e= 4

Number of triangles having $v_{n-1}v_n$ as base (n=1, 2, 3, 4, 5) = length of the path v_1-v_5 in T(P_5)

=5-1 =4
Number of triangles having $e_{n-1}e_{n-2}$ as base (n=1, 2, 3, 4, 5)
= length of the path e_1-e_{n-1} in T(P_5)= 5-2= 3
Number of triangles in T(P_5) = n-1 + n-2 = 4+3= 5+4-2=7.

Theorem 2.2
Total graph of a path graph with n+3 vertices has 4n+7 edges. i.e. e[T(P_{n+3})] = 4n+7.
Proof:

Consider a path graph ;G =P_{n+3} The vertex set of T(P_{n+3}) will contain vertices corresponding to the vertices of P_{n+3} and vertices corresponding to the edges of Pn+3. Each vertex in T(P_{n+3}) corresponding to the internal vertices of Pn+3 will be adjacent to two vertices representing the incident edges and two vertices representing the adjacent vertices. Thus the sum of the degrees of the vertices representing the internal vertices in T(P_{n+3}) is equal to 4(n+1).Since there exists n+1 internal vertices for P_{n+3}.

Similarly all the vertices of T(P_{n+3}) corresponding to the edges of P_{n+3} except the end edges has 4 degree each. i.e.; 2 contributed by the end vertices and two contributed by the edges incident to the end vertices. Therefore the sum of the degrees of these vertices in T(P_{n+3}) is equal to 4n.

Now we see that the vertices in T(P_{n+3}) representing the end vertices of P_{n+3} has 2 degrees each. Thus the sum of the degrees of these vertices is 4. Also the sum of degrees of vertices in T(P_{n+3}) representing the end edges in P_{n+3} is 3+3=6 , since these edges in P_{n+3} has got two end vertices and one edge incident to it . Thus the total sum of the degrees of vertices of T(P_{n+3}) is 4(n+1)+4n+4+6 = 8n+14. Thus by hand shaking theorem ; the number of edges in T(P_{n+3}) is (8n+14)/2 = 4n+7.

Example 2.2.1

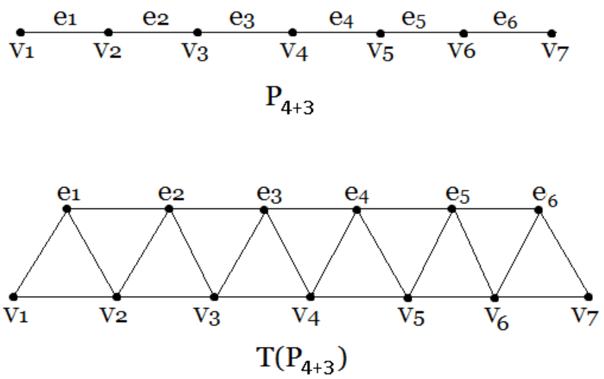

Figure 2

In the above figure n+3=4+3=7. Then e[T(P_{4+3})]=4n+7=(4×4)+7=23.

Example 2.2.2

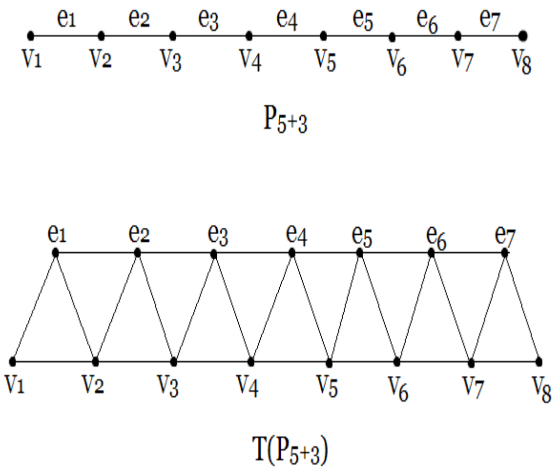

Figure 3

In the above figure n+3=5+3=8. Then e[T(P_{5+3})]=4n+7=(4×5)+7=27.

Observation 2.3
T(P_{n+3}) has total degree 8n+14

Observation 2.4
V[T(P_n)]= 2n-1

Observation 2.5
Number of edges of total graph of a path graphP_n is e[T(P_n)]=4n-5

III. On Properties of Total Graph of a Complete Graph

Theorem 3.1
Total graph of a complete graph is 2(n-1) regular.
Proof:
Consider a complete graph K_n. Let $v_1, v_2, \ldots, v_n$ be vertices of K_n and $v_1v_2, v_1v_3, v_1v_n, \ldots, v_nv_{n-1}, \ldots$ etc be edges of K_n. Thus the total graph of K_n, T(K_n) will have n+${}^nC_2$ vertices which represents the vertices and edges of K_n.

Consider the vertex v_i of T(K_n). We see that these are n-1 vertices in K_n adjacent to vertex v_i. Thus all those vertices are adjacent to the vertex v_i in T(K_n). That contributes n-1 to the degree of the vertex v_i, we also see that there are n-1 edges incident on the vertex v_i connecting it with the rest n-1 vertices. These edges are vertices in T(K_n). Hence those vertices v_iv_j in T(K_n) will be adjacent to v_i in K_n. This also contributes n-1 to the degree of the vertex v_i. Thus we see that degree of vertices v_i is 2(n-1).

Now consider the vertex v_iv_j of T(K_n). It is adjacent to the vertex v_i and v_j of T(K_n). Thus it contributes 2 to the degree of the vertex v_iv_j of T(K_n). We also see that there are n-2 vertices adjacent to v_i in K_n except v_j. The corresponding edges connecting v_i to the other vertices other than v_j are the vertices adjacent to v_i in T(K_n). Thus it contributes n-2 to the degree of the vertex v_iv_j in T(K_n).

Similarly there are n-2 vertices adjacent to v_j in K_n except v_i. The corresponding edges connecting v_j to the other vertices other than v_i are the vertices are the vertices adjacent to v_j in T(K_n). Thus it also contributes n-2 to the degree of the vertex v_iv_j in T(K_n).
Hence the degree of the vertices of the form v_iv_j is 2+(n-2)+(n-2)=2(n-1). Thus we see that the total graph of a complete graph is 2(n-1) regular.

Example 3.1.1

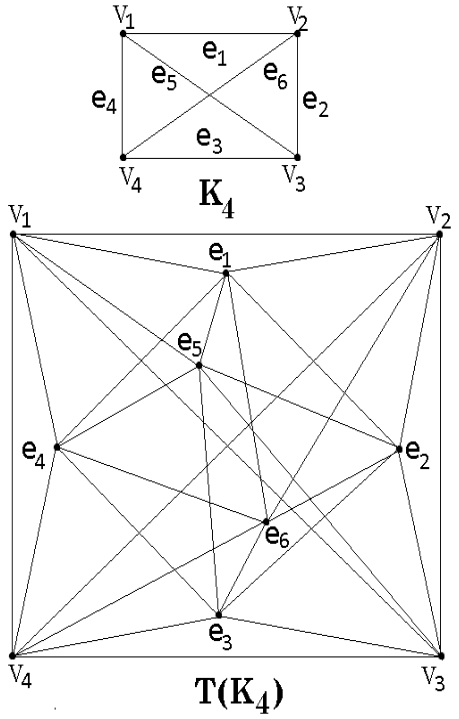

Figure 4

When n=4
Regularity of T(K_4)= 2(n-1)= 2(4-1)= 6.

Corollary 3.2
Total Graph of a complete graph K_n has (n+${}^{n}C_2$)(n-1) edges.
Proof:
We know that a complete graph K_n has n vertices and ${}^{n}C_2$ edges. Thus total graph of K_n has (n+ ${}^{n}C_2$) vertices

From previous theorem we see that T(K_n) is 2n-1 regular. Thus the sum of degrees of all the vertices of T(K_n) is (n+ ${}^{n}C_2$)(2n-2). Hence by Hand shaking theorem; the total number of edges of the graph T(K_n) is $\frac{(n+{}^{n}C_2)(2n-2)}{2}$= (n+ ${}^{n}C_2$)(n-1).

Example 3.2.1
In figure 4 we see that total graph of K_4 say T(K_4) has (4+${}^{4}C_2$)(4-1)= 30 edges.

IV. ACKNOLEDGEMENT

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V. CONCLUSION

Study on total graph is very advanced area of research. In this paper we had studied the total graph of a few standard graphs say, path graph, complete graphs and cyclic graphs and found out the regularity of the total graph of a complete graph, number of triangles of total graph of a path graph and the formulas to calculate the number of edges of above mentioned standard graphs.

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