Some Stronger Forms of βwg-Continuous Maps in Topological Spaces

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Abstract: In this paper, we introduce and study some stronger forms of βwg-continuous functions namely, strongly βwg-continuous, perfectly βwg-continuous and completely βwg-continuous functions in topological spaces. Further we introduce the concepts of strongly βwg-closed and strongly βwg-open maps and obtain some of their properties.
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I. INTRODUCTION

In 1983, Monsef et. al. [1] introduced β-open sets and β-continuity in topology. In 1986, Andrijevic [2], introduced semipre-open sets. In 1970, Levine [9] introduced the new class of generalized closed (briefly, g-closed) sets in topological spaces. The generalized continuity was studied in recent years by Balachandran et.al. Devi et.al, Maki et.al, [4, 6, 10]. Levine [7], Noiri [19] and Arya and Gupta [3] introduced and investigated the concept of strongly continuous, perfectly continuous and completely continuous functions respectively which are stronger than continuous functions. Later, Sundaram [21] defined and studied strongly g-continuous functions and perfectly g-continuous functions in topological spaces. Generalized closed (g-closed) maps were introduced by Malghan [12]. In 2013, P.G.Patil et al. [20] studied the concept of stronger forms of wα- continuous functions in topological spaces. Recently G.B.Navalgi et.al., [16],[17] introduced and studied the concept of βwg-closed sets, βwg-continuity, βwg-irresolute functions, βwg-closed maps, βwg-open maps and βwgT_b-spaces for general topology.

In this paper, we define and study some stronger forms of βwg-continuous functions namely, strongly βwg-continuous, perfectly βwg-continuous and completely βwg-continuous functions in topological spaces. Further, we introduce the concepts of stronlgly βwg-closed and strongly βwg-open maps and obtain some of their properties.

II. PRELIMINARIES

Throughout this paper (X,τ) , (Y,σ) and (Z,η) represent non-empty topological spaces on which no separation axioms are assumed unless explicitly stated and they are simply written as X, Y and Z respectively. For a subset A of a topological space (X,τ), the closure of A and the interior of A with respect to τ are denoted by cl(A) and int(A) respectively. The complement of A is denoted by A^c. The α-closure (resp. pre-closure and β-closure) of A is the smallest α-closed (resp. pre-closed and β-closed) set containing A and is denoted by αcl(A) (resp. pcl(A) and βcl(A)).

Before entering into our work we recall the following definitions from various authors.

Definition 2.1: A subset A of a topological space (X,τ) is called
(i) preopen [13] if $A \subseteq int(cl(A))$ and preclosed if $cl(int(A)) \subseteq A$.
(ii) α-open [18] if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$.
(iii) semipre-open [2] (β-open[1]) if $A \subseteq cl(int(cl(A)))$ and
semipre-closed (β-closed) if $int(cl(int(A))) \subseteq A$.

Definition 2.2: A subset A of a topological space (X,τ) is called a
(i) g-closed [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
(ii) g*p-closed [22] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X,τ).
(iii) βwg-closed [16] if $\beta cl(A) \subseteq U$) whenever $A \subseteq U$ and U is αg-open) in (X,τ).

Definition 2.3: A of a topological space (X, τ) is called a $_{\beta wg}T_b$ -space [17] if every βwg-closed set is closed.

Definition 2.4: A function f: (X,τ) → (Y,σ) is called pre-continuous [13] (resp. g-continuous [4], βwg-continuous [17] and g*p-continuous [22]) if $f^{-1}(V)$ is pre-closed (resp. g-closed, βwg-closed and g*p-closed) in X for every closed set V in Y.

Definition 2.5: A function f: (X, τ) → (Y, σ) is called a irresolute [5] (resp. gc-irresolute [6] and βwg-irresolute [17]) if if $f^{-1}(V)$ is semi-closed (resp. g-closed and βwg-closed) in X for every semi-closed (resp. g-closed and βwg-closed) set V of Y.

Definition 2.6: A function f: (X, τ) → (Y, σ) is called a

(i) Strongly continuous [7] if $f^{-1}(V)$ is both open and closed in X for each subset V in Y

(ii) Perfectly continuous [19] if $f^{-1}(V)$) is both open and closed in X for each open set V in Y.

(iii) Completely continuous [3] if $f^{-1}(V)$ is regular-open in X for each open set V in Y.

(iv) Strongly g-continuous [21] if $f^{-1}(V)$ is open in X for each g-open set V in Y.

(v) Perfectly g-continuous [21] if $f^{-1}(V)$ is both open and closed in X for each g-open set V in Y.

Definition 2.7: A function f: (X, τ) → (Y, σ) is called a

(i) M-preopen (resp. M-preclosed) [15] if f (V) is preopen (resp. preclosed) set in Y for every preopen (resp. preclosed) set V of X.

(ii) M-β-open (resp. M-β-closed) if f (V) is β-open (resp. β-closed) set in Y for every β-open (resp. β-closed) set V of X.

(iii) βwg-open (resp. βwg-closed) [17] if f (V) is βwg-open (resp. βwg-closed) in Y for each open (resp. closed) set V in Y.

III. ON STRONGLY βwg-CONTINUOUS FUNCTIONS

We define and study the following

Definition 3.1: A function f: X → Y is called strongly βwg-continuous if the inverse image of every ßwg-closed set in Y is closed in X.

Theorem 3.2: A function f: X → Y is a strongly βwg-continuous if and only if the inverse image of every βwg-closed set in Y is closed in X.

Proof: Suppose f: X → Y is a strongly βwg-continuous. Let K be a βwg-closed set in Y. Then Y-K is βwg-open set in Y. Since f is a strongly βwg-continuous, f^{-1} (Y-K) is βwg-open in X. But $f^{-1}(Y\text{-}K) = X\text{-}f^{-1}(K)$.Thus $f^{-1}(K)$ is closed set in X.

Conversely, suppose that the inverse image of every βwg-closed set in Y is closed in X. Let U be a βwg-open set in Y. Then Y-U is βwg-closed set in Y. By theory $f^{-1}(Y\text{-}U)$ is closed set in X. However $f^{-1}(Y\text{-}U) = X - f^{-1}(U)$ is closed set in X. Therefore, $f^{-1}(U)$ is open in X and consequently, f is a strongly βwg-continuous function.

Theorem 3.3: Every strongly βwg-continuous function is continuous and thus strongly pre-continuous, sp-continuous and βwg-continuous.

Proof: The proof follows from the definitions.

Theorem 3.4: Every strongly continuous function is strongly βwg-continuous but not conversely.

Proof: Follows from the definitions.

Example 3.5: Let X = Y = {a, b, c} with topologies, τ = {X, φ, {a}, {b}, {a, b}, {a, c}} and σ = {Y, φ, {a}}. Define a function f: X → Y by f(a) = a, f(b) = c, and f(c) = b. Then f is strongly βwg-continuous function but not strongly continuous, since for the closed subset {b,c} in Y, f^{-1} ({b,c}) = (b,c} is closed but not open in X.

Theorem 3.6: Every strongly βwg-continuous function is a βwg-irresolute and thus every strongly continuous function is a βwg-irresolute but not conversely.

Proof: Let f: X → Y be a strongly βwg-continuous function and let V be a βwg-closed set in Y. Then f^{-1} (V) is closed and hence βwg-closed in X, since every closed set is βwg-closed. Hence f is a βwg-irresolute. By Theorem 3.4, f is βwg-irresolute function.

Example 3.7: Let X = Y = {a,b,c} with topologies τ = {φ, X, {a,b}} and σ = {φ, Y, {a}, {b}, {a,b}}. Define a function f: X → Y by f (a) = b, f (b) = a and f(c) = c. Then f is βwg-irresolute function but not strongly βwg - continuous, since for the βwg-closed subset {a,c} of Y, f^{-1} ({a, c}) = (b, c} is closed but it is βwg-closed in X.

Theorem 3.8: Let (X, τ) be any topological spaces, (Y, σ) is a $_{\beta wg}T_b$ - space and let f: X → Y be any function. Then the following are equivalent:

(i) f is strongly βwg-continuous

(ii) f is continuous

Proof: (i) ⇒ (ii): Follows from the Theorem 3.3.

(ii) ⇒ (i): Let U be any βwg-open set in Y. Since Y is a $_{\beta wg}T_b$ – space, U is open in Y. Again since f is continuous, then we have $f^{-1}(U)$ is open in X. Therefore f is strongly βwg- continuous.

Definition 3.9: A topological space (X, τ) is called a βwg - space if the every subset in it is βwg-closed. i.e., βwgC(X, τ) = P(X).

Example 3.10: Let X = {a, b, c}, τ = {Ø, X, {a}, {b, c}} Then the space (X, τ) is βwg-space, because βwgC(X, τ) = P(X).

Theorem 3.11: Let X be discrete topological space and Y be a βwg-space and f: X → Y be a function.
Then the following statements are equivalent:

(i) f is strongly continuous.

(ii) f is strongly βwg-continuous.

Proof: (i) ⇒ (ii): Follows from the Theorem 3.6.

(ii) ⇒ (i): Let V be any βwg-open set in Y . Since Y is βwg-space, V is a βwg-open subset of Y and by hypothesis, $f^{-1}(V)$ is open in X. But X is a discrete topological space and so $f^{-1}(V)$ is also closed in X. That is $f^{-1}(V)$ is both open and closed in X and hence f is strongly continuous.

Remark 3.12: The concept of strongly βwg-continuity and strongly g-continuity are independent of each other as shown in the following examples.

Example 3.13: Let X = {a, b, c} = Y with topologies, τ = {φ, X, {a}, {b}, {a, b}, {b, c}}} and σ = {φ, Y, {a}}. Define a function f: X → Y by f (a) = b, f (b) = a and f (c) = c. Then f is strongly βwg-continuous but not strongly g-continuous, since for the g-closed subset {a,b} in Y, f^{-1} ({a, b}) = (a,b} is not closed in X.

Example 3.14: Let X = Y = {a,b,c}, τ = {φ, X, {a}, {b}, {a,b}} and σ = {φ, Y, {a}, {a,b}}. Define a function f: X → Y by f (a) = b, f (b) = a and f (c) = c. Then f is strongly g - continuous but not strongly βwg - continuous, since for the βwg-closed set {b} in Y, f^{-1} ({b}) = {a} is not closed in X.

Now, we derive decomposition of strongly βwg-continuous functions in the following

Theorem 3.15: The composition of two strongly βwg - continuous functions is again a strongly βwg - continuous function.
Proof: Let f: X → Y and g: Y → Z be any two strongly βwg - continuous functions. Let U be βwg - closed set in Z. Since g is strongly βwg-continuous, g^{-1} (U) is closed set in Y. Again, since f is strongly βwg - continuous, f^{-1} (g^{-1} (U)) closed in X. But
f^{-1} (g^{-1} (U)) = $(g \circ f)^{-1}$ (U). Thus, g o f is strongly βwg-continuous function.

Theorem 3.16: Let f: X → Y and g: Y → Z be any two functions. Then their composition gof: X → Z is strongly βwg-continuous.

- (i) if g is strongly βwg - continuous and f if is strongly g-continuous.
- (ii) if g is βwg-irresolute and f if is strongly βwg-continuous.
- (iii) if g is strongly βwg-continuous and f is continuous.

Proof: The proof follows from the definitions.

Theorem 3.17: Let f: X → Y and g: Y → Z be any two functions such that gof: X → Z. Then

- (i) gof is βwg-irresolute if f is βwg-continuous and g is strongly βwg-continuous.
- (ii) gof is strongly βwg-continuous if g is strongly βwg-continuous and f is strongly βwg– continuous.
- (iii) gof is continuous if g is continuous and f is strongly βwg-continuous.
- (iv) gof is βwg-irresolute if g is strongly βwg-continuous and f is βwg-irresolute.

Proof: Obvious

We define the following

Definition 3.18: A function f: X → Y is called perfectly βwg-continuous if the inverse image of every βwg-closed set in Y is both open and closed in X.

Theorem 3.19: Every perfectly βwg-continuous function is strongly βwg-continuous but not conversely.
Proof: Let f: X → Y be a perfectly βwg-continuous function. Let U be a βwg - open set in Y. Then f^{-1} (U) is clopen set in X. Therefore f^{-1} (U) open set in X. Hence, f is strongly βwg - continuous function.

Example 3.20: Let (X,τ) be as in the Example 2.3.114.Define a function f: X → Y by f (a) = a, f(b) = c, and f(c) = b. Then the function f is strongly βwg - continuous but not perfectly βwg - continuous function, since for the βwg - closed set {b, c} in Y,
f^{-1} ({b,c}) = (b,c} is closed but not open in X.

Theorem 3.21: Every perfectly βwg-continuous function is continuous but not conversely.
Proof: Consider a perfectly βwg-continuous function f: X → Y. Let V be an open set in Y. Since f is perfectly βwg-continuous function, f^{-1} (V) is clopen set in X. Therefore f^{-1} (V) is open set in X. Hence f is continuous function.

Example 3.22: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {a, c}} and σ = {φ, Y, {a}}. Then the identity function f: X → Y is continuous but not perfectly βwg-continuous function, since for the βwg-closed set {c} in Y, f^{-1}({c}) = {c} is closed but not clopen in X.

Theorem 3.23: Every perfectly βwg-continuous function is perfectly continuous but not conversely.
Proof: Consider a perfectly βwg-continuous function f: X → Y. Let U be an open set in Y. Since f is perfectly βwg-continuous function, f^{-1} (U) is clopen set in X. Hence f is perfectly continuous function.

Example 3.24: Let X = Y = {p, q, r}, τ = {φ, X, {p}, {q, r}} and σ = {φ, Y, {p}}. Define an identity function f: X → Y by f (p) = p, f(q) = q, and f(r) = r. Then the function f is perfectly continuous but not perfectly βwg-continuous function, since for the βwg-open set {p, q} in Y, f^{-1} ({p, q}) = {p, q} is neither closed nor open set in X.

Theorem 3.25: If a function f: X → Y is perfectly continuous and Y is $_{\beta wg}T_b$ –space, then f is perfectly βwg-continuous function.
Proof: Assume that G is a βwg-open set in Y. Then G is open set in Y as Y is $_{\beta wg}T_b$ – space. Then f^{-1} (G) is clopen set in X as f is perfectly continuous function. Therefore f is perfectly βwg-continuous function.

Theorem 3.26: For a discrete topological space X and Y be any topological space. Then for the function f: X → Y the following results are equivalent:

(i) f is perfectly βwg-continuous.
(ii) f is strongly βwg-continuous.

Proof: (i) ⇒ (ii): Follows from the Theorem 3.19.

(ii) ⇒ (i): Let B be any βwg - open set in Y. By hypothesis, $f^{-1}(B)$ is open set in X. But X is a discrete topological space and so $f^{-1}(B)$ is closed in X. Therefore is $f^{-1}(B)$ is both open and closed in X. Hence f is perfectly βwg-continuous function.

Theorem 3.27: Let f: X → Y and g: Y → Z be any two functions. Then their composition gof: X → Z is perfectly βwg-continuous.
(i) if f and g are perfectly βwg-continuous function.
(ii) if g is strongly βwg-continuous and f is perfectly βwg-continuous.
(iii) if f is perfectly βwg-continuous and g is βwg-irresolute

Proof: The proof is straight forward.

Definition 3.28: A function f: X → Y is called completely βwg-continuous if the inverse image of every βwg-closed set in Y is regular closed in X.

Theorem 3.29: A function f: X → Y is called completely βwg-continuous if and only if the inverse image of every βwg-open set in Y is regular-open in X.
Proof: Obvious.

Theorem 3.30: If a function f: X → Y is called completely βwg-continuous then f is continuous.
Proof: Let f: X → Y be a function. Let H be an open set in Y. Since f is completely βwg-continuous function, f^{-1} (H) is regular open set in X. So f^{-1}(H) is open set in X. Hence f is continuous function.

However the reverse implication is not possible as seen from the following example.

Example 3.31: Let X = {a, b, c} = Y with topology, τ = {X, φ, {a}, {a, b}} and σ = {Y, φ, {a, b}}. Then the identity function f: X → Y is continuous but not completely βwg-continuous function, since for the βwg-closed set {c} in Y, f^{-1} ({c}) = {c} is not regular closed set in X but it is closed in X.

Theorem 3.32: Every completely βwg-continuous function is completely continuous function.
Proof: Let f: X → Y be a completely βwg-function. Let U be an open set in Y. Then U an open set in Y. Since f is completely βwg-continuous function, f^{-1} (U) is regular open set in X. Hence f is completely continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.33: Let X = {a, b, c} = Y, τ = {φ, X, {a},{b},{a,b}} and σ = {φ, Y, {a}} be the topologies on X and Y respectively. Then the identity function f: X → Y is completely continuous but not completely βwg-continuous. Since for the βwg-open set {c} in Y, f^{-1} ({c}) = {c} is not regular open in X.

Theorem 3.34: Every completely βwg-continuous then f is strongly βwg-continuous but not conversely.

Proof: Let f: X → Y be a completely βwg- continuous function. Let U be a βwg-open set in Y. Since f is completely βwg-continuous function, f^{-1} (U) is regular open set in X. But every regular open set is open and hence f^{-1} (U) is open in X. Hence f is strongly βwg-continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.35: Let (X, τ) be as in the Example 3.5. Define a function f: X → Y by f(a) = a, f(b) = c, and f(c) = b. Then the function f is strongly βwg - continuous but not completely βwg-continuous function, since for the βwg-closed set {b, c} in Y, f^{-1} ({b,c}) = (b,c} is not regular closed but it is closed in X.

Theorem 3.36: If a function f: X → Y is completely continuous and Y is ${}_{\beta wg}T_b$ -space, then f is completely βwg-continuous.
Proof: Assume that G is a βwg-open set in Y. Then G is open set in Y as Y is ${}_{\beta wg}T_b$ – space. Then f^{-1} (G) is clopen set in X as f is perfectly continuous function. Therefore f is perfectly βwg-continuous function.

Theorem 3.37: Let f: X → Y and g: Y → Z be any two functions such that gof: X → Z. Then
(i) gof is completely βwg-continuous if f is completely continuous and g is completely βwg-continuous.
(ii) gof is completely βwg-continuous if g is βwg-irresolute and f is completely βwg-continuous.
(iii) gof is completely βwg-continuous if f is completely βwg-continuous and g is strongly βwg-continuous.

Proof: Follows from the definitions.

IV. STRONGLY βwg-CLOSED FUNCTIONS AND STRONGLY βwg-OPEN FUNCTIONS

Next, we introduce strongly βwg-closed and strongly βwg-open maps and investigate some of their properties in the following

Definition 4.1: A function f: X → Y is called strongly βwg-closed (resp. strongly βwg-open) function if the image of every βwg-closed (resp. βwg-open) set in X is βwg-closed (resp. βwg-open) set in Y.

Theorem 4.2: If a function f: X → Y is called strongly βwg-closed then it is βwg-closed but not conversely.
Proof: Obvious.

Example 4.3: Let X = {a, b, c} = Y, τ = {φ, X, {a,b}} and σ = {φ, Y, {a}}. Then the identity function f: X → Y is completely continuous but not completely βwg-continuous. Since for the βwg-closed set {a,c} in X, f ({a,c}) = {a,c} is not closed set in Y.

Theorem 4.4: The composition of two strongly βwg-closed functions is again a strongly βwg-closed function.

Remark 4.5: Strongly βwg-closed functions and βwg-irresolute functions are independent of each other as seen from the following examples.

Example 4.6: Let X = Y = {a, b, c} with topologies, τ = {φ, X, {a}, {a, b}} and σ = {φ, Y, {a}, {b}, {a, b}}. Then the identity function f: X → Y is strongly βwg-closed but not βwg-irresolute. Since for the βwg-closed set {c} in Y, f ({c}) = {c} is not βwg-closed in X.

Example 4.7: Let X = Y = {a, b, c} with topologies τ = {φ, X, {a},{b}, {a,b}} and σ = {φ, Y, {a}}. Define a function f: X → Y by f(a) = a, f(b) = c, and f(c) = b. Then f is βwg-irresolute but not strongly βwg-closed. Since for the βwg-closed set {a,c} in X,
f ({a, c}) = {a,b} is not βwg-closed in Y.

Theorem 4.8: A function f: X → Y is a strongly βwg-closed if and only if for each subset B of Y and for each βwg-closed set U of X containing f^{-1} (B), there exists a βwg-open V set such that B ⊆ Y and f^{-1} (V) ⊆ U.
Proof: Necessary: Suppose f: X → Y is a strongly βwg -closed function. Let B be any subset of Y and U be a βwg-closed set of X containing f^{-1} (B). Put V = Y - f^{-1} (U) Then V is βwg-open set in Y containing B such that f^{-1} (V) ⊆ U.
Sufficiency: Let F be any βwg - closed set of X. Then f^{-1}(Y-f(U)) ⊆ X-F. Put B = Y - f^{-1} (F). We have f^{-1} (B) ⊆ X-F. Also X-F is βwg-open in X and f^{-1} (V) ⊆ X-F. Therefore we have f^{-1}(F) = Y-V and hence f(F) is βwg-closed set in Y. Therefore f is a strongly βwg-closed function.

Theorem 4.9: If f: X → Y is αg-irresolute and pre-β-closed, then f is strongly βwg-closed function.
Proof: Let A be a βwg-closed set in X. Let V be any αg-open set in Y containing f(A). Then A ⊆ f^{-1}(V). Since f is αg-irresolute, f^{-1}(V) is αg-open set in X. Again since A is βwg-closed in X, cl(A) ⊆ f^{-1}(V) and hence f(A) ⊆ f(βcl(A)) ⊆ V. As f is pre-β-closed and βcl(A) is β-closed in X, f(βcl(A)) is β-closed in Y and hence βcl(f(A)) ⊆ βcl(f(βcl(A))) ⊆ V. This shows that f(A) is βwg-closed set in Y . Hence f is strongly βwg-closed.

Theorem 4.10: Let f: X → Y and g: Y → Z be two functions such that gof: X → Z. Then
- **(i)** gof is βwg-closed if f is closed and g is strongly βwg-closed.
- **(ii)** gof is βwg-closed if f is closed and g is αg-irresolute and pre-β-closed.
- **(iii)** g is βwg-closed if f is continuous surjection and gof is strongly βwg-closed.
- **(iv)** gof is βwg-closed if f is strongly βwg-closed and g is strongly βwg-closed.

Proof: (i) By Theorem 4.2, g is βwg-closed function. Let F be a closed set in X. Since f is closed, then f(F) is closed in Y . Therefore g(f(F)) is βwg-closed in Z as g is βwg-closed function. That is gof(F) is βwg-closed set in Z. Hence gof is βwg-closed function.

(ii) By Theorem 4.9, g is strongly βwg-closed function. Hence by (1), gof is βwg-closed.

(iii) Let F be a closed set of Y. Since f is continuous, f^{-1}(F) is closed in X and hence f^{-1}(F) is βwg-closed in X. Since gof is strongly βwg-closed, (gof)(f^{-1}(F))) is βwg-closed in Z. Again since f is surjective, g(F) is βwg-closed in Z. Hence g is βwg-closed.

(iv) Follows by (i).

Further, we define βwg-regular and βwg-normal spaces in the following

Definition 4.11: A space (X, τ) is said to be βwg-regular if for every closed set F and a point x ∉ F, there exist disjoint βwg-open sets U and V such that F ⊆ U and x ∈ V.

Theorem 4.12: Every βwg-regular space is regular space.
Proof: obvious.

The converse of the above theorem need not be true as seen from the following example.

Example 4.13: Let X = {a,b,c}, τ ={X, φ, {a}, {b,c}}. Then the space (X, τ) is regular but not βwg-regular space.

Also, we define the followings

Definition 4.14: A topological space (X,τ) is said to be βwg-normal if for every pair of disjoint closed sets A and B, there exist disjoint βwg-open sets U and V such that A ⊆ U and B ⊆ V .

Theorem 4.15: Every βwg-normal space is normal, but converse is not true in general.
Proof: Let X is a βwg-normal space. Let A and B be a pair of disjoint closed sets in X. Since every closed set is βwg-closed. Therefore A and B are βwg-closed sets in X. Again, since X is βwg-normal, there exists a pair of disjoint open sets G and H such that A ⊆ G and B ⊆ H .Hence X is βwg-normal space.

The converse of the above Theorem need not be true as seen from the following example

Example 4.16: Let (X,τ) be as in the Example 4.13. Then, clearly the space (X,τ) is normal space. But, it is not βwg-normal space.

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