A study of linear programming technique

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Abstract
This paper focuses on one of the most used techniques of operations research i.e. linear programming.The linear programming is a mathematical method to provide an optimal solution for the problems where objective and the requirements both are linear. The limelight during the second World War. The rapid useof linear programming in various fields started after the Second World War. This study provides insights on the assumptions and properties of the linear programming. It also explains the process of formulating any problem as a linear programming problem if all the assumptions are satisfied. The limitations and mathematical formulation are also problem if all the assumptions are satisfied. The limitations and mathematical formulation are also explained. Some practical applications of linear programming are discussed. This study also provides insight on the latest applications of linear programming problem in various fields like sports, lean manufacturing, financial planning and radiotherapy.

Keywords: Linear programming problem, limitations, assumption, applications

Introduction

"No dreams come true until you wake up and go for work" similarly any problem cannot be resolved unless we fix the problem and try to solve it. Most of the business or planning problems involving resources can be converted into a mathematical problem the only vital condition is all information should be available. These problems involved consist of two major components, one is objective function and the other is constraints. If both objective function and constraints are linear, then they can be framed as a linear programming problem. Here linear means that variable involved in constraints and objective function should not have any power. There are some standard types of Linear programming formulations given in this paper,but the list is indicative not exhaustive as after formulating 1000 problems, there would be another problem where the formulation might be entirely different

Properties and Assumptions in Linear Programming Problem

A Linear programming model is based on the assumption of proportionality, additivity, continuity, certainty, and finite choices. These are given detailed below.

- **Proportionality:** The rate of change (slope) of the objective function and the constraintequations with respect to decision variable is constant.

- **Continuity:** Decision fractional value and variables can take on any are thereforecontinuous as opposed to integer in nature.

- **Certainty:** Values of all model parameters are assumed to be known with certainty.

Formulation of Linear Programming problem

Steps of Linear Programming Formulation
- Determine the objective of the problem
- Identify decision variables and conditions involved
- Formulate objective function
- Formulate constraints
- Express the non-negativity constraints
- Weather the all the conditions are satisfied

Limitations of linear programming

- Linear programming linear. But it is not true situations. Considers all relationships as in many real life and business.
- As the number of variables and constraints increases the problem becomes very complex and very Difficult to solve.
- Linear programming gives its solution in fractions which are not suitable in many practical situations. For example- if it can give a solution as construction of 25/4 apartments and 35/3 shops as optimum solution, which is not possible for any construction company.
- Factor pertaining to uncertainty such as, strike, absenteeism of labor, weather conditions, etc. are not taken into deliberation during formulation.
- LPP considers only a single objective function; where asin real life and business situations, there may be more thanone objective.

Mathematical Formulation

The Linear Programming problem can be put in the followingform, which is also known as standard form of LPP.

Maximize (or Minimize)

$$z = c_1x_1 + c_2x_2 + c_3x_3 + - - - - + c_nx_n$$

Objective FunctionSubject to the constraints;

$$a_{11}x_1 + a_{12}x_2 + \ - - - - + a_{1n}(\leq, \geq, =)b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ - - - - + a_{2n}(\leq, \geq, =)b_2$$

$a_{m1}x_1 + a_{m2}x_2 + ----+a_{mn}(\leq, \geq, =)b_m$

$where\ x_{1,2}, ---- x_m \geq 0$

Some Examples of Linear Programming Problem

Production Mix Problem

Example on use of linear programming to decide the product mix i.e. to use the quantity of products to be manufactured to get the maximum profit

Formulate the following as a linear programming problem Publishing house publishes three weekly magazines- Daily life, Agriculture today, and Surf's up. Publication of one issueof each of these magazines requires the following amounts of production time and paper.

Table 1: Gives requirements of each product

Magazine	Production (hour)	Paper (kg) Daily Life
Agriculture Today	0.03	0.05
Surf's Up	0.02	0.03

Each week the publisher has available 120 hours ofproduction time and 3,000kg of paper. Total circulation for allthree magazines must exceed 5,000 issues per week if the company is to keep it's advertised. The selling price per issue is \$. 22.50 For Daily life, \$. 40.00 For Agriculture Today and

\$. 15 for Surf's up. Based on past sales, the publisher knows that the maximum weekly demand for Daily Life is 3,000 issues; for Agriculture Today it is 2,000 issues while forSurf's Up it is 6,000 issues. The production manager wants to know the number of issues of each magazine to produce per week in order to maximize total sales revenue.

Let publishing house publishes x1 copies of Daily Life, x2copies of Agriculture Today and x3 copies of Surf's up. Here x_1, x_2 and x_3 are decision variables

The Objective Function- here objective of the publishing house is to maximize the total sales revenue. Thus, the objective function will be.

Max Z = $22.5x_1+40x_2+15x_3$

The Constraints-

- Paper (Kg)-$0.2x_1+0.5x_2+0.3x_3 \leq 3,000$
- Circulation (issues)–$x_1+x_2+x_3 \geq 5,000$
- Production Hours-$0.01x_1+0.03x_2+0.02x_3 \leq 120$
- Maximum demand for Daily Life–$x_1 \leq 3,000$
- Maximum demand for AgricultureToday-$x_2 \leq 2,000$
- Maximum demand for Surf'sUp-$x_3 \leq 6000$

Non-Negativity condition-As, x_1, x_2 and x_3 are the Number of unit produced, they cannot be negative. Thus, both of them can assume values only greater-than-or-equal-to zero. This can be stated as.

$x_1 \geq 0$, $x_2 \geq 0$ and $x_3 \geq 0$Subject To

Hence the mathematical formulation of the problem is

$0.01x_1+0.03x_2+0.02x_3 \leq 120$

$0.2x_1+0.5x_2+0.3x_3 \leq 3,000 x_1+x_2+x_3 \geq 5,000 x_1 \leq 3,000 x_2 \leq 2,000 x_3 \leq 6000$

x_1,x_2 and $x_3 \geq 0$

Portfolio Management

Example on use of linear programming in financial planning,a mutual fund has \$ 20 million available for investment in Government bonds, blue chips stocks, and speculative tock sand short term bank deposits. The annual expected return andrisk factor are given as follows:

Table 2: Provides annual expected return and risk factor for eachinvestment

Types of investment	Annual expected return	Risk factor (0 to 100) Government bonds
Blue chip stocks	19%	24
Speculative stocks	23%	48
Short term deposits	12%	6

Mutual Fund is required to keep at least \$ 2 million in short term deposits and not to exceed an average risk factor of 42.Speculative stocks must be at most 20percent of the total invested. How should Mutual Fund invest the funds so as to maximize its total expected annual return?? Formulate this asa linear programming problem. Solution- let mutual fund makes the following investments, \$ x_1 in government bonds, \$$x_2$ in blue chip stocks, \$ x_3 in speculative stocks and \$. x_4 in short term deposits.

Objective function – Max Z =$.14x_1+.19x_2+.23x_3+.12x_4$Subject to

$x_1+x_2+x_3+x_4 \leq 20,00,000$(Total investment)

$x_4 \geq 2,00,000$ (Minimum investment in short term deposits) $12x_1+24x_2+48x_3+6x_4/x_1+x_2+x_3+x_4 \leq 42$ (Risk factor)

Or

$30x_1+18x_2-6x_3+36x_4 \geq 0$

$x_3 \leq .2(x_1+x_2+x_3+x_4)$ (maximum investment in speculativestocks)

Or

$.8x_1-.2x_1-.2x_2-.2x_4 \leq 0$

x_1, x_2, x_3 and $x_4 \geq 0$

Research on applications of linear programming

There has been lot of research on linear programming in last few decades. Some of the latest works on the applications of linear programming in the various fields are given below.

- Mehrzad Hamidi *et al.*, 2011 have evaluated the performance of Iranian football teams utilizing linear programming.
- Oleg Viacheslavovich Yeriomin (2011) had used linear programming for calculation of standard thermodynamic potentials for Na-Zeolites.
- Konstantine Georgakakos (2012) worked on water supplyand demand sensitivities of linear programming solutions to a water all ocation problem.
- Bruno Rüttimann. Discoursed about Linear Programmingand Lean Manufacturing: Two Different Approaches witha Similar, Converging Rational
- Chongyu Jiang *et al.*, (2016) worked on Application of Linear Programming Model to Refugee Migrating Problem
- Thais R. Salvador *et al.*, (2016) worked on the application of simplex method one of the method use to solve linear programming in the radio the reply treatment.

Conclusion

This study provides complete details about the linear programming technique, one of the most applied techniques of Operations Research. This note can help students and researchers to understand the assumptions and properties of linear programming. This paper also provides the limitations of linear programming. Also, some of the latest research on applications of linear programming in various fields havebeen discussed in this study. Thus, this study will act as a catalyst for encouraging more research on applications of linear programming.

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