A Note On Cordial Labeling of Cycle Related Mixed Double Path Union

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1. Abstract: Let G_1 and G_2 be two graphs. On alternate vertices of P_m both G_1 and G_2 are fused. Resultant graph is mixed double path union on G_1 and G_2.We study it for cordial labeling by taking case1:$G_1 = C_3$ and G_2 = flag of C_3. Case2:$G_1 = C_4$ and G_2 = flag of C_4. Case3:$G_1 = C_5$ and G_2 = flag of C_5. Case 4:$G_1 = C_4$ and G_2 = flag of C_3. All these structures are observed to be cordial.

Key words: cordial, labeling, double path union, cycle, mixed path

Subject Classification: 05C78

1. Introduction. The graphs we consider are simple, finite and connected.For terminilogy and definitions we depend on Harary[5], Clark and Holton[4] and Dynamic survey of graph labeling[7].I. Cahit introduced concept of cordial [3] labeling. $f:V(G)\rightarrow\{0,1\}$ be a function. From this label of any edge (uv) is given by $|f(u)-f(v)|$. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [7]. We use the convention that $v_f(0,1) = (a,b)$ to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b. Further $e_f(0,1) = (x,y)$ we mean the number of edges labeled with o are x and number of edges labeled with 1 arey. The graph whose cordial labeling is available is called as cordial graph.
A mixed double path union on G_1 and G_2 is denoted by $Pm(G_1,G_2)$ .We take G_1 as C_k and G_2 as flag (C_k) , k = 3, 4 ,5.

3. Definitions: **Fusion of vertex**. Let G be a (p,q) graph. let $u\neq v$ be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges.[].If $u\epsilon G_1$ and $v\epsilon G_2$, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with v.

Path union of G i.e. $P_m(G)$ is obtained by taking a path P_m and m copies of graph G. Fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges, where G is a (p, q) graph. If we change the vertex on G that is fused with vertex of P_m then we generally get a path union non-isomorphic to earlier structure.

Flag of a graph G denoted by FL(G) is obtained by taking a graph G=G(p,q).At sutable vertex of G attach a pendent edge.It has p+1 vertices and q+1 edges.

Double path union on G_1 = (p1,q1) and G_2 =(p2,q2) graphs..It is denoted by Pm(G1,G2).At each vertex of a path a copy of G_1 and a copy of G_2 is fused at same fix vertex of G_1 and G_2.It has m(p1+p2) vertices and m(q1+q2)+(m-1) edges.

4. Main Results:

Theorem 1.Mixed double path union G = $P_m(G_1,G_2)$ is cordial where G_1 = C3 and G_2 = flag(C_3).

Proof: First we fuse C_3 and flag(C_3) to obtain the structure as follows. This structure is actually $P_1(G_1,G_2)$ and is fused at each vertex of path P_m. at vertex a on it.

Define a function f: V(G) → {0,1) .It produces labeled copy as in figure 4.1 below.

Fig 4.1 G = $P_1(G_1,G_2)$ A labeled copy. Vertex a is fusion vertex on G.
$v_f(0,1)$ = (3,3) , $e_f(0,1)$ = 3,4)

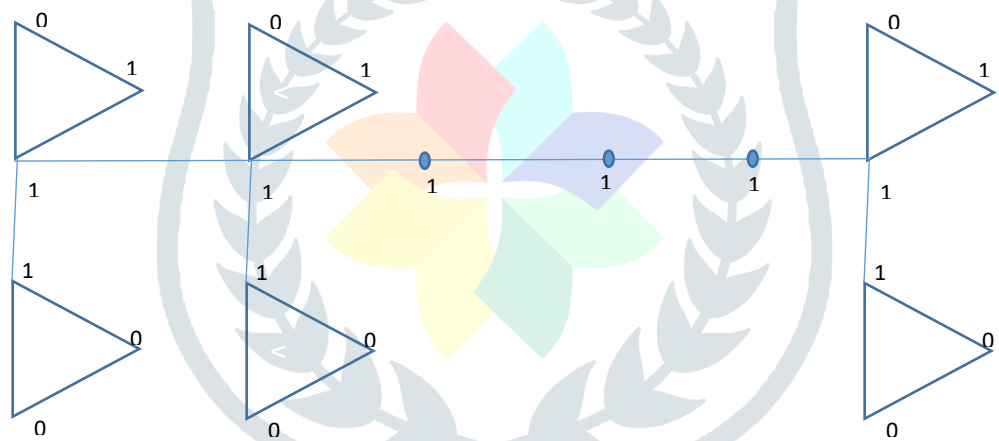

Fig 4.2 labeled copy of G = $P_m(G_1,G_2)$

Note that label of each vertex on path Pm is 0.

Label distribution on vertices is $v_f(0,1)$ = (3m,3m) for all m. For edges we have $e_f(0,1)$ = (3+4(m-1), 4+4(m-1))

Thus the graph is cordial.

Theorem 2.Mixed double path union G = $P_m(G_1,G_2)$ is cordial where G_1 = C4 and G_2 = flag(C_4).

Proof: First we fuse C_4 and flag(C_4) to obtain the structure as follows. This structure is actually $P_1(G_1,G_2)$ and is fused at each vertex of path P_m. at vertex a on it.

Define a function f: V(G) → {0,1) .It produces labeled copy as in figure 4.3 below.

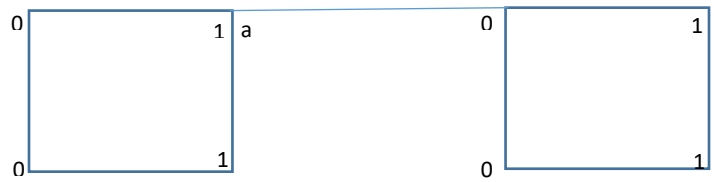

Fig 4.3 labeled copy of G = $P_m(C_4,flag(C_4)$
$v_f(0,1) = (4,4)$, $e_f(0,1) = (4,5)$

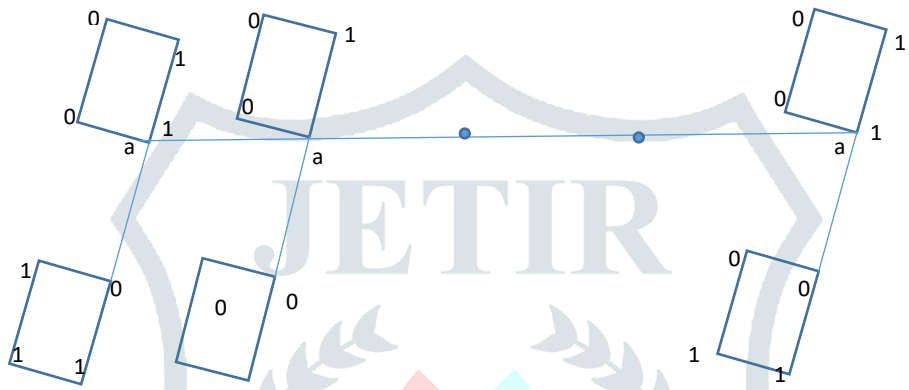

Fig 4.4 labeled copy of G = $P_m(C_4,flag(C_4))$

Note that label of each vertex on path Pm is 0.

Label distribution on vertices is $v_f(0,1) = (4m,4m)$ for all m. For edges we have $e_f(0,1) = (4+5(m-1), 4+5(m-1))$

Thus the graph is cordial.

Theorem 3.Mixed double path union G = $P_m(G_1,G_2)$ is cordial where $G_1 = C_5$ and $G_2 = flag(C_5)$.

Proof: First we fuse C_4 and $flag(C_4)$ to obtain the structure as follows. This structure is actually $P_1(G_1,G_2)$ and is fused at each vertex of path P_m. at vertex a on it.

Define a function f: V(G) → {0,1) .It produces labeled copy as in figure 4.5 below.

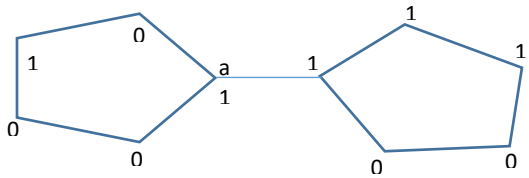

Fig 4.5 labeled copy of G = $P_m(C_5,flag(C_5)$
$v_f(0,1) = (5,5)$, $e_f(0,1) = (5,6)$

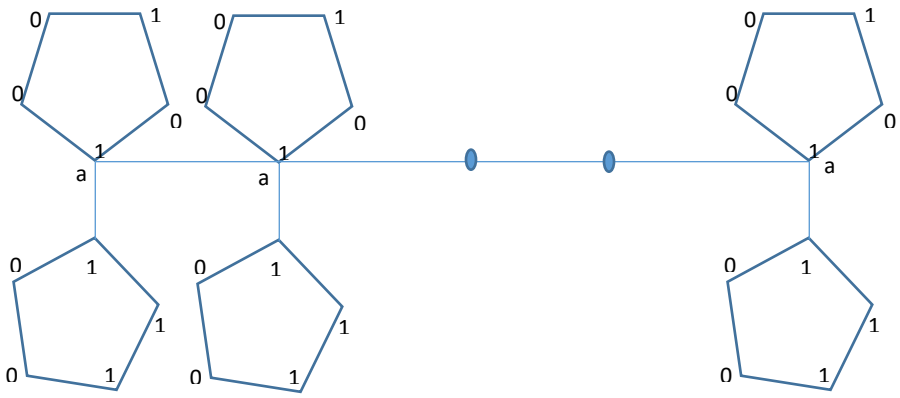

Fig 4.6 labeled copy of G = $P_m(C_5,flag(C_5))$

Note that label of each vertex on path Pm is 0.

Label distribution on vertices is $v_f(0,1)$ = (5m,5m) for all m. For edges we have $e_f(0,1)$ = (5+6(m-1), 5+6(m-1))
Thus the graph is cordial

Theorem 4.Mixed double path union G = $P_m(G_1,G_2)$ is cordial where $G_1 = C_4$ and G_2 = flag(C_3).

Proof: First we fuse C_4 and flag(C_3) to obtain the structure as follows. This structure is actually $P_1(G_1,G_2)$ and is fused at each vertex of path P_m. at vertex a on it.

Define a function f: V(G) → {0,1) .It produces labeled copy as in figure 4.6 below.

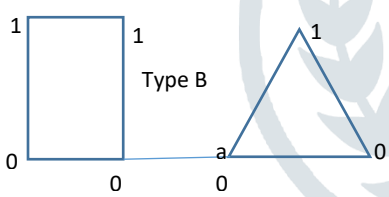

Fig 4.7 $v_f(0,1)$= (3,4), $e_f(0,1)$ = (4,4)

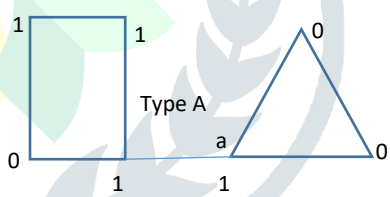

Fig 4.8 $v_f(0,1)$= (4,3), $e_f(0,1)$ = (4,4)

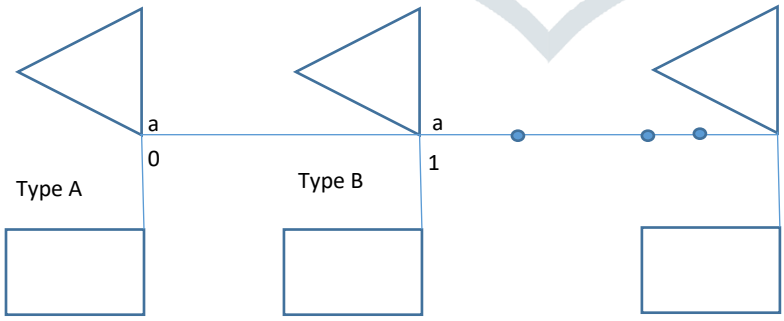

Fig 4.9 path union Pm(G1,G2) $G_1 = C_4$ and G_2 = flag(C_3).

We take a path Pm = ($v_1,v_2,\ldots v_m$). At vertex vi we fuse type A label if i ≡ 0,1 (mod 4) and type B label if i ≡ 2,3(mod 4).The fusion is taken at pont a on both type of labels.The label number distribution is $v_f(0,1)$ = (4+14x,3+14x) if m is of type 4x+1 , x =0,1,2..

$v_f(0,1) = (7x,7x)$ if m is of type 2x, x =1, 2, 3..

$v_f(0,1) = (10+14x,11+14x)$ if m is of type 3+4x, x = 0, 1, 2, 3..

$v_f(0,1) = (7x,7x)$ if m is of type 2x, x =1, 2, 3..

On edges we have $e_f(0,1) = (4+18x,4+18x)$ if m is of type 4x+1, x = 0, 1 , 2 ,..

$e_f(0,1) = (8+18x,9+18x)$ if m is of type 4x+2, x = 0, 1 , 2 ,..

$e_f(0,1) = (13+18x,13+18x)$ if m is of type 4x+3, x = 0, 1 , 2 ,..

$e_f(0,1) = (17+18x,18+18x)$ if m is of type 4x, x = 1 , 2 ,..

Thus the graph is cordial.

Conclusions In this paper we obtain path union on mixed graph.This family of graph is constructed by fusing copy each of G1 and G2 at fixed vertex of path vertex. This graph is denoted by $Pm(G_1,G_2)$.

We have proved that 1) Mixed double path union G = $P_m(G_1,G_2)$ is cordial where G_1 = C3 and G_2 = flag(C_3). 2) Mixed double path union G = $P_m(G_1,G_2)$ is cordial where G_1 = C4 and G_2 = flag(C_4).

3) Mixed double path union G = $P_m(G_1,G_2)$ is cordial where $G_1 = C_5$ and G_2 = flag(C_5).

4) Mixed double path union G = $P_m(G_1,G_2)$ is cordial where $G_1 = C_4$ and G_2 = flag(C_3).

Further it is necessary to investigate this type of families for general cases such as G1 and G2 are Cn etc.

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