PROPERTIES OF FUZZY MATCHING IN SET THEORY

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Abstract : The matching is a set of non-adjacent edges. In this paper, some new concept of properties like associative, distributive and Demorgan's law in set theory are applied in matching and complete matching on fuzzy labeling graph is introduced. A graph is said to be a complete fuzzy labeling graph if it has every pair of adjacent vertices of the fuzzy graph. Also we define the cardinality of matching and power set of fuzzy labeling graph.

Keywords: *Power set, Cardinal number, Associative Property, Distributive Law, Demorgan's law.*

I. INTRODUCTION

Many real world systems can be modeled using graphs. Graph represents the connections between the entities in these systems. The foundation for graph theory was laid in 1735 by Euler when he solved the Konigsberg bridge problem. Fuzzy graphs are generalization of graphs. Fuzzy graphs are countered in fuzzy set theory.

A fuzzy set was defined by L. A. Zadeh in 1965.Every element in the universal set is assigned a grade of membership ,a value in [0,1].The elements in the universal set along with their grades of membership form a fuzzy set.

In 1965 Fuzzy relations on a set was first defined by Zadeh.Among many branches of modern mathematics, the theory of sets (which was founded by G.Cantor) occupies a unique place.The mathematical concept of a set can be used as foundation for many branches of modern mathematics.

A matching is a set of edges which are non-adjacent. In this paper, some new concept of properties like associative, distributive and Demorgan's law in set theory are applied in matching and complete matching on fuzzy labeling graph .Here our results consider for fuzzy labeling graph.

II. PRELIMINARIES.

Definition: 2.1

A graph with n vertices in which every pair of distinct vertices is joined by a line is called **complete graph** on n vertices. It is denoted by K_n.

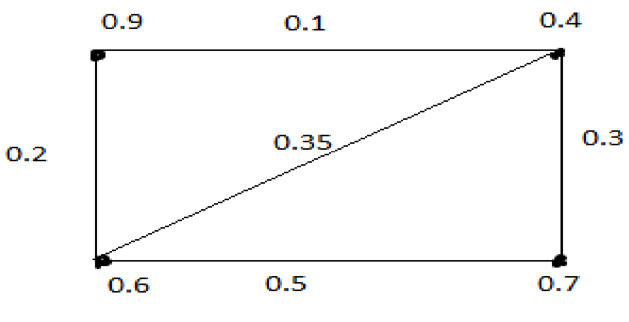
Definition 2.2

Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of U × V. A **fuzzy graph** G = (α, β) is a pair of functions $\alpha : V \to [0, 1]$ and $\beta: V \times V \to [0,1]$ where for all u,v ∈ V, we have $\beta(u,v) \le \min\{\alpha(u), \alpha(v)\}$.

Definition 2.3

A graph G = (α, β) is said to be a ***fuzzy labeling graph*** if $\alpha: V \to [0, 1]$ and $\beta: V \times V \to [0, 1]$ is a bijective such that the membership value of edges and vertices are distinct and $\beta(u, v) < \min\{\alpha(u), \alpha(v)\}$ for all u, v ∈ V.

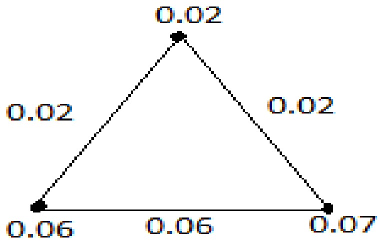
Example: 2.4



Definition: 2.5.

A fuzzy graph G = (α, β) is said to be ***complete*** if β (u, v) = min {α (u), α (v)} for all u, v ∈ V and every pair of vertices are adjacent. It is denoted by K_n[FLG].

Example 2.6



Definition: 2.7

The set of all subsets of a set A is called the power set of A and it is denoted by ρ (A).

Definition: 2.8

Let M_1 ad M_2 be two matchings in a fuzzy labeling graph with vertex set $\propto_1$ (v) ,$\propto_2$ (v) and edge set β_1 and β_2. *T*hen **Union of M_1 and M_2** ($M_1 \cup M_2$) consists of a vertex set($\alpha_1 \cup \alpha_2$)which is the subset of $\propto$(v) and edge set ($\beta_1 \cup \beta_2$) which is the subset of β.

Definition: 2.9

Let M_1 ad M_2 be two matchings in a fuzzy labeling graph with vertex set $\propto_1$ (v) ,$\propto_2$ (v) and edge set β_1 and β_2. *T*hen **intersection of M_1 and M_2** ($M_1 \cap M_2$) consists of a vertex set($\alpha_1 \cap \alpha_2$) which is the subset of $\propto$(v) and edge set ($\beta_1 \cap \beta_2$) which is the subset of β.

Definition: 2.10

Let M_1 ad M_2 be two matchings in a fuzzy labeling graph with vertex set $\propto_1$ (v) ,$\propto_2$ (v) and edge set β_1 and β_2. *T*hen **symmetric difference between M_1 and M_2** ($M_1 \Delta M_2$) is defined by ($M_1 - M_2$)U ($M_2 - M_1$) .

Note:

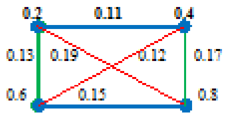
(i) The number of perfect matching in a complete graph K_{2n} = (2n) !/ 2^n n!.
(ii) For odd number of vertices, there exists no complete matching.

III . MAIN RESULTS

Definition 3.1

Let FLG be a fuzzy labeling graph and α and β be the vertex and edge set of a fuzzy labeling graph. The set of all complete matching in a fuzzy labeling graph is power set of fuzzy labeling graph. If a graph contains n vertices then the number of elements in the power set is K_{2n} = (2n) !/ 2^n n!.

Example: 3.2



(FLG)

Here the complete matchings are M_1, M_2 M_3.
M_1 ={ 0.11,0.15} , M_2 ={ 0.13,0.17} and M_3 ={ 0.19,0.12}
The power set is { M_1, M_2 M_3 }.

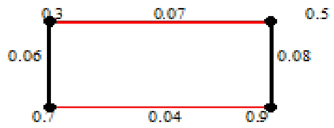
Definition 3.3

The number of edges in the complete matching is called the cardinality of the matching. In the above example3.2, cardinality of M_1, M_2 M_3 is 3.

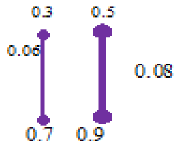
Definition: 3.4

Let M be matching which is a subset of the edge set β.The complement of M relative to β denoted by M^c is defined to be the set of all elements in β which are not in M.

Example: 3.5



(FLG)



M_1

Here $M_1^c = \{0.07, 0.04\}$.

Theorem: 3.6 (Distributive Law)

For any three complete matchings M_1, M_2, M_3 of fuzzy labeling graph we have

(i) $M_1 \cup (M_2 \cap M_3) = (M1 \cup M_2) \cap (M_1 \cup M_3)$.

(ii) $M_1 \cap (M_2 \cup M_3) = (M_1 \cap M_2) \cup (M_1 \cap M_3)$.

Proof:

(i) Let $e \in M_1 \cup (M_2 \cap M_3)$.

$Then$ $e \in M_1$ or $e \in M_2 \cap M_3$

$\therefore$ $e \in M_1$ or ($e \in M_2$ and $e \in M_3$).

$\therefore$ ($e \in M_1$ or $e \in M_2$) and ($e \in M_1$ or $e \in M_3$)

$\therefore$ $e \in (M_1 \cup M_2)$ and $e \in (M_1 \cup M_3)$.

$\therefore$ $e \in (M_1 \cup M_2) \cap (M_1 \cup M_3)$.

Hence $M_1 \cup (M_2 \cap M_3) \subset (M_1 \cup M_2) \cap (M_1 \cup M_3)$ ………………(1)

Now we take $e \in (M_1 \cup M_2) \cap (M_1 \cup M_3)$.

Then $e \in (M_1 \cup M_2)$ and $e \in (M_1 \cup M_3)$.

$\therefore$ ($e \in M_1$ or $e \in M_2$) and ($e \in M_1$ or $e \in M_3$)

$\therefore$ ($e \in M_1$ or ($e \in M_2$ and $e \in M_3$)

$\therefore$ $e \in M_1$ or ($e \in M_2 \cup M_3$)

$e \in M_1 \cup (M_2 \cap M_3)$.

Hence $(M_1 \cup M_2) \cap (M_1 \cup M_3) \subset M_1 \cup (M_2 \cap M_3)$ …………….(2)

From (1) and (2) we get $M_1 \cup (M_2 \cap M_3) = (M_1 \cup M_2) \cap (M_1 \cup M_3)$.

(ii) Let $e \in M_1 \cap (M_2 \cup M_3)$

Then $e \in M_1$ and ($e \in M_2$ or $e \in M_3$).

$\therefore$ ($e \in M_1$ and $e \in M_2$) or ($e \in M_1$ and $e \in M_3$)

$\therefore$ $e \in (M_1 \cap M_2)$ and $e \in (M_1 \cap M_3)$.

$\therefore$ $e \in (M_1 \cap M_2) \cup (M_1 \cap M_3)$.

Hence $M_1 \cap (M_2 \cup M_3) \subset (M_1 \cap M_2) \cup (M_1 \cap M_3)$ …………… (3)

Now we take $e \in (M_1 \cap M_2) \cup (M_1 \cap M_3)$

Then $e \in (M_1 \cap M_2)$ or $e \in (M_1 \cap M_3)$.

$\therefore$ ($e \in M_1$ and $e \in M_2$) or ($e \in M_1$ and $e \in M_3$)

$\therefore$ ($e \in M_1$ and ($e \in M_2$ or $e \in M_3$)

$\therefore$ $e \in M_1$ and ($e \in M_2 \cup M_3$)

$e \in M_1 \cap (M_2 \cup M_3)$.

Hence $(M_1 \cap M_2) \cup (M_1 \cap M_3 \subset M_1 \cap (M_2 \cup M_3)$ ……………(4)

From (3) and (4) we get

$$M_1 \cap (M_2 \cup M_3) = (M_1 \cap M_2) \cup (M_1 \cap M_3).$$

Theorem: 3.7 (Associative Law)

For any three complete matchings M_1, M_2, M_3 of fuzzy labeling graph we have
$M_1 \cup (M_2 \cup M_3) = (M_1 \cup M_2) \cup M_3$.

Proof:

Let $e \in M_1 \cup (M_2 \cup M_3)$. Then $e \in M_1$ or $e \in (M_2 \cup M_3)$.
$\therefore e \in M_1$ or ($e \in M_2$ or $e \in M_3$).
$\therefore$ ($e \in M_1$ or $e \in M_2$). or $e \in M_3$
$\therefore e \in (M_1 \cup M_2)$ or $e \in M_3$.
$\therefore e \in (M_1 \cup M_2) \cup M_3$.
Hence $M_1 \cup (M_2 \cup M_3) \subset (M_1 \cup M_2) \cup M_3$ …………(1)
Now we take $e \in (M_1 \cup M_2) \cup M_3$.
Then $e \in M_1$ or ($e \in M_2$ or $e \in M_3$).
$\therefore e \in M_1 \cup (M_2 \cup M_3)$.
Hence$(M_1 \cup M_2) \cup M_3 \subset M_1 \cup (M_2 \cup M_3)$ ……………(2)
From (1) and (2) we get

$$M_1 \cup (M_2 \cup M_3) = (M_1 \cup M_2) \cup M_3.$$

Theorem: 3.8 (Demorgan's Law)

For any two matchings M_1 and M_2 of a fuzzy labeling graph,

(i) $(M_1 \cup M_2)^c = M_1^c \cap M_2^c$

(ii) $(M_1 \cap M_2)^c = M_1^c \cup M_2^c$

Proof:

Let $e \in (M_1 \cup M_2)^c$
Then $e \notin (M_1 \cup M_2)$. $\therefore e \notin M_1$ and $e \notin M_2$
Hence $e \in M_1^c$ and $e \in M_2^c$
$\therefore e \in M_1^c \cap M_2^c$
Hence $(M_1 \cup M_2)^c \subseteq M_1^c \cap M_2^c$ ………………..(1)
Next we take $e \in M_1^c \cap M_2^c$
Hence $e \in M_1^c$ and $e \in M_2$
$\therefore e \notin M_1$ or $e \notin M_2$. Then $e \notin (M_1 \cup M_2)$.
Hence $e \in (M_1 \cup M_2)^c$
Hence $M_1^c \cap M_2^c \subseteq (M_1 \cup M_2)^c$ ………………..(2)
From (1) and (2) we get

$$(M_1 \cup M_2)^c = M_1^c \cap M_2^c$$

(ii) To prove $(M_1 \cap M_2)^c = M_1^c \cup M_2^c$
Let $e \in (M_1 \cap M_2)^c$
Then $e \notin (M_1 \cap M_2)$. $\therefore e \notin M_1$ and $e \notin M_2$
Hence $e \in M_1^c$ or $e \in M_2^c$. $\therefore e \in M_1^c \cup M_2^c$
Hence $(M_1 \cap M_2)^c \subseteq M_1^c \cup M_2^c$ ………………..(3)
Next we take $e \in M_1^c \cup M_2^c$
Hence $e \in M_1^c$ or $e \in M_2^c$
$\therefore e \notin M_1$ and $e \notin M_2$. Then $e \notin (M_1 \cap M_2)$.
Hence $e \in (M_1 \cap M_2)^c$. $\therefore$ $M_1^c \cup M_2^c \subseteq (M_1 \cap M_2)^c$ ………………..(4)
From (3) and (4) we get , $(M_1 \cap M_2)^c = M_1^c \cup M_2^c$

$$(M_1 \cup M_2)^c = M_1^c \cap M_2^c$$

Note: (Commutative Law)

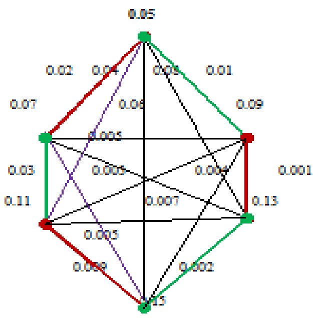
For any two matchings M_1 and M_2 of a fuzzy labeling graph, the following are also statisfied.

(i) $M_1 \cup M_2 = M_2 \cup M_1$

(ii) $M_1 \cap M_2 = M_2 \cap M_1$

Remark:

All the above results verified by using the following example.



K_6

Here the edge set is { 0.02, 0.04, 0.08,0.01, 0.06, 0.005, 0.03,0.003, 0.004,0.001, 0.007, 0.005, 0.009,0.002}
The following complete matching are exists
$M_1 = \{0.02, 0.009, 0.001\}$
$M_2 = \{0.03, 0.002, 0.01\}$
$M_3 = \{0.04, 0.005, 0.001\}$
$M_4 = \{0.06, 0.03, 0.001\}$
$M_5 = \{0.08, 0.005, 0.003\}$
$M_6 = \{ 0.03, 0.004, 0.08\}$
$M_7 = \{ 0.005, 0.007, 0.01\}$
Similarly we can find the remaining matching.

$M_1 \cup (M_3 \cap M_4) = \{ 0.001 , 0.02, 0.009\}$

$(M_1 \cup M_3) \cap (M_1 \cup M_4) = \{0.02, 0.009, 0.04, 0.005, 0.001\} \cap \{0.02, 0.009, 0.001, 0.06, 0.03\}$

$= \{ 0.001 , 0.02, 0.009\}$

Hence $M_1 \cup (M_2 \cap M_3) = (M1 \cup M_2) \cap (M_1 \cup M_3)$.
Similarly we can verified associative law, Demorgan's and commutative properties.

IV. CONCLUSION

Here, we defined the cardinality of matching and power set of fuzzy labeling graph. Some properties like associative, distributive and Demorgan's law in set theory are applied in matching and complete matching on fuzzy labeling graph.

V. REFERENCES

1. S. Arumugam, Issac.A.T, Modern Algebra
2. A. Nagoorgani and J. Malarvizhi, Isomorphism on fuzzy graphs, Inter. J. Comput. Math. Sci. (2008), 200-206.
3. Narsingh Deo, Graph Theory with applications to Engineering and Computer Science.
4. K .Ranganathan, R.Balakrishnan, A Text Book of Graph Theory, Springer.
5. A. Rosenfeld, Fuzzy graphs, Fuzzy Sets and their Applications to Cognitive and Decision Process, L. A. Zadeh, K. S. Fu, K. Tanaka and M. Shimura, Academic Press, New york, 1975, pp. 75-95.
6. M. S. Sunitha and A. Vijayakumar, Complement of fuzzy graph, Indian J. Pure Appl. Math. 33(9) (2002), 1451-1464.
7. S.Yahya Mohamed and S.Suganthi Energy of Complete Fuzzy Labeling Graph through Fuzzy Complete Matching,International Journal of Mathematics Trends and Technology (IJMTT) – Volume 58 Issue 3 – June 2018.
8. S.Yahya Mohamed and S.Suganthi Operations in Fuzzy Labeling Graph through Matching and Complete Matching, International Journal of Applied Engineering Research Volume 13, Number 12 (2018) .
9. S.Yahya Mohamed and S.Suganthi , Matching in Fuzzy Labeling Graph ,International Journal of fuzzy mathematical Archieve, Vol. 14, No. 1, 2017,
10. L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.