ON THE CONSTRUCTION OF FOURIER MATRICES OF ORDER 11 AND 13

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Abstract: In this paper we forward methods of construction of Fourier matrices of order 11 and 13 by the multiplication of suitable permutation matrix with suitable linear combination of adjacency matrices of suitable coherent configuration**.**

Index Terms **- Coherent configuration, Fourier matrices.**

I. INTRODUCTION

COHERENT CONFIGURATION (CC) is defined as a partition $C = \{C_1, C_2, C_2, ..., C_m\}$ of $X \times X$ where X is a non-empty finite with adjacency matrices $A_1, A_2, A_3, ..., A_m$ satisfying the following conditions:

(i) There exists a subset of $\{A_1, A_2, A_3, ..., A_m\}$ with sum $I_{|X|}$ = unit matrix;

(ii) The set $\{A_1, A_2, A_3, ..., A_m\}$ is closed under matrix transposition.

(iii) The product $A_i A_j$ for all $i, j \in \{1, 2, 3, ..., m\}$, is some linear combination of elements of the set $\{A_1, A_2, A_3, ..., A_m\}$ with non-negative integral coefficients.

(Vide: [9]).

A square matrix F_m of order $m \times m$ given by:

$$F_m = \begin{bmatrix} 1 & 1 & 1 & \dots & \dots & 1 \\ 1 & \omega & \omega^2 & \dots & \dots & \omega^{m-1} \\ 1 & \omega^2 & \omega^4 & \dots & \dots & \omega^{2(m-1)} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & \omega^{m-1} & \omega^{2(m-1)} & \dots & \dots & \omega^{(m-1)2} \end{bmatrix}$$

$$\text{where} \quad \omega = e^{(\frac{2\pi i}{m})}$$

is called Fourier matrix.

(Vide: [8]).

Fourier matrices are used in modular data and modular data is useful in rational conformal field theory. The rational field theory has some major applications in Physics. The modular data are also useful in fusion rings and $C-$algebras. (Vide: [1], [2], [8], [16] and [18])

There are many methods of the construction of Fourier Matrices and related theorems given in [10], [11] and [13**].**

II. MAIN WORKS:

In [4], [5], [6] and [7] methods of construction of weighing/conference matrices of order 6, 10 and 14 and 18 and 26 and 30 is given with the help of suitable Coherent Configuration. In [3] Manjhi and Kumar introduced methods of construction of Fourier matrices of order 2, 3, 5 and 7 with the help of adjacency matrices of suitable Coherent Configuration. In this paper we forward methods of construction of Fourier matrices of order 11 and 13 with the help of adjacency matrices of suitable Coherent Configuration.

2.1. Construction of Fourier Matrix of Orders 11:

Let us consider $X=\{i:i=1,2,3,4,5,6,7,8,9,10,11\}$ and a partition $C=\{C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8,C_9,C_{10},C_{11},C_{12},C_{13}\}$. Where: $C_1=\{(i,i):i=1\}$,

$C_2=\{(1,i):i=2,3,4,5,6,7,8,9,10,11,12,13\}$, $C_3=\{(i,1):i=2,3,4,5,6,7,8,9,10,11,12,13\}$,

$C_4=\{(i,i):i=2,3,4,5,6,7,8,9,10,11,12,13\}$,

$C_5=\{\{(2,i):i=7\}\bigcup\{(3,i):i=2\}\bigcup\{(4,i):i=8\}\bigcup\{(5,i):i=3\}\bigcup\{(6,i):i=9\}\bigcup\{(7,i):i=4\}\bigcup\{(8,i):i=10\}$
$\bigcup\{(9,i):i=5\}\bigcup\{(10,i):i=11\}\bigcup\{(11,i):i=6\}\}$.

$C_6=\{\{(2,i):i=5\}\bigcup\{(3,i):i=9\}\bigcup\{(4,i):i=2\}\bigcup\{(5,i):i=6\}\bigcup\{(6,i):i=10\}\bigcup\{(7,i):i=3\}\bigcup\{(8,i):i=7\}$
$\bigcup\{(9,i):i=11\}\bigcup\{(10,i):i=4\}\bigcup\{(11,i):i=8\}\}$.

$C_7=\{\{(2,i):i=4\}\bigcup\{(3,i):i=7\}\bigcup\{(4,i):i=10\}\bigcup\{(5,i):i=2\}\bigcup\{(6,i):i=5\}\bigcup\{(7,i):i=8\}\bigcup\{(8,i):i=11\}$
$\bigcup\{(9,i):i=3\}\bigcup\{(10,i):i=6\}\bigcup\{(11,i):i=9\}\}$.

$C_8=\{\{(2,i):i=10\}\bigcup\{(3,i):i=8\}\bigcup\{(4,i):i=6\}\bigcup\{(5,i):i=4\}\bigcup\{(6,i):i=2\}\bigcup\{(7,i):i=11\}\bigcup\{(8,i):i=9\}$
$\bigcup\{(9,i):i=7\}\bigcup\{(10,i):i=5\}\bigcup\{(11,i):i=3\}\}$.

$C_9=\{\{(2,i):i=3\}\bigcup\{(3,i):i=5\}\bigcup\{(4,i):i=7\}\bigcup\{(5,i):i=9\}\bigcup\{(6,i):i=11\}\bigcup\{(7,i):i=2\}\bigcup\{(8,i):i=4\}$
$\bigcup\{(9,i):i=6\}\bigcup\{(10,i):i=8\}\bigcup\{(11,i):i=10\}\}$.

$C_{10}=\{\{(2,i):i=9\}\bigcup\{(3,i):i=6\}\bigcup\{(4,i):i=3\}\bigcup\{(5,i):i=11\}\bigcup\{(6,i):i=8\}\bigcup\{(7,i):i=5\}\bigcup\{(8,i):i=2\}$
$\bigcup\{(9,i):i=10\}\bigcup\{(10,i):i=7\}\bigcup\{(11,i):i=4\}\}$.

$C_{11}=\{\{(2,i):i=8\}\bigcup\{(3,i):i=4\}\bigcup\{(4,i):i=11\}\bigcup\{(5,i):i=7\}\bigcup\{(6,i):i=3\}\bigcup\{(7,i):i=10\}\bigcup\{(8,i):i=6\}$
$\bigcup\{(9,i):i=2\}\bigcup\{(10,i):i=9\}\bigcup\{(11,i):i=5\}\}$

$C_{12}=\{\{(2,i):i=6\}\bigcup\{(3,i):i=11\}\bigcup\{(4,i):i=5\}\bigcup\{(5,i):i=10\}\bigcup\{(6,i):i=4\}\bigcup\{(7,i):i=9\}\bigcup\{(8,i):i=3\}$
$\bigcup\{(9,i):i=8\}\bigcup\{(10,i):i=2\}\bigcup\{(11,i):i=7\}\}$.

$C_{13}=\{\{(2,i):i=11\}\bigcup\{(3,i):i=10\}\bigcup\{(4,i):i=9\}\bigcup\{(5,i):i=8\}\bigcup\{(6,i):i=7\}\bigcup\{(7,i):i=6\}\bigcup\{(8,i):i=5\}$
$\bigcup\{(9,i):i=4\}\bigcup\{(10,i):i=3\}\bigcup\{(11,i):i=2\}\}$.

Then adjacency matrices $A_1,A_2,A_3,A_4,A_5,A_7,A_8,A_9,A_{10},A_{11},A_{12}\ and\ A_{13}$ of $C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8,C_9,C_{10},C_{11},C_{12}\ and\ C_{13}$ respectively are given below:

$$A_1 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&1&0&0&0&0&0 \end{bmatrix}, \quad A_6 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \end{bmatrix}$$

$$A_7 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \end{bmatrix}, A_8 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$A_9 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \end{bmatrix}, A_{10} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \end{bmatrix}$$

$$A_{11} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \end{bmatrix}, A_{12} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \end{bmatrix}$$

$$A_{13} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

We see that:

$(i).\ A_1 + A_4 = I_{11}.$

$(ii).\ A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9 + A_{10} + A_{11} + A_{12} + A_{13} = J_{11}.$

$(iii).\ A_1^{'} = A_1, A_2^{'} = A_3, A_3^{'} = A_2, A_4^{'} = A_4, A_5^{'} = A_9, A_6^{'} = A_7, A_7^{'} = A_6, A_8^{'} = A_{12},$

$A_9^{'} = A_5, A_{10}^{'} = A_{11}, A_{11}^{'} = A_{10}, A_{12}^{'} = A_8, A_{13}^{'} = A_{13}.$

We see the following calculations:

$(i).\ A_1^2 = A_1, A_1A_2 = A_2, A_2A_1 = 0, A_1A_3 = 0, A_3A_1 = A_3, A_1A_4 = 0, A_4A_1 = 0, A_1A_5 = 0, A_5A_1 = 0, A_1A_6 = 0, A_6A_1 = 0,$
$A_1A_7 = 0, A_7A_1 = 0, A_1A_8 = 0, A_8A_1 = 0, A_1A_9 = 0, A_9A_1 = 0, A_1A_{10} = 0, A_{10}A_1 = 0, A_1A_{11} = 0, A_{11}A_1 = 0, A_1A_{12} = 0,$
$A_{12}A_1 = 0, A_1A_{13} = 0, A_{13}A_1 = 0.$

$(ii).\ A_2^2 = 0, A_2A_3 = 10A_1, A_3A_2 = A_4 + A_5 + A_6 + A_7 + A_8 + A_9 + A_{10} + A_{11} + A_{12} + A_{13}, A_2A_4 = A_2, A_4A_2 = 0,$
$A_2A_5 = A_2, A_5A_2 = 0, A_2A_6 = A_2, A_6A_2 = 0, A_2A_7 = A_2, A_7A_2 = 0, A_2A_8 = A_2, A_8A_2 = 0, A_2A_9 = A_2, A_9A_2 = 0,$
$A_2A_{10} = A_2, A_{10}A_2 = 0, A_2A_{11} = A_2, A_{11}A_2 = 0, A_2A_{12} = A_2, A_{12}A_2 = 0, A_2A_{13} = A_2, A_{13}A_2 = 0.$

$(iii).\ A_3^2 = 0, A_3A_4 = 0, A_4A_3 = A_3, A_3A_5 = 0, A_5A_3 = A_3, A_3A_6 = 0, A_6A_3 = A_3, A_3A_7 = 0, A_7A_3 = A_3, A_3A_8 = 0,$
$A_8A_3 = A_3, A_3A_9 = 0, A_9A_3 = A_3, A_3A_{10} = 0, A_{10}A_3 = A_3, A_3A_{11} = 0, A_{11}A_3 = A_3, A_3A_{12} = 0, A_{12}A_3 = A_3, A_3A_{13} = 0,$
$A_{13}A_3 = A_3.$

$(iv).\ A_4^2 = A_4, A_4A_5 = A_5, A_5A_4 = A_5, A_4A_6 = A_6, A_6A_4 = A_6, A_4A_7 = A_7, A_7A_4 = A_7, A_4A_8 = A_8, A_8A_4 = A_8, A_4A_9 = A_9,$
$A_9A_4 = A_9, A_4A_{10} = A_{10}, A_{10}A_4 = A_{10}, A_4A_{11} = A_{11}, A_{11}A_4 = A_{11}, A_4A_{12} = A_{12}, A_{12}A_4 = A_{12}, A_4A_{13} = A_{13}, A_{13}A_4 = A_{13}.$

$(v).\ A_5^2 = A_7, A_5A_6 = A_9, A_6A_5 = A_9, A_5A_7 = A_{11}, A_7A_5 = A_{11}, A_5A_8 = A_{13}, A_8A_5 = A_{13}, A_5A_9 = A_4, A_9A_5 = A_4,$
$A_5A_{10} = A_6, A_{10}A_5 = A_6, A_5A_{11} = A_8, A_{11}A_5 = A_8, A_5A_{12} = A_{10}, A_{12}A_5 = A_{10}, A_5A_{13} = A_{12}, A_{13}A_5 = A_{12}.$

$(vi).\ A_6^2 = A_{12}, A_6A_7 = A_4, A_7A_6 = A_4, A_6A_8 = A_7, A_8A_6 = A_7, A_6A_9 = A_{10}, A_9A_6 = A_{10}, A_6A_{10} = A_{13}, A_{10}A_6 = A_{13},$
$A_6A_{11} = A_5, A_{11}A_6 = A_5, A_6A_{12} = A_8, A_{12}A_6 = A_8, A_6A_{13} = A_{11}, A_{13}A_6 = A_{11}.$

$(vii).\ A_7^2 = A_8, A_7A_8 = A_{12}, A_8A_7 = A_{12}, A_7A_9 = A_5, A_9A_7 = A_5, A_7A_{10} = A_9, A_{10}A_7 = A_9, A_7A_{11} = A_{13}, A_{11}A_7 = A_{13},$
$A_7A_{12} = A_6, A_{12}A_7 = A_6, A_7A_{13} = A_{10}, A_{13}A_7 = A_{10}.$

$(viii).\ A_8^2 = A_6, A_8A_9 = A_{11}, A_9A_8 = A_{11}, A_8A_{10} = A_5, A_{10}A_8 = A_5, A_8A_{11} = A_{10}, A_{11}A_8 = A_{10}, A_8A_{12} = A_4,$
$A_{12}A_8 = A_4, A_8A_{13} = A_9, A_{13}A_8 = A_9.$

$(ix).\ A_9^2 = A_6, A_9A_{10} = A_{12}, A_{10}A_9 = A_{12}, A_9A_{11} = A_7, A_{11}A_9 = A_7, A_9A_{12} = A_{13}, A_{12}A_9 = A_{13}, A_9A_{13} = A_8, A_{13}A_9 = A_8.$

$(x).\ A_{10}^2 = A_8, A_{10}A_{11} = A_4, A_{11}A_{10} = A_4, A_{10}A_{12} = A_{11}, A_{12}A_{10} = A_{11}, A_{10}A_{13} = A_7, A_{13}A_{10} = A_7.$

$(xi).\ A_{11}^2 = A_{12}, A_{11}A_{12} = A_9, A_{12}A_{11} = A_9, A_{11}A_{13} = A_6, A_{13}A_{11} = A_6.$ $(xii).\ A_{12}^2 = A_7, A_{12}A_{13} = A_5, A_{13}A_{12} = A_5.$
$(xiii).\ A_{13}^2 = A_4.$

Hence, product of any two adjacency matrices is some linear combinations of adjacency matrices. Thus, the set $C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}\}$ is Coherent Configuration. Consider the matrix

$$F_{11} = 1.A_1 + 1.A_2 + 1.A_3 + \omega.A_4 + \omega^2 A_5 + \omega^3 A_6 + \omega^4 A_7 + \omega^5 A_8 + \omega^6 A_9 + \omega^7 A_{10} + \omega^8 A_{11} + \omega^9 A_{12} + \omega^{10} A_{13}$$

$$F_{11} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^6 & \omega^4 & \omega^3 & \omega^9 & \omega^2 & \omega^8 & \omega^7 & \omega^5 & \omega^{10} \\ 1 & \omega^2 & \omega & \omega^8 & \omega^6 & \omega^7 & \omega^4 & \omega^5 & \omega^3 & \omega^{10} & \omega^9 \\ 1 & \omega^3 & \omega^7 & \omega & \omega^9 & \omega^5 & \omega^6 & \omega^2 & \omega^{10} & \omega^4 & \omega^8 \\ 1 & \omega^4 & \omega^2 & \omega^5 & \omega & \omega^3 & \omega^8 & \omega^{10} & \omega^6 & \omega^9 & \omega^7 \\ 1 & \omega^5 & \omega^8 & \omega^9 & \omega^4 & \omega & \omega^{10} & \omega^7 & \omega^2 & \omega^3 & \omega^6 \\ 1 & \omega^6 & \omega^3 & \omega^2 & \omega^7 & \omega^{10} & \omega & \omega^4 & \omega^9 & \omega^8 & \omega^5 \\ 1 & \omega^7 & \omega^9 & \omega^6 & \omega^{10} & \omega^8 & \omega^3 & \omega & \omega^5 & \omega^2 & \omega^4 \\ 1 & \omega^8 & \omega^4 & \omega^{10} & \omega^2 & \omega^6 & \omega^5 & \omega^9 & \omega & \omega^7 & \omega^3 \\ 1 & \omega^9 & \omega^{10} & \omega^3 & \omega^5 & \omega^4 & \omega^7 & \omega^6 & \omega^8 & \omega & \omega^2 \\ 1 & \omega^{10} & \omega^5 & \omega^7 & \omega^8 & \omega^2 & \omega^9 & \omega^3 & \omega^4 & \omega^6 & \omega \end{bmatrix}$$

Where: $\exp(2\pi i/11)$ .So $w^{11} = 1$. Which is equivalent to Fourier matrix of order 11.

2.2. Construction of Fourier Matrix of Orders 13:

Let us consider $X = \{i, i: 1,2,3,4,5,6,7,8,9,10,11,12,13\}$ and a partition $C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}\}$ of $X \times X$. Where:

$C_1 = \{(i,i): i = 1\}$, $C_2 = \{(1.i): i = 2,3,4,5,6,7,8,9,10,11,12,13\}$,

$C_3 = \{(i,1): i = 2,3,4,5,6,7,8,9,10,11,12,13\}$, $C_4 = \{(i,i): i = 2,3,4,5,6,7,8,9,10,11,12,13\}$

$C_5 = \{\{(2,i): i = 8\} \cup \{(3,i): i = 2\} \cup \{(4,i): i = 9\} \cup \{(5,i): i = 3\} \cup \{(6,i): i = 10\} \cup \{(7,i): i = 4\} \cup \{(8,i): i = 11\}$
$\cup \{(9,i): i = 5\} \cup \{(10,i): i = 12\} \cup \{(11,i): i = 6\} \cup \{(12.i): i = 13\} \cup \{(13,i): i = 7\}\}.$

$C_6 = \{\{(2,i): i = 10\} \cup \{(3,i): i = 6\} \cup \{(4,i): i = 2\} \cup \{(5,i): i = 11\} \cup \{(6,i): i = 7\} \cup \{(7,i): i = 3\} \cup \{(8,i): i = 12\}$
$\cup \{(9,i): i = 8\} \cup \{(10,i): i = 4\} \cup \{(11,i): i = 13\} \cup \{(12.i): i = 9\} \cup \{(13,i): i = 5\}\}.$

$C_7 = \{\{(2,i): i = 11\} \cup \{(3,i): i = 8\} \cup \{(4,i): i = 5\} \cup \{(5,i): i = 2\} \cup \{(6,i): i = 12\} \cup \{(7,i): i = 9\} \cup \{(8,i): i = 6\}$
$\cup \{(9,i): i = 3\} \cup \{(10,i): i = 13\} \cup \{(11,i): i = 10\} \cup \{(12.i): i = 7\} \cup \{(13,i): i = 4\}\}.$

$C_8 = \{\{(2,i): i = 9\} \cup \{(3,i): i = 4\} \cup \{(4,i): i = 12\} \cup \{(5,i): i = 7\} \cup \{(6,i): i = 2\} \cup \{(7,i): i = 10\} \cup \{(8,i): i = 5\}$
$\cup \{(9,i): i = 13\} \cup \{(10,i): i = 8\} \cup \{(11,i): i = 3\} \cup \{(12.i): i = 11\} \cup \{(13,i): i = 6\}\}.$

$C_9 = \{\{(2,i): i = 12\} \cup \{(3,i): i = 10\} \cup \{(4,i): i = 8\} \cup \{(5,i): i = 6\} \cup \{(6,i): i = 4\} \cup \{(7,i): i = 2\} \cup \{(8,i): i = 13\}$
$\cup \{(9,i): i = 11\} \cup \{(10,i): i = 9\} \cup \{(11,i): i = 7\} \cup \{(12.i): i = 5\} \cup \{(13,i): i = 3\}\}.$

$C_{10} = \{\{(2,i): i = 3\} \cup \{(3,i): i = 5\} \cup \{(4,i): i = 7\} \cup \{(5,i): i = 9\} \cup \{(6,i): i = 11\} \cup \{(7,i): i = 13\} \cup \{(8,i): i = 2\}$
$\cup \{(9,i): i = 4\} \cup \{(10,i): i = 6\} \cup \{(11,i): i = 8\} \cup \{(12.i): i = 10\} \cup \{(13,i): i = 12\}\}.$

$C_{11} = \{\{(2,i): i = 6\} \cup \{(3,i): i = 11\} \cup \{(4,i): i = 3\} \cup \{(5,i): i = 8\} \cup \{(6,i): i = 13\} \cup \{(7,i): i = 5\} \cup \{(8,i): i = 10\}$
$\cup \{(9,i): i = 2\} \cup \{(10,i): i = 7\} \cup \{(11,i): i = 12\} \cup \{(12.i): i = 4\} \cup \{(13,i): i = 9\}\}.$

$C_{12} = \{\{(2,i): i = 4\} \cup \{(3,i): i = 7\} \cup \{(4,i): i = 10\} \cup \{(5,i): i = 13\} \cup \{(6,i): i = 3\} \cup \{(7,i): i = 6\} \cup \{(8,i): i = 9\}$
$\cup \{(9,i): i = 12\} \cup \{(10,i): i = 2\} \cup \{(11,i): i = 5\} \cup \{(12.i): i = 8\} \cup \{(13,i): i = 11\}\}.$

$C_{13} = \{\{(2,i): i = 5\} \cup \{(3,i): i = 9\} \cup \{(4,i): i = 13\} \cup \{(5,i): i = 4\} \cup \{(6,i): i = 8\} \cup \{(7,i): i = 12\} \cup \{(8,i): i = 3\}$
$\cup \{(9,i): i = 7\} \cup \{(10,i): i = 11\} \cup \{(11,i): i = 2\} \cup \{(12.i): i = 6\} \cup \{(13,i): i = 10\}\}.$

$C_{14}=\{\{(2,i):i=7\}\bigcup\{(3,i):i=13\}\bigcup\{(4,i):i=6\}\bigcup\{(5,i):i=12\}\bigcup\{(6,i):i=5\}\bigcup\{(7,i):i=11\}\bigcup\{(8,i):i=4\}$
$\bigcup\{(9,i):i=10\}\bigcup\{(10,i):i=3\}\bigcup\{(11,i):i=9\}\bigcup\{(12.i):i=2\}\bigcup\{(13,i):i=8\}\}.$
$C_{15}=\{\{(2,i):i=13\}\bigcup\{(3,i):i=12\}\bigcup\{(4,i):i=11\}\bigcup\{(5,i):i=10\}\bigcup\{(6,i):i=9\}\bigcup\{(7,i):i=8\}\bigcup\{(8,i):i=7\}$
$\bigcup\{(9,i):i=6\}\bigcup\{(10,i):i=5\}\bigcup\{(11,i):i=4\}\bigcup\{(12.i):i=3\}\bigcup\{(13,i):i=2\}\}.$

Then adjacency matrices $A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9,A_{10},A_{11},A_{12},A_{13},A_{14}\ and\ A_{15}$ of $C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8,C_9,C_{10},C_{11},C_{12},C_{13},C_{14}and\,C_{15}$ respectively are given below:

$$A_1=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix},\ A_2=\begin{bmatrix}0&1&1&1&1&1&1&1&1&1&1&1&1\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix},$$

$$A_3=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix},\ A_4=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&0&0&0&0&1\end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \end{bmatrix}, A_6 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \end{bmatrix}, A_8 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \end{bmatrix},$$

$$A_9 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}, A_{10} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \end{bmatrix},$$

$$A_{11}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&1&0&0\\0&0&1&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&1\\0&0&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&1&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&1&0\\0&0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0&0\end{bmatrix},\ A_{12}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&1\\0&0&1&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&1&0\\0&1&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&1&0&0\end{bmatrix},$$

$$A_{13}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&1\\0&0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&1&0\\0&0&1&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&1&0&0&0\end{bmatrix},\ A_{14}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&1\\0&0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&1&0\\0&0&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&1&0&0\\0&0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&1&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0&0\end{bmatrix},$$

$$A_{15}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}$$

We see that:

$(i).\ A_1 + A_4 = I_{13}.$

$(ii).\ A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9 + A_{10} + A_{11} + A_{12} + A_{13} + A_{14} + A_{15} = J_{13}.$

$(ii).\ A_1^{'} = A_1, A_2^{'} = A_3, A_3^{'} = A_2, A_4^{'} = A_4, A_5^{'} = A_{10}, A_6^{'} = A_{12}, A_7^{'} = A_{13}, A_8^{'} = A_{11}, A_9^{'} = A_{14},$

$A_{10}^{'} = A_5, A_{11}^{'} = A_8, A_{12}^{'} = A_6, A_{13}^{'} = A_7, A_{14}^{'} = A_9, A_{15}^{'} = A_{15}.$

We see the following calculations:

$(i).\ A_1^2 = A_1, A_1A_2 = A_2, A_2A_1 = 0, A_1A_3 = 0, A_3A_1 = A_3, A_1A_4 = 0, A_4A_1 = 0, A_1A_5 = 0, A_5A_1 = 0, A_1A_6 = 0, A_6A_1 = 0,$

$A_1A_7 = 0, A_7A_1 = 0, A_1A_8 = 0, A_8A_1 = 0, A_1A_9 = 0, A_9A_1 = 0, A_1A_{10} = 0, A_{10}A_1 = 0, A_1A_{11} = 0, A_{11}A_1 = 0,$

$A_1A_{12} = 0, A_{12}A_1 = 0, A_1A_{13} = 0, A_{13}A_1 = 0, A_1A_{14} = 0, A_{14}A_1 = 0, A_1A_{15} = 0, A_{15}A_1 = 0.$

$(ii).\ A_2^2 = 0, A_2A_3 = 12A_1, A_3A_2 = A_4 + A_5 + A_6 + A_7 + A_8 + A_9 + A_{10} + A_{11} + A_{12} + A_{13} + A_{14} + A_{15}, A_2A_4 = A_2,$

$A_4A_2 = 0, A_2A_5 = A_2, A_5A_2 = 0, A_2A_6 = A_2, A_6A_2 = 0, A_2A_7 = A_2, A_7A_2 = 0, A_2A_8 = A_2, A_8A_2 = 0, A_2A_9 = A_2, A_9A_2 = 0,$

$A_2A_{10} = A_2, A_{10}A_2 = 0, A_2A_{11} = A_2, A_{11}A_2 = 0, A_2A_{12} = A_2, A_{12}A_2 = 0, A_2A_{13} = A_2, A_{13}A_2 = 0, A_2A_{14} = A_2, A_{14}A_2 = 0,$

$A_2A_{15} = A_2, A_{15}A_2 = 0.$

$(iii).\ A_3^2 = 0, A_3A_4 = 0, A_4A_3 = A_3, A_3A_5 = 0, A_5A_3 = A_3, A_3A_6 = 0, A_6A_3 = A_3, A_3A_7 = 0, A_7A_3 = A_3, A_3A_8 = 0,$

$A_8A_3 = A_3, A_3A_9 = 0, A_9A_3 = A_3, A_3A_{10} = 0, A_{10}A_3 = A_3, A_3A_{11} = 0, A_{11}A_3 = A_3, A_3A_{12} = 0, A_{12}A_3 = A_3, A_3A_{13} = 0,$

$A_{13}A_3 = A_3, A_3A_{14} = 0, A_{14}A_3 = A_3.A_3A_{15} = 0, A_{15}A_3 = A_3.$

$(iv).\ A_4^2 = A_4, A_4A_5 = A_5, A_5A_4 = A_5, A_4A_6 = A_6, A_6A_4 = A_6, A_4A_7 = A_7, A_7A_4 = A_7, A_4A_8 = A_8, A_8A_4 = A_8, A_4A_9 = A_9,$

$A_9A_4 = A_9, A_4A_{10} = A_{10}, A_{10}A_4 = A_{10}, A_4A_{11} = A_{11}, A_{11}A_4 = A_{11}, A_4A_{12} = A_{12}, A_{12}A_4 = A_{12}, A_4A_{13} = A_{13},$

$A_{13}A_4 = A_{13}, A_4A_{14} = A_{14}, A_{14}A_4 = A_{14}, A_4A_{15} = A_{15}, A_{15}A_4 = A_{15}.$

$(v).\ A_5^2 = A_7, A_5A_6 = A_9, A_6A_5 = A_9, A_5A_7 = A_{11}, A_7A_5 = A_{11}, A_5A_8 = A_{13}, A_8A_5 = A_{13}, A_5A_9 = A_{15}, A_9A_5 = A_{15},$

$A_5A_{10} = A_4, A_{10}A_5 = A_4, A_5A_{11} = A_6, A_{11}A_5 = A_6, A_5A_{12} = A_8, A_{12}A_5 = A_8, A_5A_{13} = A_{10}, A_{13}A_5 = A_{10}, A_5A_{14} = A_{12},$

$A_{14}A_5 = A_{12}, A_5A_{15} = A_{14}, A_{15}A_5 = A_{14}.$

$(vi).\ A_6^2 = A_{12}, A_6A_7 = A_{15}, A_7A_6 = A_{15}, A_6A_8 = A_5, A_8A_6 = A_5, A_6A_9 = A_8, A_9A_6 = A_8, A_6A_{10} = A_{11}, A_{10}A_6 = A_{11},$

$A_6A_{11} = A_{14}, A_{11}A_6 = A_{14}, A_6A_{12} = A_4, A_{12}A_6 = A_4, A_6A_{13} = A_7, A_{13}A_6 = A_7, A_6A_{14} = A_{10}, A_{14}A_6 = A_{10}, A_6A_{15} = A_{13},$

$A_{15}A_6 = A_{13}.$

$(vii).\ A_7^2 = A_6, A_7A_8 = A_{10}, A_8A_7 = A_{10}, A_7A_9 = A_{14}, A_9A_7 = A_{14}, A_7A_{10} = A_5, A_{10}A_7 = A_5, A_7A_{11} = A_9, A_{11}A_7 = A_9,$

$A_7A_{12} = A_{13}, A_{12}A_7 = A_{13}, A_7A_{13} = A_4, A_{13}A_7 = A_4, A_7A_{14} = A_8, A_{14}A_7 = A_8, A_7A_{15} = A_{12}, A_{15}A_7 = A_{12}.$

$(viii).\ A_8^2 = A_{15}, A_8A_9 = A_7, A_9A_8 = A_7, A_8A_{10} = A_{12}, A_{10}A_8 = A_{12}, A_8A_{11} = A_4, A_{11}A_8 = A_4, A_8A_{12} = A_9, A_{12}A_8 = A_9,$

$A_8A_{13} = A_{14}, A_{13}A_8 = A_{14}, A_8A_{14} = A_6, A_{14}A_8 = A_6, A_8A_{15} = A_{11}, A_{15}A_8 = A_{11}.$

$(ix).\ A_9^2 = A_{13}, A_9A_{10} = A_6, A_{10}A_9 = A_6, A_9A_{11} = A_{12}, A_{11}A_9 = A_{12}, A_9A_{12} = A_5, A_{12}A_9 = A_5, A_9A_{13} = A_{11}, A_{13}A_9 = A_{11},$

$A_9A_{14} = A_4, A_{14}A_9 = A_4, A_9A_{15} = A_{10}, A_{15}A_9 = A_{10}.$

$(x).\ A_{10}^2 = A_{13}, A_{10}A_{11} = A_7, A_{11}A_{10} = A_7, A_{10}A_{12} = A_{14}, A_{12}A_{10} = A_{14}, A_{10}A_{13} = A_8, A_{13}A_{10} = A_8, A_{10}A_{14} = A_{15},$

$A_{14}A_{10} = A_{15}, A_{10}A_{15} = A_9, A_{15}A_{10} = A_9.$

$(xii).\ A_{11}^2 = A_{15}, A_{11}A_{12} = A_{10}, A_{12}A_{11} = A_{10}, A_{11}A_{13} = A_5, A_{13}A_{11} = A_5, A_{11}A_{14} = A_{13}, A_{14}A_{11} = A_{13}, A_{11}A_{15} = A_8,$

$A_{15}A_{11} = A_8.$

$(xii).\ A_{12}^2 = A_6, A_{12}A_{13} = A_{15}, A_{13}A_{12} = A_{15}, A_{12}A_{14} = A_{11}, A_{14}A_{12} = A_{11}, A_{12}A_{15} = A_7, A_{15}A_{12} = A_7.$

$(xiii).\ A_{13}^2 = A_{12}, A_{13}A_{14} = A_9, A_{14}A_{13} = A_9, A_{13}A_{15} = A_6, A_{15}A_{13} = A_6.$ $(xiv).\ A_{14}^2 = A_7, A_{14}A_{15} = A_5, A_{15}A_{14} = A_5.$ $(xv).\ A_{15}^2 = A_4.$

Hence, product of any two adjacency matrices is some linear combinations of adjacency matrices. Thus the set $C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}\}$ is Coherent Configuration. Consider the matrix

$$F_{13} = A_1 + A_2 + A_3 + \omega A_4 + \omega^2 A_5 + \omega^3 A_6 + \omega^4 A_7 + \omega^5 A_8 + \omega^6 A_9 + \omega^7 A_{10} + \omega^8 A_{11} + \omega^9 A_{12} + \omega^{10} A_{13} + \omega^{11} A_{14} + \omega^{12} A_{15}.$$

$$F_{13} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^7 & \omega^9 & \omega^{10} & \omega^8 & \omega^{11} & \omega^2 & \omega^5 & \omega^3 & \omega^4 & \omega^6 & \omega^{12} \\ 1 & \omega^2 & \omega & \omega^5 & \omega^7 & \omega^3 & \omega^9 & \omega^4 & \omega^{10} & \omega^6 & \omega^8 & \omega^{12} & \omega^{11} \\ 1 & \omega^3 & \omega^8 & \omega & \omega^4 & \omega^{11} & \omega^7 & \omega^6 & \omega^2 & \omega^9 & \omega^{12} & \omega^5 & \omega^{10} \\ 1 & \omega^4 & \omega^2 & \omega^{10} & \omega & \omega^6 & \omega^5 & \omega^8 & \omega^7 & \omega^{12} & \omega^3 & \omega^{11} & \omega^9 \\ 1 & \omega^5 & \omega^9 & \omega^6 & \omega^{11} & \omega & \omega^3 & \omega^{10} & \omega^{12} & \omega^2 & \omega^7 & \omega^4 & \omega^8 \\ 1 & \omega^6 & \omega^3 & \omega^2 & \omega^8 & \omega^9 & \omega & \omega^{12} & \omega^4 & \omega^5 & \omega^{11} & \omega^{10} & \omega^7 \\ 1 & \omega^7 & \omega^{10} & \omega^{11} & \omega^5 & \omega^4 & \omega^{12} & \omega & \omega^9 & \omega^8 & \omega^2 & \omega^3 & \omega^6 \\ 1 & \omega^8 & \omega^4 & \omega^7 & \omega^2 & \omega^{12} & \omega^{10} & \omega^3 & \omega & \omega^{11} & \omega^6 & \omega^9 & \omega^5 \\ 1 & \omega^9 & \omega^{11} & \omega^3 & \omega^{12} & \omega^7 & \omega^8 & \omega^5 & \omega^6 & \omega & \omega^{10} & \omega^2 & \omega^4 \\ 1 & \omega^{10} & \omega^5 & \omega^{12} & \omega^9 & \omega^2 & \omega^6 & \omega^7 & \omega^{11} & \omega^4 & \omega & \omega^8 & \omega^3 \\ 1 & \omega^{11} & \omega^{12} & \omega^8 & \omega^6 & \omega^{10} & \omega^4 & \omega^9 & \omega^3 & \omega^7 & \omega^5 & \omega & \omega^2 \\ 1 & \omega^{12} & \omega^6 & \omega^4 & \omega^3 & \omega^5 & \omega^2 & \omega^{11} & \omega^8 & \omega^{10} & \omega^9 & \omega^7 & \omega \end{bmatrix},$$

Where: $\exp(2\pi i/13)$. So $w^{13} = 1$. Which is equivalent to Fourier matrix of order 13.

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