A BRIEF STUDY ON DOMINATION NUMBER OF A NON-ZERO ZERO DIVISOR GRAPH FORMED FROM THE CARTESIAN PRODUCT OF TWO STAR ZERO DIVISOR GRAPHS

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Abstract:
In this paper, we have established the domination number of cartesian product of two non-zero zero divisor graph.Also we studied the bounds for domination of complement of a zero divisor graph.

Key Words:
Zero divisor graph,Complement of a zero divisor graph,star graph,Cartesian product of zero divisor graphs,Domination number.

1. Introduction:
Let R be a Commutative ring with unit element and let Z(R) be its set of zero divisor.The zero divisor graph of R is denoted by $[\Gamma(Z(R))]$and it is defined as for any two distinct vertices x and y are adjacent iff xy =0 for all x,y in Z(R)\{0}. All the graphs considered here are simple, finite, connected and undirected graph.
The concept of the zero divisor graph was first introduced by I.Beck in 1988 and further developed by D.D.Anderson and M.Naseer.The aim of this paper is to study the domination number of Cartesian product of star zero divisor graph and its complement graph.We have taken a non-zero zero divisor graph which has only composite number but not a prime number.It is denoted by Z_{2_p}

2. Basic Definitions:

In this section, we discussed some basic definitions ,notations and its meanings

Definition 2.1: [2]

A field is a ***commutative ring*** where every non-zero element 'a' is invertible. That is, if R has a multiplication inverse 'b' such that ab = 1 therefore, by definition any field is a commutative ring.

Definition 2.2: [3]

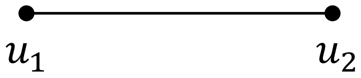
A ***zero divisor*** of commutative ring R is non-zero element 'r' such that,
rs = 1for some other non-zero element s of the ring. If the ring R is commutative, then rs = 0 $\Leftrightarrow$ sr = 0.

Definition 2.3: [4]

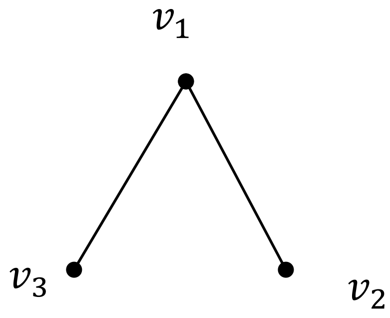
In graph theory, the ***Cartesian product***$G_1 \times G_2$ of graphs G_1 and G_2 is a graph such that,

- The vertex set of $G_1 \times G_2$ is the Cartesian product V(G_1) × V(G_2) and
- Any two vertices (u ,u^1) and (v,v^1) are adjacent in $G_1 \times\ G_2$ if and only if either,
  i. u = v and u^1 is adjacent with v^1 in G_2.
  ii. u^1=v^1 and u adjacent with v in G_1.

Example 2.1:

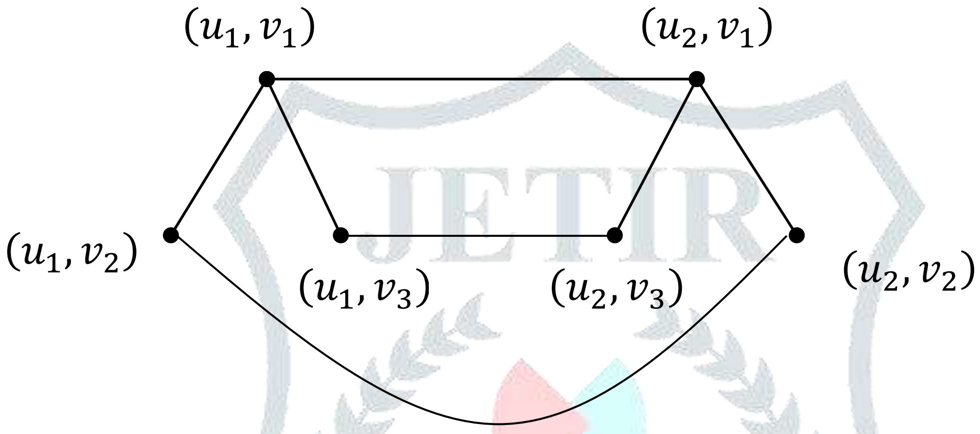


G_1



G_2

Vertex set of the Cartesian product of
$G_1 \times G_2 = \{(u_1, v_1)(u_1, v_2)(u_1, v_3)(u_2, v_1)(u_2, v_2)(u_2, v_3)\}$

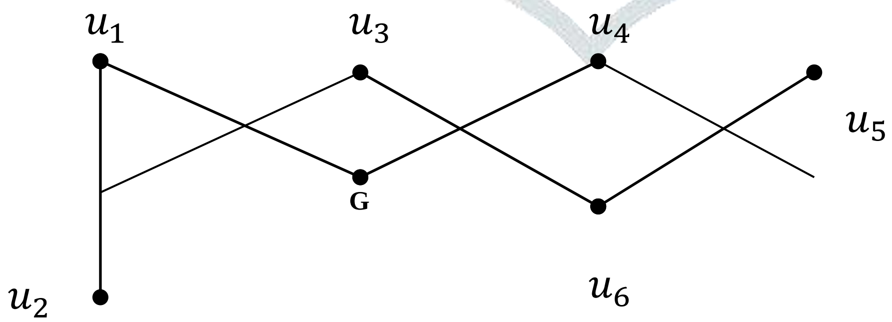


$G_1 \times G_2$

Definition 2.5: [6]

Let G = (V, E) be a graph. Then a subset D of V (The vertex set of G) is said to be a ***Dominating set*** of G if for each v ∈ V either v ∈ D or v is adjacent to some vertex in D.

Example 2.2:



G

The dominating sets are,

- $D_1 = \{u_3, u_5\}$
- $D_2 = \{u_1, u_4, u_6\}$
- $D_3 = \{u_6, u_4, u_2\}$

Definition 2.6: [6]

The minimum cardinality of a minimal dominating set in G is called a ***domination number*** of G and is denoted by $\gamma(G)$.

Example 2.3:

From example 2.2,
The set D = $\{v_3, v_5\}$ is the minimum dominating set of G.
Therefore,γ(G) = 2**.**

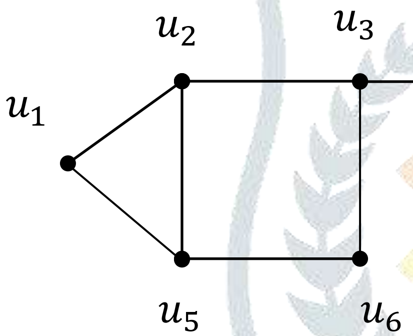
Definition 2.7: [7]

The dominating set D is called a ***perfect dominating set***, if every vertex in D $\subseteq$ V in adjacent to exactly one vertex in D. The perfect domination number $\gamma_{pr}(G)$ is the minimum cardinality of a minimal perfect dominating set .

Definition 2.8: [7]

The ***connected dominating set*** D $\subseteq$ V, in a connected graph G is a dominating set such that $\langle D \rangle$ is connected.
The minimum cardinality of a minimal connected dominating set is denotes by $\gamma_c(G)$ and it is called the connected domination number.

Example 2.4:



The connected dominating set is $D = \{u_2, u_3\}$
$\therefore$ $\boldsymbol{\gamma_c(G)}$ **= 2.**

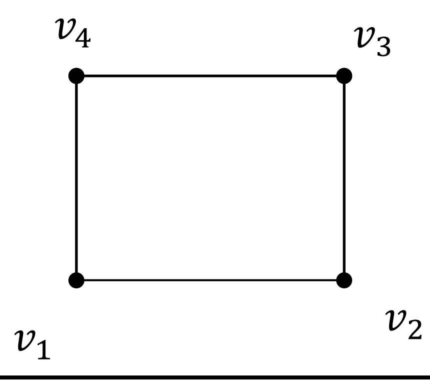
Definition 2.9: [7]

The dominating set D $\subseteq$ V of a graph G is called a ***global dominating set*** , if D is also a dominating of $\bar{G}$.
The minimum cardinality of a minimal global dominating set is denoted by $\boldsymbol{\gamma_g(G)}$ and it is called the global domination number.

Definition 2.10: [3]

A dominating set D in which no two vertices are adjacent is called an ***independent dominating set*** of G. The induced subgraph $\langle D \rangle$ is a null graph, if D is an independent dominating set.
The minimum cardinality of a minimal independent dominating set is called the ***independent domination number*** of G and it is denoted by $\gamma_i(G)$.

Example 2.5:



Independent dominating sets are,

- $D_1 = \{v_1\}$
- $D_2 = \{v_2\}$
- $D_3 = \{v_3\}$
- $D_4 = \{v_4\}$

$\therefore\ \boldsymbol{\gamma_i(G) = 1}$

Definition 2.11: [2]

(Beck) The ***zero divisor graph*** of a commutative ring R (with 1) is a simple graph whose set of vertices consists of all element of the ring, with an edge defined between 'a' and 'b' if and only if ab = 0.

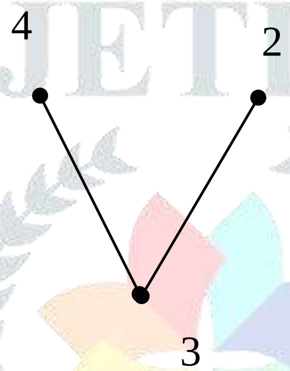
3. Example for a Zero Divisor Graph: Non-zero zero divisor graph which has only composite number and do not have a prime number:

Example:3.1

$\Gamma(Z_4) = \{2\}$

Example:3.2

$\Gamma(Z_6) = \{2,3,4\}$

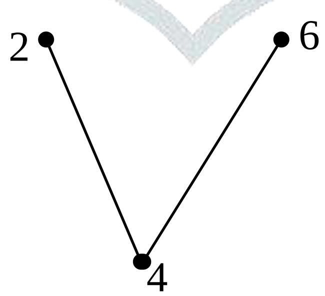


Example:3.3

$\Gamma(Z_9) = \{3,6\}$

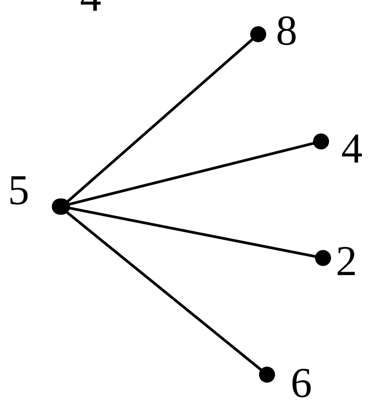
Example:3.4

$\Gamma(Z_8) = \{2,4,6\}$



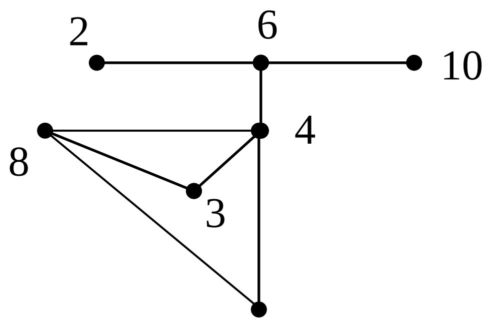
Example:3.5

$\Gamma(Z_{10}) = \{2,4,5,6,8\}$



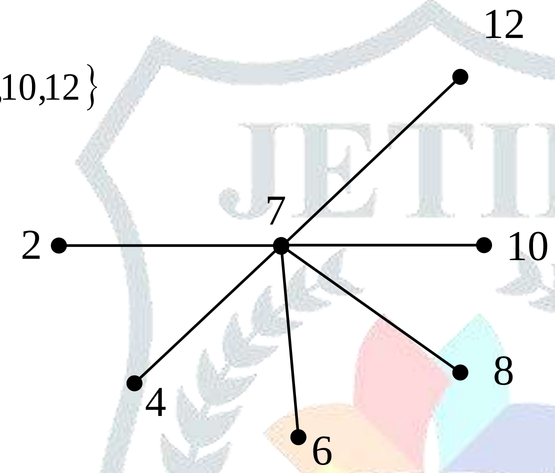
Example:3.6

$$\Gamma(Z_{12}) = \{2,3,4,6,8,9,10\}$$



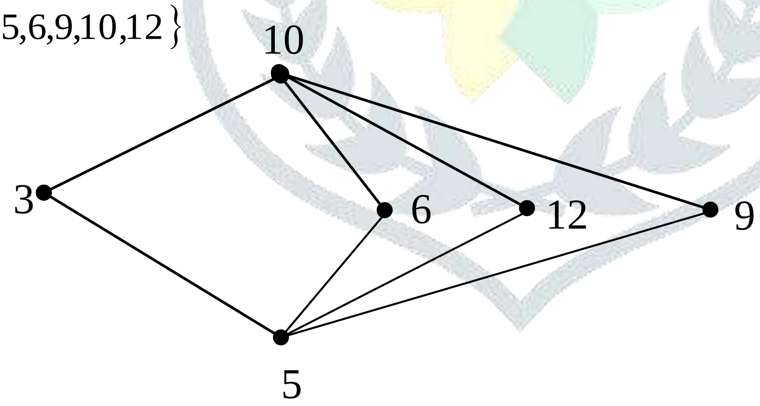
Example:3.7

$$\Gamma(Z_{14}) = \{2,4,6,7,8,10,12\}$$



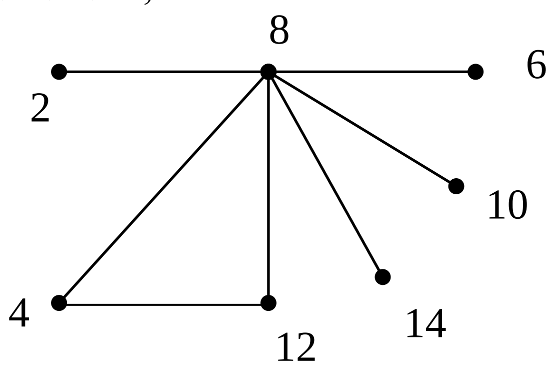
Example:3.8

$$\Gamma(Z_{15}) = \{3,5,6,9,10,12\}$$



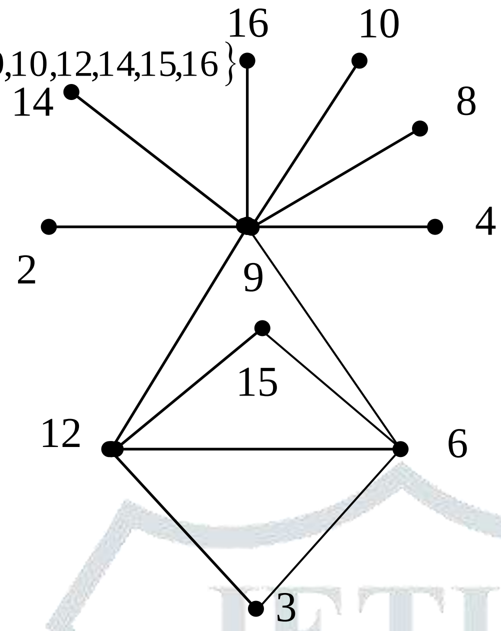
Example:3.9

$$\Gamma(Z_{16}) = \{2,4,6,8,10,12,14\}$$



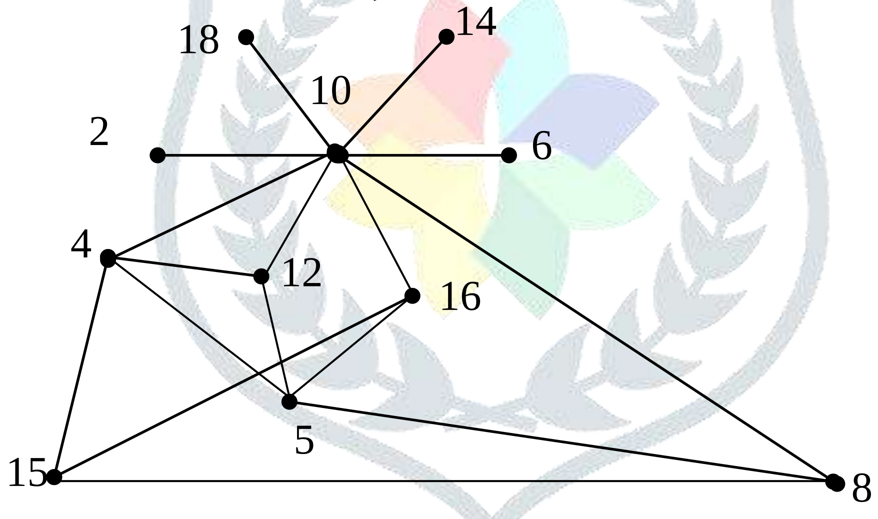
Example:3.10

$$\Gamma(Z_{18}) = \{2,3,4,6,8,9,10,12,14,15,16\}$$



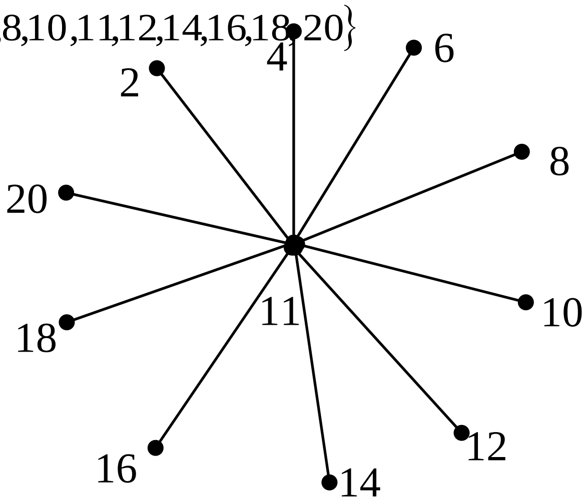
Example:3.11

$$\Gamma(Z_{20}) = \{2,4,5,6,8,10,12,14,15,16,18\}$$



Example:3.12

$$\Gamma(Z_{22}) = \{2,4,6,8,10,11,12,14,16,18,20\}$$



Notations with their meanings:

- $\Gamma(Z(G))$ — Non-zero zero divisor graph.
- $\Gamma(Z_{2p}(G))$ — Star zero divisor graph, where p is prime.
- $\gamma(\Gamma Z_{2p}(G))$ — Domination number of star zero divisor Graph, where p is a prime number.
- $\Gamma(Z_{2p_1}(G)\times Z_{2p_2}(G))$ — Cartesian product of two star zero divisor graph, where p_1 and p_2 are prime numbers and $p_1 < p_2$.
- $\gamma(\Gamma(Z_{2p_1}(G)\times Z_{2p_2}(G)))$ — Domination number of Cartesian product of star zero divisor Graphs ,wherep_1 and p_2 are prime numbers and $p_1 < p_2$.
- $\Gamma(\overline{Z(G)})$ — Complement of a non-zero zero divisor graph.
- $\Gamma(\overline{Z_{2p}(G)})$ — Complement of a star zero divisor Graph,where p is prime.
- $\gamma(\Gamma(\overline{Z_{2p}(G)}))$ — Domination number of complement of a star zero divisor graph.
- $\Gamma(\overline{Z_{2p_1}(G)\times Z_{2p_2}(G)})$ — Cartesian product of complement of star zero divisor graphs, where p_1 and $p_2$are prime numbers and $p_1 < p_2$.
- $\gamma(\Gamma(\overline{Z_{2p_1}(G)\times Z_{2p_2}(G)}))$ — Domination number of Cartesian product of complement of star zero divisor graphs,where p_1 and p_2 are prime number,$p_1 < p_2$.
- $\gamma_U\Gamma(Z_{2p_1}(G)\times Z_{2p_2}(G))$ — Universal Domination number of Cartesian product of star zero divisor graphs.
- $\gamma_G\Gamma(Z_{2p_1}(G)\times Z_{2p_2}(G))$ — Global Domination number of Cartesian product of star zero divisor graphs.

4. Types of a Zero Divisor Graph [6]

There are several types of zero divisor graphs. In this paper, we discussed about some of them. Let p, q and r represented three distinct primes then the following cases arise:

One prime 4.1

When drawing the ***Anderson and Livingston*** graph this is trivial because we eliminated the zero divisors 0 and 1 such that these graphs have neither vertices nor edges.

Two prime 4.2

Let this be represented by **pq**. There are two case:

a) p and q can be distinct.

b) P andq can be the same.

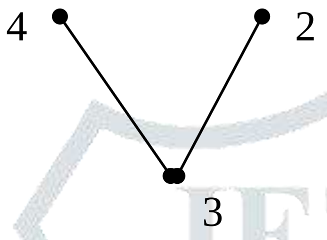
a)The distinct case

If p and q are distinct then the graph will be a complete bipartite graph.

Example:4.3

When p = 2 and q = 3

Then, $\Gamma(Z_{2\times 3}) = \Gamma(Z_6) = \{2,3,4\}$

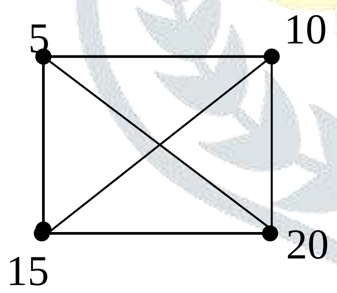


The non-distinct case

If p and q take the same value ,then pq becomes p^2.This will be a complete graph with p-1 vertices, every vertex will be a multiple of $p < p^2$.

Example:4.4

$\Gamma(Z_{5^2}) = \Gamma(Z_{25}) = \{5,10,15,20\}$



Three primes:4.5

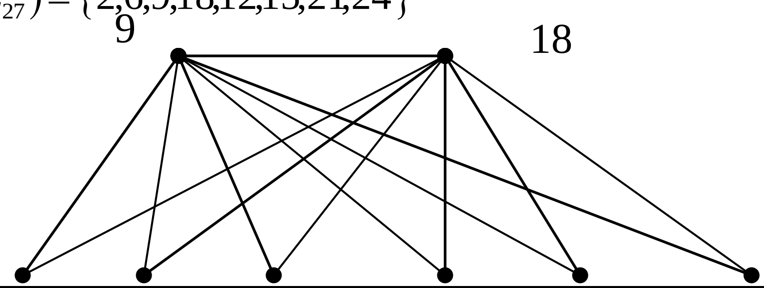
There are three types of prime numbers which are represented by p^3, p^2q, pqr.

Case (i) p^3 when p is a prime

In this case, the graph will be a complete bipartite graph with one side having one set of vertices that are multiples of p^2 the other set of vertices being all remaining multiples of p.

Example:4.6

$\Gamma(Z_{3^2}) = \Gamma(Z_{27}) = \{2,6,9,18,12,15,21,24\}$



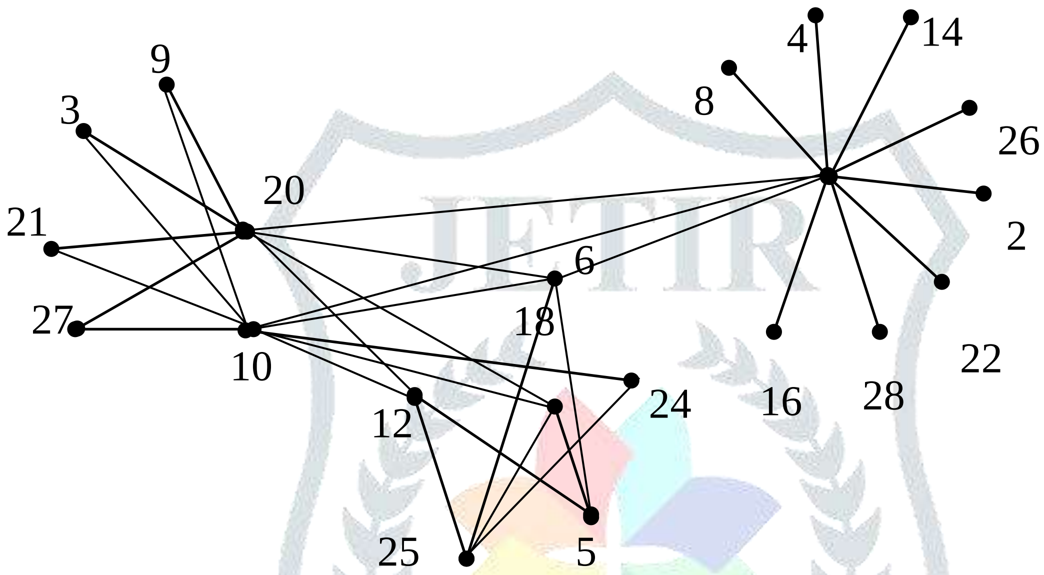
3 24 12 15 6 21

Case (ii) pqr when, p, q, and r are primes and $p < q < r$

Each grouping of terms is a group of factors that have to do with the multiples of p, not associated with q or r, then the multiples of r associated with p or q and then the multiples of pq.

Example:4.7

$$\Gamma\left(Z_{2\times3\times5}\right)=\Gamma\left(Z_{30}\right)=\{2,3,4,5,6,8,9,10,12,14,16,18,20,21,22,24,25,26,27,28\}$$

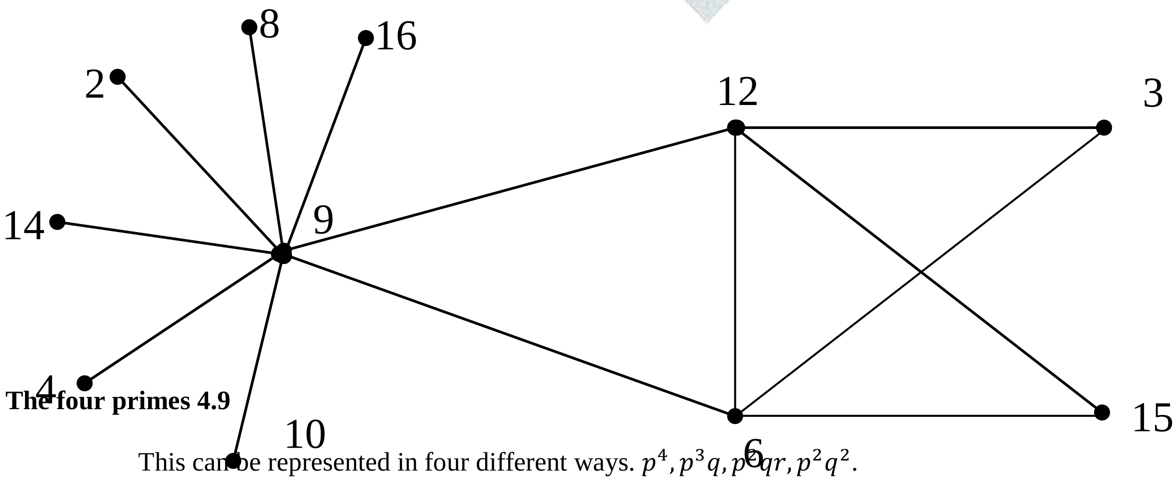


Case (iii) p^2q **when p and q are primes**

The centre of this graph will be $\frac{(p^2q)}{2}$ the vertices connected to $\frac{(p^2q)}{2}$ will be multiples of pq these multiples will form a complete bipartite graph with the multiple of p that are remaining connected to $\frac{(p^2q)}{2}$ will be all remaining multiples of q. The trail vertices are rq where, r is relatively prime to q. The center node is p^2. The gill vertices are multiples of pq.

Example:4.8

$$\Gamma\left(Z_{2^2\times7}\right)=\Gamma\left(Z_{28}\right)=\{2,3,4,6,8,9,10,12,14,15,16\}$$



The four primes 4.9

This can be represented in four different ways. p^4, p^3q, p^2qr, p^2q^2.

The five primes 4.10

This can be represented in three different ways that are p^5, p^4q, p^3q^2.

5. DOMINATION OF CARTESIAN PRODUCT OF A STAR ZERO DIVISOR GRAPHS AND ITS COMPLEMENT

This section deals with domination of Cartesian product of two star zero divisor graph and its complement graph and we studied the bounds for these graphs.

5.1 Preliminaries

Theorem : 5.1.1 [3]

If n = 2p where p is an odd prime, Then the non-zero zero divisor graph is a star graph and it is also called complete bipartite graph $(K_{1,p-1})$.

Theorem :5.1.2 [3]

If n = 2p where p is an odd prime. Then the dominating number of star zero divisor graph is 1.

5.2 Domination Number of Cartesian Product Of the Star Zero Divisor Graph with Illustrations:

Theorem: 5.2.1

If $G_1 = \Gamma(Z_{2p_1})$ and $G_2 = \Gamma(Z_{2p_2})$ are two star zerodivisor graphs. Let $G = G_1 \times G_2$ be the Cartesian product of star zero divisor graphs. Then the domination number of G is given by,

i. $\gamma(G) = 1$ if $p_1 = 2$ and $p_2 = 3$. $[p_1 < p_2]$
ii. $\gamma(G) = p_1$ if $p_1 < p_2$ when p_1 and p_2 are odd primes.

Proof:

Let $G_1 = \Gamma(Z_{2p_1})$ and $G_2 = \Gamma(Z_{2p_2})$ be two star zero divisor graphs.
And $\gamma(G_1) = \gamma(G_2) = 1$ by theorem 3.2.2,
Let $G = G_1 \times G_2$, when $p_1 < p_2$ and p_1 and p_2 are odd primes.
To find, the value of $\gamma(G_1 \times G_2)$:

Case (i):

Let $p_1 = 2$ and $p_2 = 3$.
We have,

$$G = \Gamma(Z_{2(2)} \times Z_{2(3)}) = \Gamma(Z_4 \times Z_6) = \Gamma(Z_4) \times \Gamma(Z_6)$$

Since, V(G_1) = { 2 }. The dominating set $D_1 \subseteq V(G)$ and $D_1 = \{2\}$
Also, | V(G_1)| = 1.
V(G_2) = { 2,3,4 }. The dominating set D_2 of G_2 is $D_2 = \{3\}$
And | V(G_2)| = 3.
Now, the vertex set of G is given by,
$V(G) = \{(2,2), (2,3), (2,4)\}$ and its order is 3.
By the definition 2.2.5,
The set of pair of adjacent vertices in $V_1(G) = \{((2,2)(2,3)), ((2,3)(2,4))\}$
The set of non-adjacent vertices in $V_2(G) = \{(2,2), (2,4)\}$. (by definition of Non-zero Zero divisor graph)
The dominating set of G is $D = \{(2,3)\}$ forms a minimal dominating set with cardinality 1.
Hence $\gamma(G) = 1$.
Case (ii):
Sub case (i):

When $p_1 < p_2$. To find, the value of $\gamma(G_1 \times G_2)$. This can be proved by using induction hypothesis,
For $p_1 = 3$ and $p_2 = 5$
We have,

$$G = \Gamma\left(Z_{2(3)} \times Z_{2(5)}\right) = \Gamma\left(Z_6 \times Z_{10}\right) = \Gamma\left(Z_6\right) \times \Gamma\left(Z_{10}\right)$$

Since, V(G_1) = { 2,3,4 } and D(G_1) = { 3 }

$|V(G_1)| = 3 = p_1$

V(G_2) = { 2,4,5,6,8 } and D(G_2) = { 5 }

$|V(G_2)| = 5 = p_2$

The vertex set of G is given by

$$V(G) = \begin{Bmatrix} (2,2), (2,4), (2,5), (2,6), (2,8) \\ (3,2), (3,4), (3,5), (3,6), (3,8) \\ (4,2), (4,4), (4,5), (4,6), (4,8) \end{Bmatrix}$$ and its order is 15.

By the definition of Domination,

Any two vertices (u, v) $\left(ie., u = v = \{ 2,3,4 \} \in \Gamma\left(Z_6\right)\right)$ and (u^1, v^1) $\left(ie., u^1 = v^1 = \{ 2,4,6,5,8 \} \in \Gamma\left(Z_{10}\right)\right)$ are adjacent in $\Gamma\left(Z_6 \times Z_{10}\right)$ if and only if,

i. u = v in Z_6 and every odd integer u^1adjacent with even integer v^1 in Z_{10}.

ii. $u^1 = v^1$ in Z_{10} and every odd integer u adjacent with even integer v in Z_6.

The set of pair of adjacent vertices in G is given by,

$$V_1(G) = \begin{Bmatrix} ((2,2)(2,5)) \dots \dots ((2,4)(2,5)) \dots \dots ((2,8)(2,5)) \dots \dots \\ ((3,2)(3,5)) \dots \dots ((3,4)(3,5)) \dots \dots ((3,8)(3,5)) \dots \dots \\ ((4,2)(4,5)) \dots \dots ((4,8)(4,5)) \dots \dots ((2,2)(3,2)) \dots \dots \\ ((2,8)(3,8)) \dots \dots \dots \dots ((4,2)(3,2)) \dots \dots ((4,8)(3,8)) \dots \dots \end{Bmatrix}$$

And $|v_1(G)| = 22$

The set of pair of non-adjacent vertices in G is given by,

$V_2(G) = V(G) - V_1(G)$

The dominating set of G is

$$D = \{(2,3)(3,5)(4,5)\}$$

And $V - D = \begin{Bmatrix} (2,2)(2,4)(2,6)(2,8) \\ (3,2)(3,4)(3,6)(3,8) \\ (4,2)(4,4)(4,6)(4,8) \end{Bmatrix}$

Since every vertex of V-D is adjacent to at least one vertex of D.

Then $D = \{(2,3)(3,5)(4,5)\}$ forms a minimal dominating set with cardinality 3.

Hence, $\gamma(G) = \gamma(\Gamma(Z_6 \times Z_{10})) = \gamma(\Gamma(Z_6) \times \Gamma(Z_{10}))$

$$\therefore \ \gamma(G) = 3 = p_1 .$$

Sub case (ii):

When $p_1 < p_2$,

For $p_1 = 5$ and $p_2 = 7$

Let $G_1 = \Gamma\left(Z_{2(5)}\right)$ and $G_2 = \Gamma\left(Z_{2(7)}\right)$

Let $G = \Gamma\left(Z_{2(5)} \times Z_{2(7)}\right) = \Gamma\left(Z_{10} \times Z_{14}\right) = \Gamma\left(Z_{10}\right) \times \Gamma\left(Z_{14}\right)$

Since, V(G_1) = { 2,3,5,6,8 } and D(G_1) = { 5 }

And $|V(G_1)| = 5 = p_1$

V(G_2) = { 2,4,6,7,8,10,12 } and D(G_2) = { 7 }

And $|V(G_2)| = 7 = p_2$

The set of vertices in G is given by

$$V(G) = \begin{Bmatrix} (2,2), (2,4), (2,6), (2,8), (2,10)(2,12) \\ (4,2), (4,4), (4,6), (4,8), (4,10)(4,12) \\ (5,2), (5,4), (5,6), (5,8), (5,10)(5,12) \\ (6,2), (6,4), (6,6), (6,8), (6,10)(6,12) \\ (8,2), (8,4), (8,6), (8,8), (8,10)(8,12) \end{Bmatrix}$$ and its order is 35.

By the definition of Cartesian product ,

Any two vertices (u, v) $\left(ie., u = v = \{ 2,4,5,6,8 \} \in \Gamma\left(Z_{10}\right)\right)$ and (u^1, v^1) $\left(ie., u^1 = v^1 = \{ 2,4,6,7,8,10,12 \} \in \Gamma\left(Z_{14}\right)\right)$ are adjacent in $\Gamma\left(Z_{10} \times Z_{14}\right)$ if and only if,

i. u = v in Z_{10} and every odd integer u^1 adjacent with even integer v^1 in Z_{14}.

ii. $u^1 = v^1$ in Z_{14} and every odd integer u adjacent with even integer v in Z_{10}.

∴ The set of all pair of adjacent vertices in G is given by,

$$V_1(G) = \left\{ \begin{array}{c} ((2,2)(2,7)) \dots \dots ((2,12)(2,7)) \dots \dots ((4,2)(4,7)) \dots \dots ((4,12)(4,7)) \\ \dots \dots ((5,2)(5,7)) \dots \dots ((5,12)(5,7)) \dots \dots ((6,2)(6,7)) \\ \dots \dots ((6,12)(6,7)) \dots ((8,2)(8,7)) \dots \dots ((8,12)(8,7)) \dots \dots ((2,2)(5,2)) \\ \dots \dots ((2,12)(5,12)) \dots \dots ((4,2)(5,2)) \dots \dots ((8,2)(5,2)) \\ \dots \dots ((6,2)(5,2)) \dots \dots ((6,12)(5,12)) \dots \dots \end{array} \right\}$$

The set of al pair of non-adjacent vertex in G is given by, $V_2(G) = V(G) - V_1(G)$

The dominating set of D is given by,

$D = \{(2,7)(4,7)(5,7)(6,7)(8,7)\}$ which is the minimum dominating set of V(G)

$$\text{and } V - D = \left\{ \begin{array}{c} (2,2)(2,4)(2,6)(2,8)(2,10)(2,12) \\ (4,2)(4,4)(4,6)(4,8)(4,10)(4,12) \\ (5,2)(5,4)(5,6)(5,8)(5,10)(5,12) \\ (6,2)(6,4)(6,6)(6,8)(6,10)(6,12) \\ (8,2)(8,4)(8,6)(8,8)(8,10)(8,12) \end{array} \right\}$$

By the definition of dominating set, $D \subseteq V$ and every vertex of V-D is adjacent with at least one vertex of D.

Then $D = \{(2,7)(4,7)(5,7)(6,7)(8,7)\}$is a minimal dominating set with cardinality 5.

Hence, $\gamma(G) = \gamma(\Gamma(Z_{10} \times Z_{14})) = \gamma(\Gamma(Z_{10}) \times \Gamma(Z_{14}))$

∴ $\gamma(G) = 5 = p_1$

Proceedings in this way for order n,

we have

$|V(G_1)| = p_1$

$|V(G_2)| = p_2$ when, $p_1 < p_2$

The vertex set of Cartesian product of star zero divisor graph

$$V(G) = \left\{ \begin{array}{c} (2,2)(2,4) \dots \dots (2, p_1) \\ (4,2)(4,4) \dots \dots (4, p_2) \\ \dots \;\; \dots \;\; \dots \;\; \dots \;\; \dots \;\; \dots \;\; \dots \\ \dots \;\; \dots \;\; \dots \;\; \dots \;\; \dots \;\; \dots \;\; \dots \\ (p_1, 2)(p_1, 4) \dots \dots (p_1, p_2) \end{array} \right\}$$

And $|V(G)| = p_1 \times p_2$

The set of pair of vertices which are adjacent in G is given by,

$$V_1(G) = \left\{ \begin{array}{c} ((2,2),(2,p_1)) \dots ((2,p_2-2),(2,p_1)) \dots ((p_1-2,2),(p_1-2,p_1)) \dots \\ ((p_1-2,p_2-2),(p_1-2,p_1)) \dots ((2,2),(p_1^1,2)) \dots ((p_1-2,2),(p_1^1,2)) \\ \dots ((2,p_2-2),(p_1^1,p_2-2)) \dots ((p_1-2,p_2-2)(p_1^1,p_2-2)) \dots \dots \end{array} \right\}$$

Where, p_1^1 is domination number of $\Gamma(Z_{2p_1})$.

The remaining vertices are the pairs of non-adjacent vertices in G.

By definition, The dominating set D of G = $\{(2,p_1),(4,p_1),\dots\dots,(p_1,p_2)\}$

Which forms a minimal dominating set with cardinality p_1.

Hence,

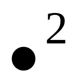
$$\gamma(G) = \gamma(G_1 \times G_2) = \gamma(\Gamma(Z_{2p_1} \times Z_{2p_2}))$$
$$= \gamma(\Gamma(Z_{2p_1}) \times \Gamma(Z_{2p_2}))$$
$$\gamma(G) = p_1$$

Where p_1 and p_2 are odd prime, when $p_1 < p_2$.

Example:5.2.2

Consider $\Gamma(Z_4)$- non-zero zero divisor graph,

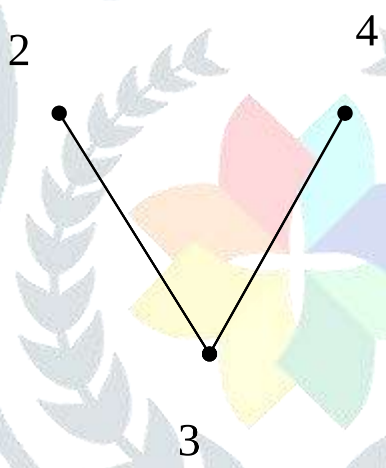
- Set of all vertices of $\Gamma(Z_4)$ is {2}
- Dominating set D is {2}
- $\gamma(\Gamma(Z_4))=1$



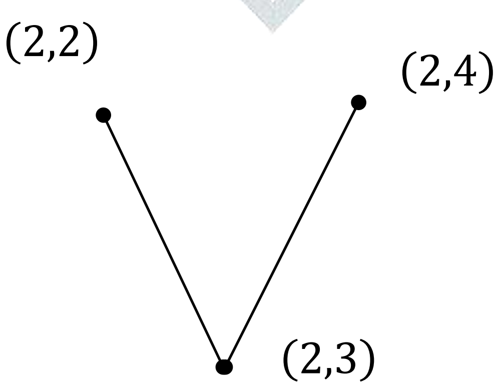
Star zero divisor graph of $\Gamma(Z_4)$

Consider $\Gamma(Z_6)$- non-zero zero divisor graph,

- Set of all vertices of $\Gamma(Z_6)$is {2,3,4}
- Dominating set D is {3}
- $\gamma(\Gamma(Z_6))=1$



Star zero divisor graph of $\Gamma(Z_6)$



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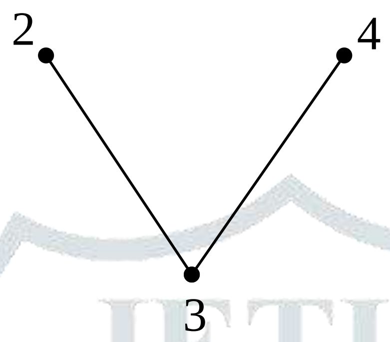
Example:5.2.3

Let $\Gamma(Z_6)$ be the star zero divisor graph of order 3,

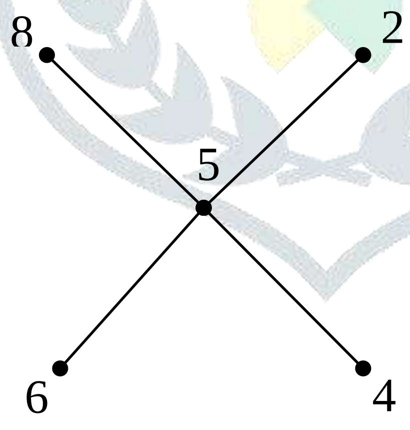
- Set of all vertices of $\Gamma(Z_6)$ is{2,3,4}
- Dominating set D is {3}
- $\gamma(\Gamma(Z_6)) = 1$



Star zero divisor graph of $\Gamma(Z_6)$

Similarly,order of the star zero divisor of $\Gamma(Z_{10})$ is 5,

- Set of all vertices of $\Gamma(Z_{10})$ is{2,4,5,6,8}
- Dominating set D is {5}
- $\gamma(\Gamma(Z_{10})) = 1$



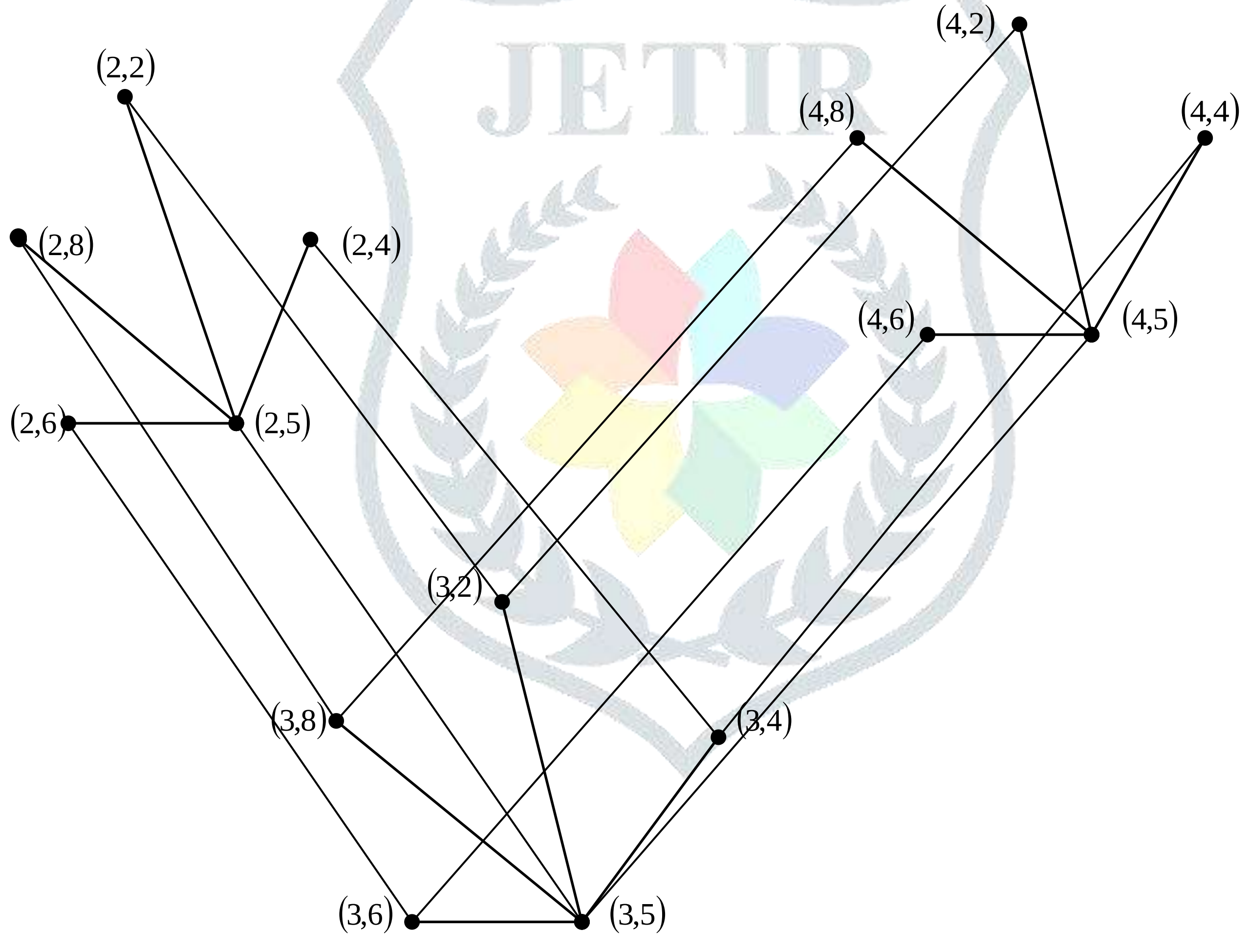
Star zero divisor graph of $\Gamma(Z_{10})$

Consider the Cartesian product of star zero divisor graph $\Gamma(Z_6 \times Z_{10})$ whose vertex set is given by,

- Set of all vertices of $\Gamma(Z_6 \times Z_{10})$ is

$$\begin{Bmatrix} (2,2),(2,4),(2,5)(2,6)(2,8) \\ (3,2)(3,4)(3,5)(3,6)(3,8) \\ (4,2)(4,4)(4,5)(4,6)(4,8) \end{Bmatrix}$$

- Dominating set D is {(2,5)(3,5)(4,5)}
- $\gamma(\Gamma(Z_6 \times Z_{10})) = 3$

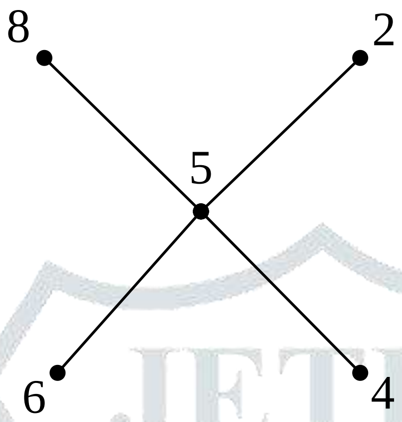


Cartesian product of star zero divisor graph of $\Gamma(Z_6 \times Z_{10})$

Example:3.3

Let $\Gamma(Z_{10})$ be the star zero divisor graph of order 5,

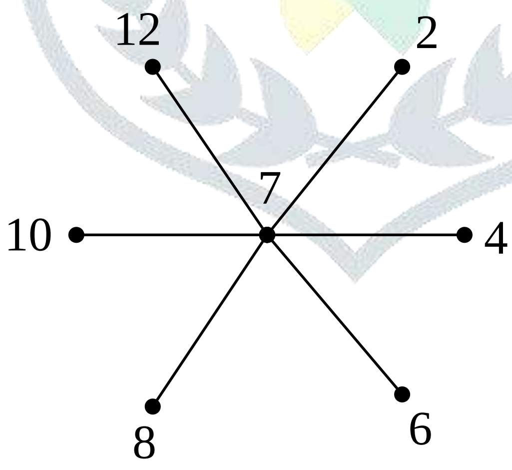
- Set of all vertices of $\Gamma(Z_{10})$ is {2,4,5,6,8}
- Dominating set D is {5}
- $\gamma(\Gamma(Z_{10})) = 1$



Star zero divisor graph of $\Gamma(Z_{10})$

Similarly, order of the star zero divisor graph of $\Gamma(Z_{14})$ is 7,

- Set of all vertices of $\Gamma(Z_{14})$ are {2,4,6,7,8,10,12}
- Dominating set is {7}
- $\gamma(\Gamma(Z_{14})) = 1$



Star zero divisor graph of $\Gamma(Z_{14})$

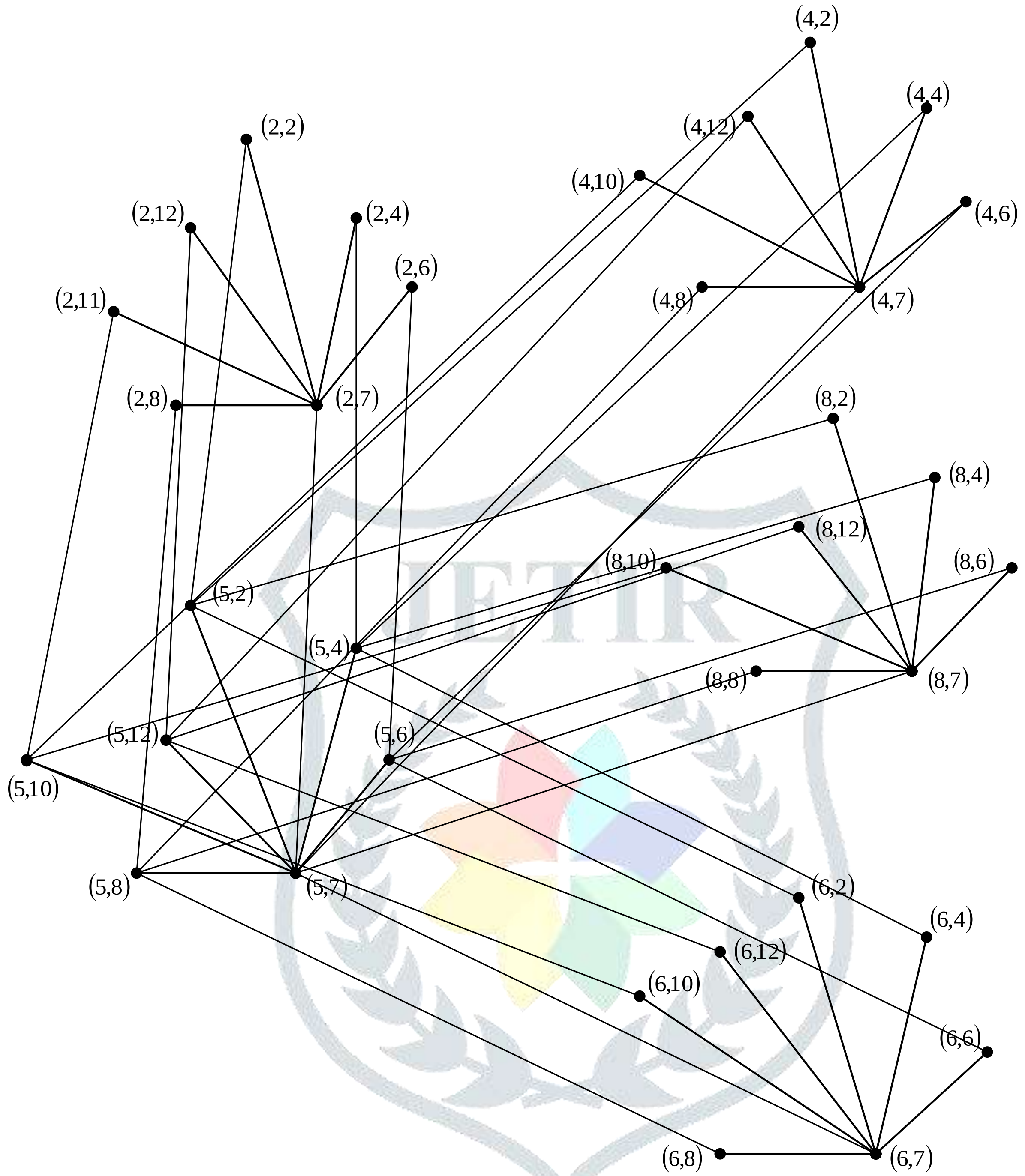
$(4,2)$

Consider the Cartesian product of star zero divisor graph $\Gamma(Z_{10} \times Z_{14})$ whose vertex set is given by,

- Set of all vertices of $\Gamma(Z_{10} \times Z_{14})$ is

$$\left\{\begin{matrix} (2,2),(2,4),(2,6)(2,7)(2,8)(2,10)(2,12) \\ (4,2)(4,4)(4,6)(4,7)(4,8)(4,10)(4,12) \\ (5,2)(5,4)(5,6)(5,7)(5,8)(5,10)(5,12) \\ (6,2)(6,4)(6,6)(6,7)(6,8)(6,10)(6,12) \\ (8,2)(8,4)(8,6)(8,7)(8,8)(8,10)(8,12) \end{matrix}\right\}$$

- Dominating set D is$\{(2,7)(4,7)(5,7)(6,7)(8,7)\}$
- $\gamma(\Gamma(Z_{10} \times Z_{14})) = 5$

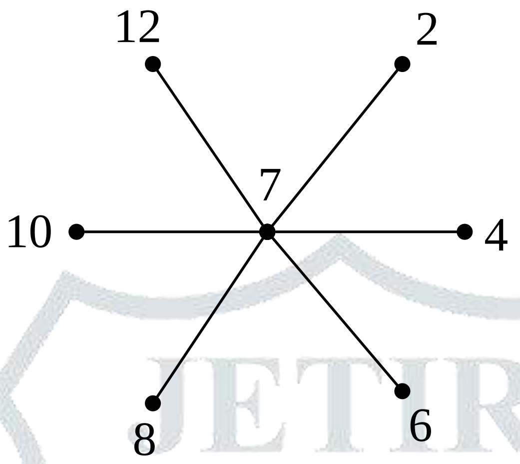


Cartesian product of star zero divisor graph of $\Gamma(Z_{10} \times Z_{14})$

Example:3.4

Let $\Gamma(Z_{14})$ be the star zero divisor graph of order 7,

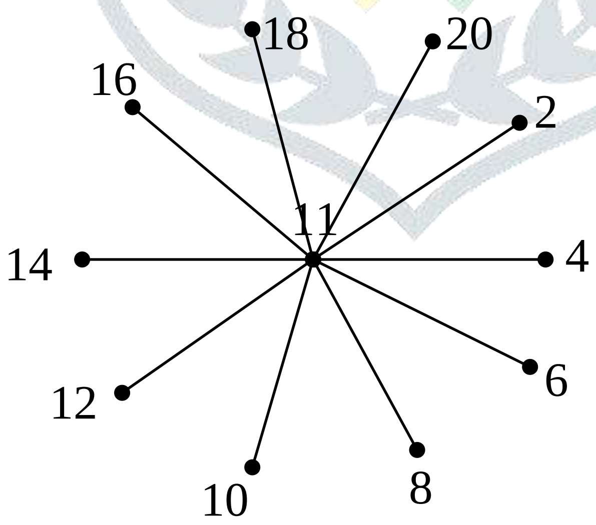
- Set of all vertices of $\Gamma(Z_{14})$ are {2,4,6,7,8,10,12}
- Dominating set is {7}
- $\gamma(\Gamma(Z_{14}))=1$



Star zero divisor graph of $\Gamma(Z_{14})$

Similarly, order of the star zero divisor graph of $\Gamma(Z_{22})$ is 11,

- Set of all vertices of $\Gamma(Z_{22})$ is {2,4,6,8,10,11,12,14,16,18,20}
- Dominating set D is {11}
- $\gamma(\Gamma(Z_{22}))=1$



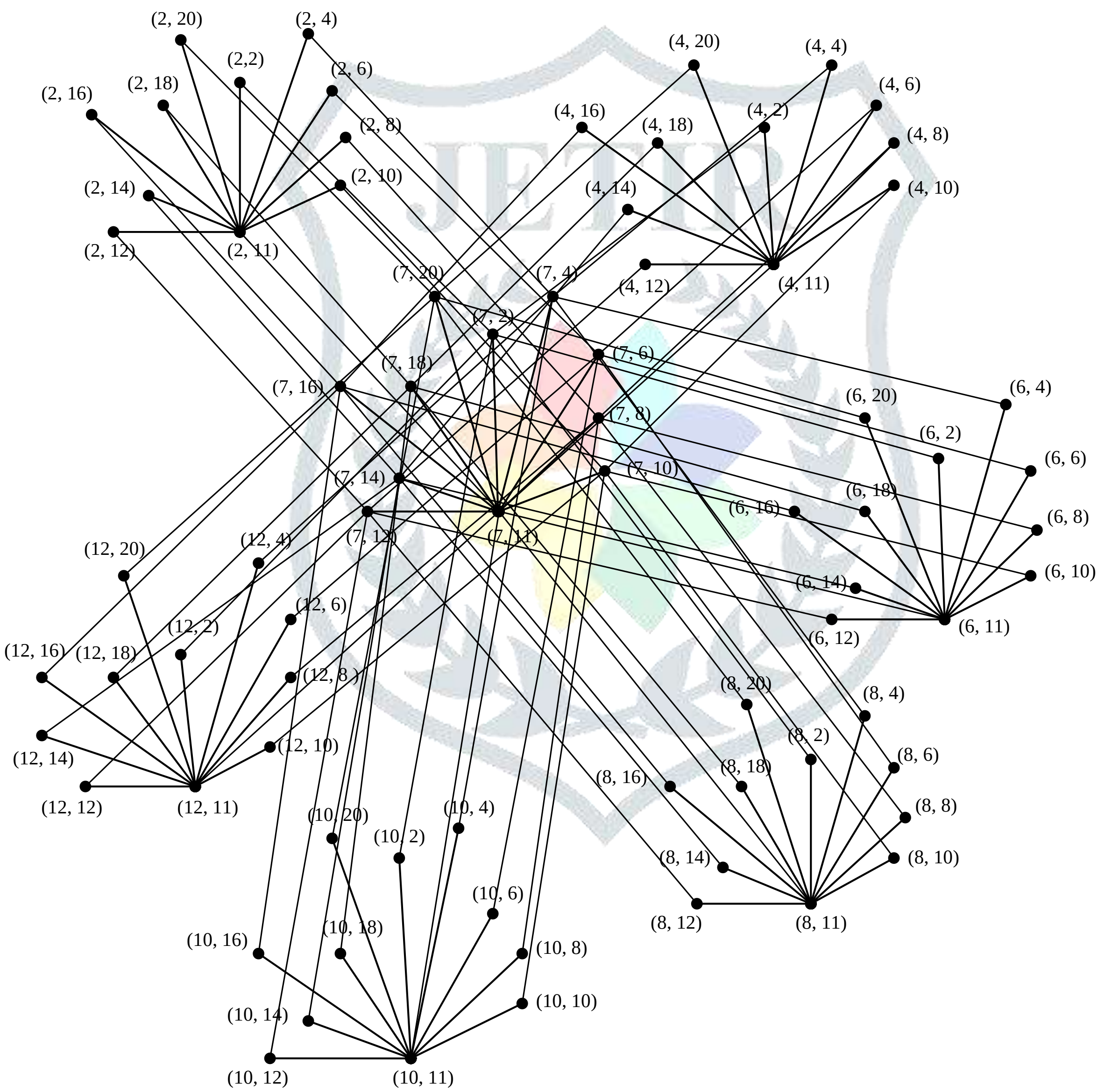
Star zero divisor graph of $\Gamma(Z_{22})$

Consider the Cartesian product of star zero divisor graph $\Gamma(Z_{14} \times Z_{22})$ whose set is given by,

- Set of all vertices of $\Gamma(Z_{14} \times Z_{22})$ is

$$\left\{\begin{array}{c} (2,2),(2,4),(2,6)(2,8)(2,10)(2,11)(2,12)(2,14)(2,16)(2,18)(2,20) \\ (4,2),(4,4),(4,6)(4,8)(4,10)(4,11)(4,12)(4,14)(4,16)(4,18)(4,20) \\ (6,2),(6,4),(6,6)(6,8)(6,10)(6,11)(6,12)(6,14)(6,16)(6,18)(6,20) \\ (8,2),(8,4),(8,6)(8,8)(8,10)(8,11)(8,12)(8,14)(8,16)(8,18)(8,20) \\ (7,2),(7,4),(7,6)(7,8)(7,10)(7,11)(7,12)(7,14)(7,16)(7,18)(7,20) \\ (10,2),(10,4),(10,6)(10,8)(10,10)(10,11)(10,12)(10,14)(10,16)(10,18)(10,20) \end{array}\right\}$$

- Dominating set D is{(2,11)(4,11)(7,11)(6,11)(8,11)(10,11)(12,11)}
- $\gamma(\Gamma(Z_{14} \times Z_{22})) = 7$



Cartesian product of star zero divisor graph of $\Gamma(Z_{14} \times Z_{22})$

6.Domination number of Complement Of Cartesian Product Of the Star Zero Divisor Graph with Illustration

In this section we obtained the bounds for the domination of complement of Cartesian product of star zero divisor graphs

Theorem :6.1

If $\overline{G}_1 = \Gamma\left(\overline{Z_{2p_1}}\right)$ and $\overline{G}_2 = \Gamma\left(\overline{Z_{2p_2}}\right)$ are the complement of star zerodivisor graphs. Let $\overline{G} = \overline{G_1 \times G_2}$ be a complement of Cartesian product of star zero divisor graphs. Then the domination number $\overline{G}$ of is given by,

$\gamma\left(\overline{G}\right) = \sum_{i=1}^{4} \gamma(H_i)$ where each H_i is component of $\overline{G}$, $\forall i = 1\ to\ 4$ and[$p_1 < p_2$] when p_1 and p_2 are primes.

Proof:

Let $\overline{G}_1 = \Gamma\left(\overline{Z_{2p_1}}\right)$ and $\overline{G}_2 = \Gamma\left(\overline{Z_{2p_2}}\right)$ be the complement of star zerodivisor graphs.

Let $\overline{G} = \overline{G_1 \times G_2}$

To find the domination of $\overline{G}$, when $p_1 < p_2$ and p_1 and p_2 are odd primes.
This Can be proved by using induction hypothesis,

Case:1

For $p_1 = 3$ and $p_2 = 5$
We have,

$\overline{G}_1 = \Gamma\left(\overline{Z_{2(3)}}\right)$ and $\overline{G}_2 = \Gamma\left(\overline{Z_{2(5)}}\right)$ are the complement of star zerodivisor graphs of $\Gamma\left(\overline{Z_6}\right)$ and $\Gamma\left(\overline{Z_{10}}\right)$ respectively

Each complement graph has 2 components say H_1 and H_2,
Let $V_1\overline{(G_1)} = V_1\overline{(H_{11})} \cup V_1\overline{(H_{12})}$
$V_1\overline{(G_1)} = \{4,2\} \cup \{3\}$
$V_1\overline{(G_1)} = \{ 3,2,4 \}$
and $D\overline{(G_1)} = \{2,3\}$
$\left|\overline{\gamma(G_1)}\right| = 3 = p_1$
Let $V_2\overline{(G_2)} = V_2\overline{(H_{21})} \cup V_1\overline{(H_{22})}$
$V_2\overline{(G_2)} = \{2,4,6,8\} \cup \{5\}$
$V_2\overline{(G_2)} = \{ 2,4,5,6,8 \}$
And $D\overline{(G_2)} = \{5,8\}$
$\left|\overline{\gamma(G_2)}\right| = 5 = p_2$
Let $\overline{G} = \overline{G_1 \times G_2}$
It has 4-components.
$V\left(\overline{G_1 \times G_2}\right) = V\left(\overline{G}\right) = V(H_1) \cup V(H_2) \cup V(H_3) \cup V(H_4)$
Where H_1, H_2, H_3, H_4 are component of $\overline{G}$.

That is, $V\left(\overline{G}\right) = \sum_{i=1}^{4} V(H_i)$

$$V\left(\overline{G}\right) = \begin{Bmatrix} (2,2), (2,4), (2,6), (2,8), \\ (4,2), (4,4), (4,6), (4,8) \end{Bmatrix} \cup \{(4,5), (2,5)\} \cup \{(3,5)\} \cup \{(3,2), (3,4)(3,6), (3,8)\}$$

$$V\left(\overline{G}\right) = \begin{Bmatrix} (2,2), (2,4), (2,5), (2,6), (2,8), \\ (3,2), (3,4), (3,5), (3,6), (3,8) \\ (4,2), (4,4), (4,5), (4,6), (4,8) \end{Bmatrix}$$

$\left|V\left(\overline{G}\right)\right| = 15.$
By definition of Cartesian product ,

The set of all pair of non-adjacent vertices of $\overline{G}$ is given by,

$$V_1(\overline{G}) = \begin{Bmatrix} ((2,2),(2,5)),((4,5),(4,2)),((3,2),(3,5)),((2,2),(3,2)) \\ ((3,4),(2,4)),\dots\dots,((4,2),(3,2)),((4,4),(3,4)),\dots\dots, \end{Bmatrix}$$

The set of all pair of adjacent vertices of $(\overline{G})$,

$V_2(\overline{G}) = V(\overline{G}) - V_1(\overline{G})$

The dominating set of $(\overline{G})$ is given by,

$D_1(H_1) = \{(2,8),(4,4)\}$
$D_2(H_2) = \{(4,5)\}$
$D_3(H_3) = \{(3,5)\}$
$D_4(H_4) = \{(3,6)\}$

$$D(\overline{G}) = \bigcup_{i=1}^{4} D_i(H_i) \quad \text{or} \quad H = \bar{G}$$

$D(\bar{G}) = \{(2,8)(4,4)(4,5)(3,5)(3,6)\}$

By definition of dominating set,

Since D is a minimum dominating set with cardinality 5,

That is, $\gamma(\overline{G}) = \gamma\left(\Gamma(\overline{Z_6 \times Z_{10}})\right) = 5$.

Case:2

For $p_1 = 5$ and $p_2 = 7$

We have,

$\overline{G}_1 = \Gamma(\overline{Z_{2(5)}})$ and $\overline{G}_2 = \Gamma(\overline{Z_{2(7)}})$ are the complement of star zerodivisor graphs of $\Gamma(\overline{Z_{10}})$ and $\Gamma(\overline{Z_{14}})$ respectively

Each complement graph has 2 components say H_1 and H_2,

Let $V_1\overline{(G_1)} = V_1\overline{(H_{11})} \cup V_1\overline{(H_{12})}$

$V_1\overline{(G_1)} = \{2,4,6,8\} \cup \{5\}$

$V_1\overline{(G_1)} = \{ 2,4,5,6,8 \}$

And $D\overline{(G_1)} = \{5,4\}$

$|\overline{\gamma(G_1)}| = 5 = p_1$

Let $V_2\overline{(G_2)} = V_2\overline{(H_{21})} \cup V_1\overline{(H_{22})}$

$V_2\overline{(G_2)} = \{2,4,6,8,10,12\} \cup \{7\}$

$V_2\overline{(G_2)} = \{ 2,4,6,7,8,10,12 \}$

And $D\overline{(G_2)} = \{8,7\}$

$|\overline{\gamma(G_2)}| = 7 = p_2$

Let $\overline{G} = \overline{G_1 \times G_2}$

It has 4-component.

$V\overline{(G_1 \times G_2)} = V(\overline{G}) = V(H_1) \cup V(H_2) \cup V(H_3) \cup V(H_4)$

Where, H_1, H_2, H_3, H_4 are component of $\overline{G}$.

That is, $\mathrm{V}(\overline{\mathrm{G}}) = \sum_{i=1}^{4} V(H_i)$

$$V(\overline{G}) = \begin{Bmatrix} (2,2),(2,4),(2,6),(2,8),(2,10),(2,12) \\ (4,2),(4,4),(4,6),(4,8),(4,10),(4,12) \\ (6,2),(6,4),(6,6),(6,8),(6,10),(6,12) \\ (8,2),(8,4),(8,6),(8,8),(8,10),(8,12) \end{Bmatrix} \cup \{(4,7),(2,7),(6,7),(8,7)\}$$
$$\cup \{(5,7)\} \cup \{(5,2),(5,4),(5,6),(5,8),(5,10),(5,12)\}$$

$$V(\overline{G}) = \begin{Bmatrix} (2,2),(2,4),(2,6),(2,7),(2,8),(2,10),(2,12) \\ (4,2),(4,4),(4,6),(4,7),(4,8),(4,10),(4,12) \\ (5,2),(5,4),(5,6),(5,7),(5,8),(5,10),(5,12) \\ (6,2),(6,4),(6,6),(6,7),(6,8),(6,10),(6,12) \\ (8,2),(8,4),(8,6),(8,7),(8,8),(8,10),(8,12) \end{Bmatrix}$$

$|V(\overline{G})| = 35.$

By definition of Cartesian product,

The set of all pairs of non-adjacent vertices of $\overline{G}$ is given by,

$$V_1(\overline{G}) = \left\{ \begin{array}{c} ((2,2),(2,7))((2,4),(2,7)) \dots\dots ((2,12),(2,7)) \dots\dots ((4,2)(4,7)), \dots\dots \\ ((5,2),(5,7)) \dots\dots ((6,2),(6,7)) \dots\dots ((8,2),(8,7)) \dots\dots \\ ((5,2),(2,2)),((5,4),(2,4)) \dots\dots ((5,12),(2,12)) \dots\dots \\ \dots\dots ((5,2),(4,2)),((5,4),(4,4)) \dots\dots ((5,12),(4,12)) \dots\dots ((5,2),(6,2)) \\ \dots\dots ((5,12),(6,12)) \dots\dots ((5,2),(8,2)) \dots\dots ((5,12),(8,12)) \\ \dots\dots ((5,7),(2,7)),((5,7),(4,7)),((5,7),(6,7)),((5,7),(8,7)) \\ , \dots\dots, \end{array} \right\}$$

The set of all pairs of adjacent vertices of $(\overline{G})$,

$V_2(\overline{G}) = V(\overline{G}) - V_1(\overline{G})$

The dominating sets of $(\overline{G})$are given by,

$D_1(H_1) = \{(4,6),(2,6),(6,6),(8,6)\}$

$D_2(H_2) = \{(4,7)\}$

$D_3(H_3) = \{(5,7)\}$

$D_4(H_4) = \{(5,6)\}$

$D(H) = \bigcup_{i=1}^{4} D_i(H_i)$ where, $H = \bar{G}$

$D(H) = \{(4,6),(2,6),(5,6),(6,6),(8,6),(4,7),(5,7)\}$

By definition of dominating set

D is the minimum dominating set with cardinality 7,

That is, $\gamma(\overline{G}) = \gamma(\Gamma(\overline{Z_{10} \times Z_{14}})) = 7$.

Similarly proceeding in this way for order n,

$\overline{G_1} = \Gamma(\overline{Z_{2p_1}})$ and $\overline{G_2} = \Gamma(\overline{Z_{2p_2}})$

$|V(\overline{G_1})| = p_1$ and $|V(\overline{G_2})| = p_2$

Hence $V(\overline{G_1 \times G_2}) = |V(\overline{G_1})| \times |V(\overline{G_2})| = p_1 \times p_2$

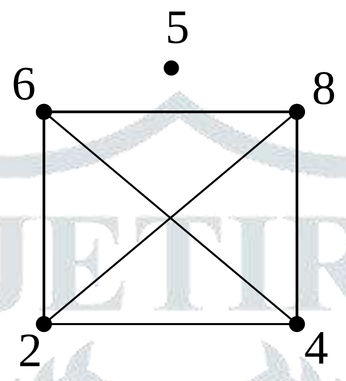
and the domination of $\overline{G}$ is given by,

$\gamma(\overline{G}) = \sum_{i=1}^{4} \gamma(H_i)$ where each H_i is component of $\overline{G}$, $\forall i = 1$ *to* 4 and($p_1 < p_2$) when p_1 and p_2 are odd primes.

Example:6.2

Let $\Gamma(\overline{Z_{10}})$be the complement of star zero divisor graph of order 5,

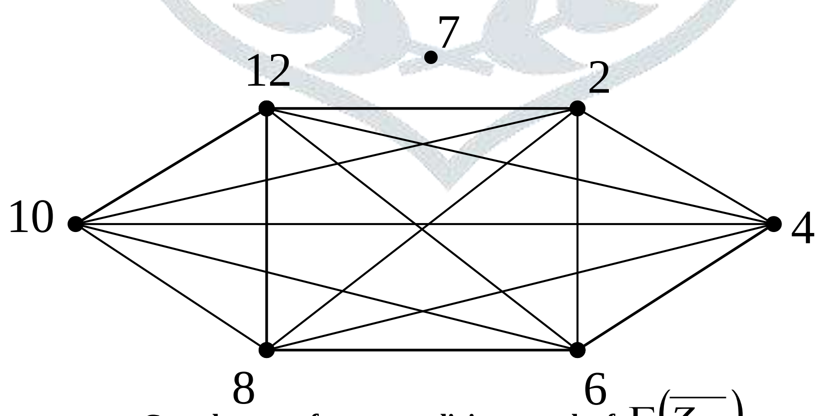
- Set of all vertices of $\Gamma(\overline{Z_{10}})$is {2,4,5,6,8}
- Dominating set D is {5,2}
- $\gamma(\Gamma(\overline{Z_{10}})) = 2$



Complement of star zero divisor graph of $\Gamma(\overline{Z_{10}})$

Similarly, order of complement of star zero divisor graph $\Gamma(\overline{Z_{14}})$is 7,

- Set of all vertices of $\Gamma(\overline{Z_{14}})$ is {2,4,6,7,8,10,12}
- Dominating set D is {7,2}
- $\gamma(\Gamma(\overline{Z_{14}})) = 2$



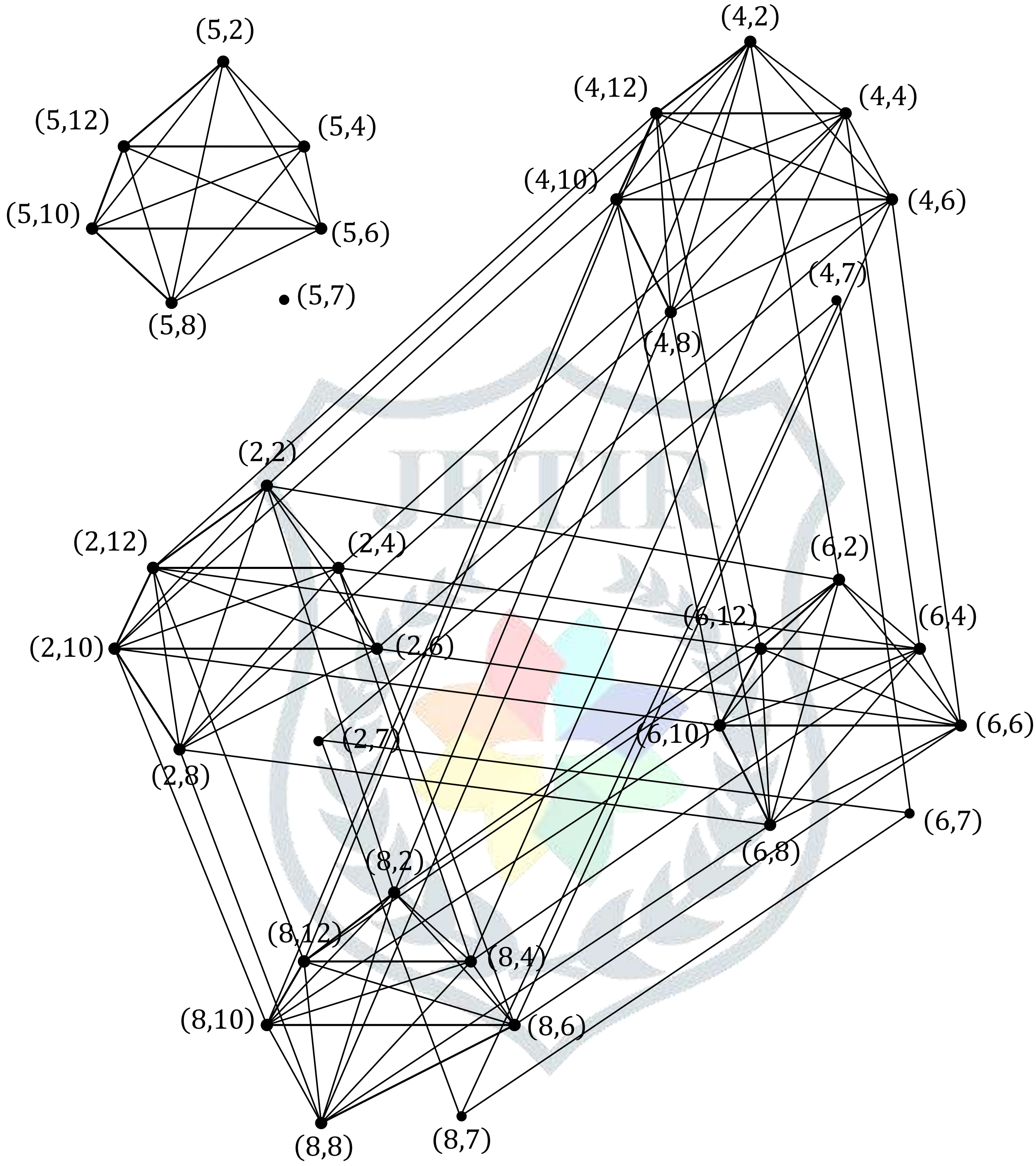
Complement of star zero divisor graph of $\Gamma(\overline{Z_{14}})$

Consider the Cartesian product of complement of star zero divisor graph $\Gamma(\overline{Z_{10} \times Z_{14}})$whose vertex set is given by,

- Set of all vertices of $\Gamma(\overline{Z_{10} \times Z_{14}})$ is

$$\begin{Bmatrix} ((2,2)2,4)(2,6)(2,7)(2,8)(2,10)(212) \\ (4,2)(4,4)(4,6)(4,7)(4,8)(4,10)(4,12) \\ (5,2)(5,4)(5,6)(5,7)(5,8)(5,10)(5,12) \\ (6,2)(6,4)(6,6)(6,7)(6,8)(6,10)(6,12) \\ (8,2)(8,4)(8,6)(8,7)(8,8)(8,10)(8,12) \end{Bmatrix}$$

- Dominating set D is{(2,2)(4,2)(4,7)(5,2)(5,7)(8,2)(6,2)}
- $\gamma\left(\Gamma\left(\overline{Z_{10} \times Z_{14}}\right)\right) = 7$
- $\gamma_g\left(\Gamma\left(Z_{10} \times Z_{14}\right)\right) = \gamma(G) + \gamma\left(\overline{G}\right) = 5 + 7 = 12$



Cartesian product of complement of star zero divisor Graph of $\Gamma\left(\overline{Z_{10} \times Z_{14}}\right)$

7.APPLICATION OF DOMINATION OF A ZERO DIVISOR GRAPH IN WI-FI SIGNALS

This Section deals with a real life application on domination of zero divisor graph in Wireless Fidelity (Wi-Fi) signals.

Domination of A Zero Divisor Graph in Wi-Fi signals

St. Joseph's College Of Arts and Science (Autonomous) has 5 blocks namely Venmani block, Durier block, Terbes block, Convention hall block, and Administrative block. An administrative block runs all maintenance, office work, Controller of Examination, and etc. Each block contains many classes, departments, and labs. Administrative block has main Wi-Fi connection in its own. In this administrative block, the main Wi-Fi connection server is available which serves the Wi-Fi signal connection to each blocks simultaneously but the range varies from strong to weak for the blocks because of the distance between main server and block. In this paper, we discussed about the application of zero – divisor graph and its domination.

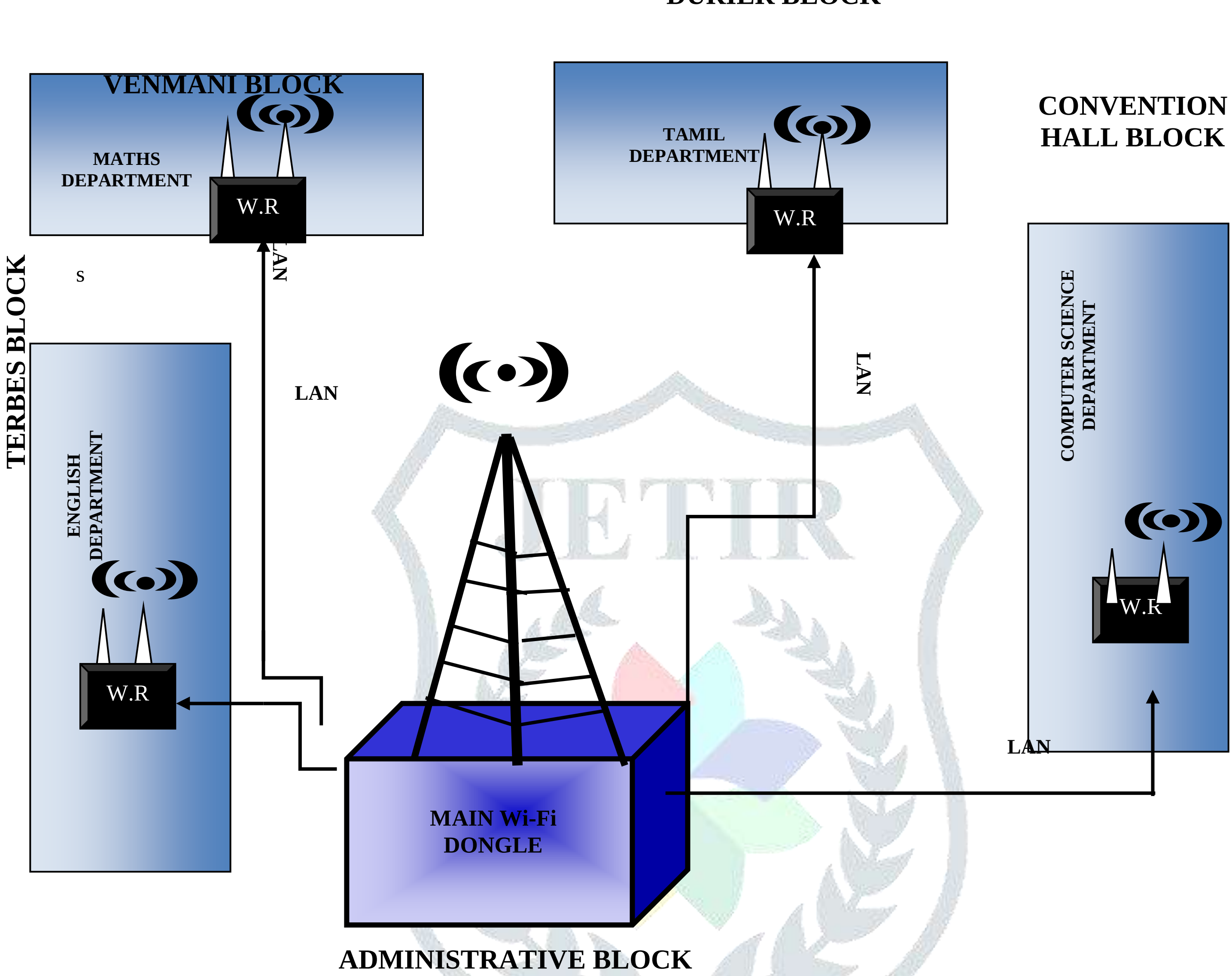
We have given a better solution for this fluctuating signal connections running in the college blocks for getting good range full strength connection of Wi-Fi to each block in the college campus. Through this project, we described how to give the good signal strength to all of the blocks near-by using main **Wi-Fi Dongle** and **Local Area Network (LAN) cables**. We can supply good Wi-Fi connection to each block through LAN cables and **Wi-Fi Router (W.R).**

In Venmani block, we can choose department of 'Mathematics'. In Terbes block, we can choose 'English' department. In Convention Hall block, we can choose 'Computer Science' department. In Durier block, we can choose 'Tamil' department, respectively.

All of the 4 blocks have their one chosen department. In this department, we fix the sub Wi-Fi Router for better signal transportation. So, the signal will pass from main server to the sub Wi-Fi Router in each block. From that sub Wi-Fi Router, other departments in that block will have their connections properly, so that every students and staffs of the St. Joseph's College will get their proper good Wi-Fi signal connection for their study purpose. The purpose of the sub Wi-Fi Router connection is to provide good range of Wi-Fi connection to all the classes and departments in blocks. If we want to cut the signal of only one block in college, we can pull off the sub Wi-Fi Router in that block, so that other blocks will receive their connection properly. The graphical representation of Wireless network of St. Joseph's College has a star zero divisor graph. With the help of Star Zero Divisor Graph and its Domination, we can get proper and good range of the Wi-Fi connection in the whole college campus.

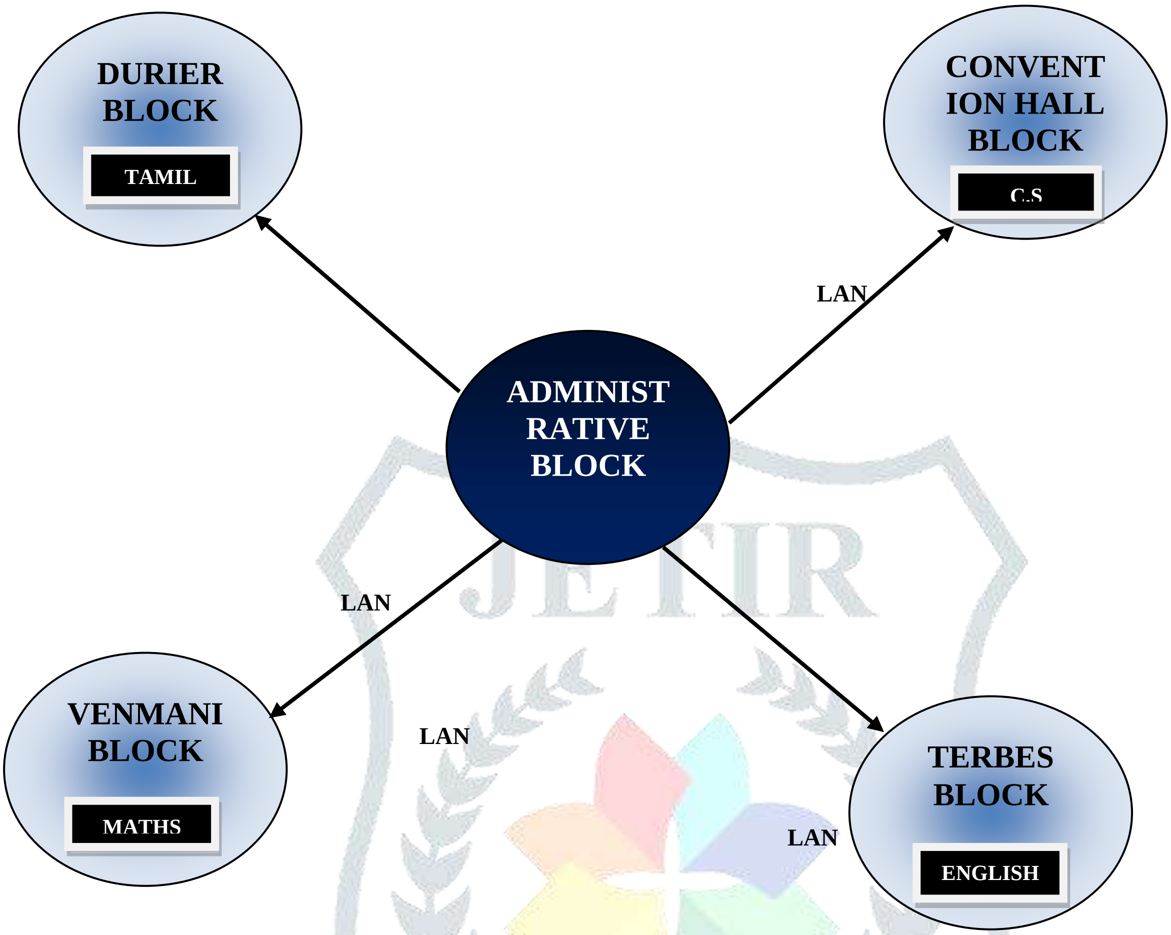
Based on star zero divisor graph $\Gamma(Z_{10})$, Wi-Fi Router gives proper signal to their adjacent blocks. The graphical representation of a Wireless network described the star zero divisor graph. It has 5 vertices, each vertex represents one block. Since the administrative block is adjacent to every other blocks namely Venmani block, Durier block, Convention Hall block, and Terbes block, Wi-Fi router produces the proper Wi-Fi signal to the specified department. Also administrative block dominates all the 4 blocks and its domination number is one.

Wireless Network of St. Joseph's College



Wireless Local Area Network (WLAN)of St. Joseph's College

Graphical Representation of Wireless Network



Star Zero Divisor Graph of $\Gamma(Z_{10})$

8. CONCLUSION

In this paper, we have briefly discussed about the domination of Cartesian product of two star zero divisor graph and established bounds for its complement . Further we discussed an application of domination in wi-fi networks and we have found that the dominating phase of Wi-Fi connection of places and good range of Wi-Fi signal connection to everywhere in the college.This work can be extended to various types of Cartesian product of zero divisor graph.

REFERENCES

[1] Adrianna Guillory, Mhel Lazo, Laura Mondello, and Thomas Naugle ***"Realizing Zero Divisor Graphs"***.

[2]David F. Anderson, Michael C. Axtell, and Joe A. Stickles, Jr, ***"Zero-divisor graphs in commutative rings"***, (2011), 23-42.

[3] David F. Anderson and Philip S. Livingston, ***"The Zero-Divisor Graph of a Commutative Ring",*** Journal of Algebra 217, 434-447 (1999).

[4] S. Arumugam, S. Ramachandran, ***"Invitation to GRAPH THEORY"***, June 2001.

[5] K. Budadoddi, ***"Some studies on domination parameters of euler totient cayley graphs, Zero divisor graphs and line graph of zero divisor graphs",*** May 2016.

[6] Carlos Lopez, Alonza Terry, and Alaina Wickboldt, ***"Zero divisor graph"***, 1991 Mathematics Subject Classification.

[7] G.Easwara Prasad and P.Suganthi, ***"Domination Parameters of*** $f_{n.r}$***",*** Global Journal of Pure and Applied Mathematics,Volume13, Number 12 (2017).

[8] Dr. R. B. Gnanajothi, R. Ezhilmary, "***Domination and global domination in graph structure"***, Mathematical Sciences International Research Journal : Volume 5 spl issue (2016).

[9] Jennifer M. Tarr, ***"Domination in graphs"***, (2010).

[10] Jim Coykendall, Sean Sather – Wagstaff, Laura Sheppardson, and Sandra Spiroff, ***"On Zero Divisor Graph"***, Progress in Commutative Algebra 2, 2012.

[11] Paul Van Dooren, Universite Catholique de Louvain, Louvain – la – Neuve, Belgium, ***"Graph Theory and applications"***, Dublin, August 2009.

[12] Sanjoy Kalita, Kuntala Patra, ***"Prime Graph of Cartesian Product of Rings",*** International Journal of Computer Applications (0975-8887), Volume69 – No.10, May2013.

[13] N. Thamaraikannan, J. Jagadeeswaran, S.Suresh, ***"Domination number of complement graph"***, International Journal of Latest Trends in Engineering and Technology, Vol.6 issue 3 January 2016.