Changing and Unchanging Strong Restrained Domination number of a Graphs

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Abstract: Let \(G = (V, E)\) be a simple graph. A Subset \(S\) of \(V\) is said to be strong restrained dominating set or restrained strong dominating set of \(G\) if for every \(u \in V - S\), there exists elements \(v \in S\) and \(w \in V - S\) such that \(v\) and \(w\) strongly dominates \(u\). The minimum cardinality of a strong restrained dominating set of \(G\) is called the strong restrained domination number of \(G\) and is denoted by \(\gamma_{sr}(G)\). In this paper, changing and unchanging strong restrained domination number of a graphs are determined.

Keywords: Domination, strong domination, restrained domination, strong restrained domination.

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1. INTRODUCTION

Throughout this paper, finite, undirected, simple graph is considered. Let \(G = (V, E)\) be a simple graph. The degree of any vertex \(u\) in \(G\) is the number of edges incident with \(u\) and is denoted by \(\deg u\). The minimum and maximum degree of a vertex is denoted by \(\delta(G)\) and \(\Delta(G)\) respectively. A vertex of degree one is called a pendant (end) vertex and a vertex which is adjacent to an end vertex is called a support vertex.

A set \(S \subseteq V\) is a dominating set of \(G\) if every vertex not in \(S\) is adjacent to a vertex in \(S\). The domination number of \(G\), denoted by \(\gamma(G)\), is the minimum cardinality of a dominating set [1]. The concept of strong domination in graphs was introduced by Sampathkumar and Puspalatha[5] and the restrained domination was introduced by Domke [2] et al. A set \(S \subseteq V(G)\) is a strong dominating set of \(G\) if every vertex \(v \in V - S\) is strongly dominated by some vertex \(u\) in \(S\). A set \(S \subseteq V(G)\) is a restrained dominating set of \(G\), if every vertex not in \(S\) is adjacent to a vertex in \(S\) and to a vertex in \(V - S\). The restrained domination number of a graph \(G\), denoted by \(\gamma_r(G)\), is the minimum cardinality of a restrained dominating set in \(G\). The strong restrained domination was introduced by Selvaloganayaki and Namasivayam [6]. For all graph theoretic terminologies and notations, Harary [3] is referred to. In this paper, changing and unchanging strong restrained domination number of a graphs are characterized.

Definition 1.1: Let \(G = (V, E)\) be a simple graph. A subset \(S\) of \(V\) is said to be a strong restrained dominating set of \(G\) if for every \(u \in V - S\), there exists \(v \in S\) and \(w \in V - S\) such that \(v\) and \(w\) strongly dominate \(u\). The minimum cardinality of a strong restrained dominating set of \(G\) is called the strong restrained domination number of \(G\) and is denoted by \(\gamma_{sr}(G)\).

The existence of a strong restrained dominating set of \(G\) is guaranteed, since \(V(G)\) is a strong restrained dominating set of \(G\).

Example 1.2: Consider the following graph \(G\),

\[
S = \{v_3, v_4\} \text{ is a strong restrained dominating set of } G. \text{ Since every vertex in } V - S \text{ has one strong neighbour in } S \text{ and one strong neighbour in } V - S.
\]
Result 1.3: For the path \( P_{n} \), \( \gamma_{ad}(P_{n}) = \left\{ \begin{array}{ll} n + 2 & \text{if } m = 3n \\ n + 3 & \text{if } m = 3n + 1 \text{ where } n \geq 1 \\ n + 4 & \text{if } m = 3n + 2 \end{array} \right. \)

Result 1.4: \( \gamma_{ad}(C_{n}) = \gamma(C_{n}) = n - 2 \left( \frac{n}{3} \right) \geq 3. \)

Result 1.5: \( \gamma_{ad}(K_{n}) = 1, n \geq 3. \)

Result 1.6: \( \gamma_{ad}(W_{n}) = 1, n \geq 4. \)

Result 1.7: For \( n \geq 1 \), \( \gamma_{ad}(K_{1,n}) = n + 1. \)

Result 1.8: For \( r, s \geq 1 \), \( \gamma_{ad}(D_{r,s}) = r + s + 2. \)

Result 1.9: Let \( G = K_{m,n} \) where \( m, n \in N. \) Then \( \gamma_{ad}(G) = \left\{ \begin{array}{ll} 2 & \text{if } m = n \\ m + n & \text{otherwise} \end{array} \right. \)

Result 1.10: Let \( G \) be a connected graph.

(i) If \( G \) has a unique full degree vertex \( u \) then any strong restrained dominating set of \( G \) contains \( u. \)

(ii) If \( G \) has two full degree vertices \( v \) and \( w, \) then any strong restrained dominating set of \( G \) contains \( v \) and \( w. \)

Result 1.11: If \( G \) is a graph with at least 3 full degree vertices, then \( \gamma_{ad}(G) = 1. \)

2. Main Result: In this chapter, the changing and unchanging values of \( \gamma_{ad} \) when a vertex is removed and an edge is removed from a graph is studied.

Definition 2.1 [4]: Following the notation used in the case of domination, we partition the vertex set \( V(G) \) into subsets \( V_{0}, V_{1}, V_{r} \) as follows:

\[
V_{0}^{+}(G) = \{ v \in V(G) : \gamma_{ad}(G) > \gamma_{ad}(G - v) \} \\
V_{1}^{+}(G) = \{ v \in V(G) : \gamma_{ad}(G) = \gamma_{ad}(G - v) \} \\
V_{r}^{+}(G) = \{ v \in V(G) : \gamma_{ad}(G) < \gamma_{ad}(G - v) \}.
\]

Theorem 2.2: Let \( G = P_{3n}, n \geq 1. \) Let \( v_{i} \) be a vertex of \( P_{3n}. \) Then \( V_{r}^{+}(G) = V(G). \)

Proof: Case i: Let \( v_{i} \) be an end vertex of \( P_{3n}. \) Thus \( P_{3n} - v_{i} = P_{3n - 1}. \) \( \gamma_{ad}(P_{3n - 1}) = n + 3 \) and \( \gamma_{ad}(P_{3n}) = n + 2. \) Therefore \( \gamma_{ad}(P_{3n} - v_{i}) > \gamma_{ad}(P_{3n}). \) Hence \( v_{i} \in V_{r}^{+}(G). \)

Case ii: Suppose \( v_{i} = v_{k} \) or \( v_{i} = v_{3n - 1}. \) Thus \( P_{3n} - v_{i} = P_{3n - 1}. \) \( \gamma_{ad}(P_{3n - 1}) = n + 2. \) Therefore \( \gamma_{ad}(P_{3n} - v_{i}) > \gamma_{ad}(P_{3n}). \) Hence \( v_{i} \in V_{r}^{+}(G). \)

Case iii: Suppose \( v_{i} = v_{3} \) or \( v_{i} = v_{3n - 2}. \) Thus \( P_{3n} - v_{i} = P_{3n - 1}. \) \( \gamma_{ad}(P_{3n - 1}) = n + 1. \) Therefore \( \gamma_{ad}(P_{3n} - v_{i}) > \gamma_{ad}(P_{3n}). \) Hence \( v_{i} \in V_{r}^{+}(G). \)

Case iv: Suppose \( v_{i} = v_{3k}, 2 \leq k \leq n - 1. \) Thus \( P_{3n} - v_{i} = P_{3k + 1} \cup P_{3k - 3k - 1}. \) \( \gamma_{ad}(P_{3k + 1}) = k + 3 \) and \( \gamma_{ad}(P_{3k - 3k - 1}) = n = k + 2. \) Hence \( \gamma_{ad}(P_{3n} - v_{i}) = n + 5. \) Therefore \( \gamma_{ad}(P_{3n} - v_{i}) > \gamma_{ad}(P_{3n}). \) Hence \( v_{i} \in V_{r}^{+}(G). \)

Case v: Suppose \( v_{i} = v_{3k + 1}, 1 \leq k \leq n - 2. \) Thus \( P_{3n} - v_{i} = P_{3k + 1} \cup P_{3k - 3k - 1}. \) \( \gamma_{ad}(P_{3k + 1}) = k + 3 \) and \( \gamma_{ad}(P_{3k - 3k - 1}) = n = k + 2. \) Hence \( \gamma_{ad}(P_{3n} - v_{i}) = n + 5. \) Therefore \( \gamma_{ad}(P_{3n} - v_{i}) > \gamma_{ad}(P_{3n}). \) Hence \( v_{i} \in V_{r}^{+}(G). \)

In all the cases, \( V_{r}^{+}(G) = V(G). \) Hence the theorem.

Theorem 2.3: \( V_{0}^{+}(P_{3n}) = \emptyset, \) where \( n = 3n + 1, 3n + 2, n \geq 1. \)

Proof: Case i: Let \( G = P_{3n + 1}. \) Suppose \( v_{i} \in V_{0}^{+}(G), \) where \( 1 \leq i \leq 3n + 1. \) Then \( \gamma_{ad}(G - v_{i}) = \gamma_{ad}(G). \)

Subcase ia: Let \( v_{i} \) be an end vertex of \( G. \) Thus \( G - v_{i} = P_{3n}. \) \( \gamma_{ad}(P_{3n}) = n + 2. \) Therefore \( \gamma_{ad}(G - v_{i}) < \gamma_{ad}(G), \) a contradiction. Therefore \( v_{i} \) cannot be an end vertex of \( G. \)

Subcase ib: Suppose \( v_{i} \neq v_{1} \) or \( v_{i} \neq v_{3n - 1}. \) Thus \( G - v_{i} = P_{3n + 1} \cup P_{3n + 3n - 1}. \) \( \gamma_{ad}(P_{3n + 1}) = n + 3 \) and \( \gamma_{ad}(P_{3n + 3n - 1}) = n + 4. \) Therefore \( \gamma_{ad}(G - v_{i}) < \gamma_{ad}(G), \) a contradiction. Therefore \( v_{i} \neq v_{3k + 1}, 1 \leq k \leq n - 1. \)

Subcase ic: Suppose \( v_{i} \neq v_{3k + 1}, 1 \leq k \leq n - 2. \) Then \( G - v_{i} = P_{3k + 1} \cup P_{3n - 3n - 1}. \) \( \gamma_{ad}(P_{3k + 1}) = k + 3 \) and \( \gamma_{ad}(P_{3n - 3n - 1}) = n = k + 2. \) Hence \( \gamma_{ad}(G - v_{i}) = n + 6. \) Therefore \( \gamma_{ad}(G - v_{i}) > \gamma_{ad}(G), \) a contradiction. Therefore \( v_{i} \neq v_{3k + 1}, 1 \leq k \leq n - 1. \)

Subcase id: Suppose \( v_{i} = v_{3k}, 2 \leq k \leq n - 1. \) Thus \( G - v_{i} = P_{3k + 1} \cup P_{3n - 3n + 1}. \) \( \gamma_{ad}(P_{3k + 1}) = k + 3 \) and \( \gamma_{ad}(P_{3n - 3n + 1}) = n = k + 2. \) Hence \( \gamma_{ad}(G - v_{i}) = n + 6. \) Therefore \( \gamma_{ad}(G - v_{i}) > \gamma_{ad}(G), \) a contradiction. Therefore \( v_{i} \neq v_{3k}, 2 \leq k \leq n - 1. \)

Subcase id: Suppose \( v_{i} = v_{3k + 1}, 1 \leq k \leq n - 1. \) Thus \( G - v_{i} = P_{3k + 1} \cup P_{3n - 3n + 1}. \) \( \gamma_{ad}(P_{3k + 1}) = k + 2 \) and \( \gamma_{ad}(P_{3n - 3n + 1}) = n = k + 3. \) Hence \( \gamma_{ad}(G - v_{i}) = n + 5. \) Therefore \( \gamma_{ad}(G - v_{i}) > \gamma_{ad}(G), \) a contradiction. Therefore \( v_{i} \neq v_{3k + 1}, 1 \leq k \leq n - 1. \)
Theorem 2.4: Let $G = C_m$, $m \geq 4$. Then $V_{sr}(G) = V(G)$.

Proof: Case i: Let $G = C_{3n}$, $n \geq 2$. Let $v \in V(G)$. Then $y_{sr}(G) = n$, $G - v$ is a path $P_{3n-1}$ and $y_{sr}(P_{3n-1}) = n + 3$. Therefore $y_{sr}(G - v) > y_{sr}(G)$.

Case ii: Let $G = C_{3n+1}$, $n \geq 1$. Let $v \in V(G)$. Then $y_{sr}(G) = n + 1$, $G - v$ is a path $P_{3n}$ and $y_{sr}(P_{3n}) = n + 2$. Therefore $y_{sr}(G - v) > y_{sr}(G)$.

Case iii: Let $G = C_{3n+2}$, $n \geq 1$. Let $v \in V(G)$. Then $y_{sr}(G) = n + 2$, $G - v$ is a path $P_{3n+1}$ and $y_{sr}(P_{3n+1}) = n + 3$. Therefore $y_{sr}(G - v) > y_{sr}(G)$. Therefore $V_{sr}(G) = V(G)$. Hence the theorem.

Remark 2.5: Let $G = C_1$. Let $v \in V(G)$. Then $y_{sr}(G) = 1$, $G - v$ is a path $P_2$ and $y_{sr}(P_2) = 2$. Therefore $y_{sr}(G - v) > y_{sr}(G)$. Therefore $V_{sr}(G) = V(G)$.

Theorem 2.6: Let $G = K_{1,n}$. Then $V_{sr}(G) = V(G)$, $n \geq 2$.

Proof: Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{v_i v_j : 1 \leq i \leq n - 1\}$. Therefore $y_{sr}(K_{1,n}) = n + 1$.

Case i: Suppose $G - v$ is a complete bipartite graph $K_{m,n}$ and $y_{sr}(K_{m,n}) = n$. Therefore $y_{sr}(G - v) < y_{sr}(G)$. Hence $v \in V_{sr}(G)$.

Case ii: Suppose $G - v$ is a cycle $C_n$ and $y_{sr}(C_n) = n - 2$. Therefore $y_{sr}(G - v) > y_{sr}(G)$. Hence $v \in V_{sr}(G)$, a contradiction.

Theorem 2.7: $V_{sr}(W_n) = \emptyset$, $n \geq 4$.

Proof: Let $G = W_n$, $n \geq 4$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and let $E(G) = \{v_i v_j : 1 \leq i \leq n - 1\}$. Therefore $y_{sr}(W_n) = 1$. Suppose $v \in V_{sr}(W_n)$. Then $y_{sr}(G - v) > y_{sr}(G)$, a contradiction. Therefore $V_{sr}(W_n) = \emptyset$. Hence the theorem.

Theorem 2.8: Let $G = K_{m,n}$, $m, n \geq 2$. Then $V(G) = \{y_{sr}(G) \text{ if } m = n \}$.

Proof: Let $G = K_{m,n}$, $m, n \geq 2$. Let $V(G) = \{v_i \in V(G) / 1 \leq i \leq m, 1 \leq j \leq n\}$. Therefore $y_{sr}(K_{m,n}) = m + n - 1$. Suppose $v \in V_{sr}(G)$. Then $y_{sr}(G - v) = y_{sr}(G)$ and $y_{sr}(G - v) < y_{sr}(G)$. Therefore $V_{sr}(G) = V(G)$.

Case i: Suppose $m = n$. Therefore $V_{sr}(G) = V(G)$.

Case ii: Suppose $m < n$.

Subcase iia: Suppose $n - m = 1$, $y_{sr}(G) = m + n$.

Subcase iiai: Suppose $G - u$ is a complete bipartite graph $K_{m-1,n}$ and $y_{sr}(G - u) = m + n - 1 < y_{sr}(G)$.

Subcase iiaii: Suppose $G - v$ is also a complete bipartite graph $K_{m-1,n}$, $m = n - 1$; then $y_{sr}(G - v) = 2 < y_{sr}(G)$.

Subcase iib: Suppose $n - m = 1$, $y_{sr}(G) = m + n$.

Subcase iib: Suppose $G - u$ is a complete bipartite graph $K_{m,n-1}$ and $y_{sr}(G - u) = m + n - 1 < y_{sr}(G)$.

Subcase iibii: Suppose $G - v$ is also a complete bipartite graph $K_{m,n-1}$, $m = n - 1$, then $y_{sr}(G - v) = m + n - 1 < y_{sr}(G)$. Therefore $V_{sr}(G) = V(G)$. Hence the theorem.

Theorem 2.9: Let $G = D_{t,r}$, $s \geq 2$. Then $V_{sr}(G) = V(G)$.

Proof: Let $v \in V(G)$, $y_{sr}(G) = r + s + 2$. Thus $G - v = K_{t,s}$ or $sK_t$ (or) $rK_t$, $K_{t,1}$, $K_{t,1}$ (or) $K_{s-1}$, $K_{s-1}$, $sK_t = r + s + 1 < y_{sr}(G)$. Therefore $v \in V_{sr}(G)$. Therefore $V_{sr}(G) = V(G)$. Hence the theorem.

Definition 2.10 [4]: Following the notation used in the case of domination, we partition the edge set $E(G)$ into subsets $E_0, E_1, E_2, E_3, E_4$. Then $E_{sr}(G) = E(G)$.

Proof: Let $G = P_{3n}$, $n \geq 2$. Let $v \in V(G)$. Then $y_{sr}(G) = y_{sr}(P_{3n}) + 1$. Hence $v \in E_{sr}(G)$.

Case i: Suppose $e_1 = e_2$ or $e_1 = e_3$. Thus $P_{3n} - e_1 = P_1 \cup P_{3n-1}$ and $y_{sr}(P_1) = n + 3$, $y_{sr}(P_{3n-1}) = n + 4$. Therefore $y_{sr}(P_{3n} - e_1) = y_{sr}(P_{3n})$. Hence $e_2 \in E_{sr}(G)$.

Case ii: Suppose $e_0 = e_2$ or $e_0 = e_3$. Thus $P_{3n} - e_0 = P_1 \cup P_{3n-2}$ and $y_{sr}(P_{3n-2}) = n + 2$, $y_{sr}(P_{3n-2}) = n + 4$. Therefore $y_{sr}(P_{3n} - e_0) = y_{sr}(P_{3n})$. Hence $e_2 \in E_{sr}(G)$.

Case iii: Suppose $e_0 = e_0$, $1 \leq k < n - 1$. Thus $P_{3n} - e_0 = P_1 \cup P_{3n-3}$ and $y_{sr}(P_{3n-3}) = n + k + 2$ and $y_{sr}(P_{3n-3}) = n + k + 2$. Therefore $y_{sr}(P_{3n} - e_0) = y_{sr}(P_{3n})$. Hence $e_2 \in E_{sr}(G)$.

Case iv: Suppose $e_0 = e_{k+1}$, $1 \leq k < n - 2$. Thus $P_{3n} - e_0 = P_1 \cup P_{3n-3}$ and $y_{sr}(P_{3n-3}) = n + k + 3$ and $y_{sr}(P_{3n-3}) = n + k + 3$. Therefore $y_{sr}(P_{3n} - e_0) = y_{sr}(P_{3n})$. Hence $e_2 \in E_{sr}(G)$. 

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Case v: Suppose $e_i = e_{3i+2}$, $1 \leq k \leq n - 2$. Thus $P_{3n} - e_i = P_{3i+2} \cup P_{3n-3k-2}$. Hence $\gamma_{srd}(P_{3n-3k-2}) = k + 4$ and $\gamma_{srd}(P_{3n}) = n + 3$. Therefore $\gamma_{srd}(P_{3n}) > \gamma_{srd}(P_{3i+2})$. Hence $e_i \in E^+_s(G)$. In all the cases, $E^+_s(G) = E(G)$. Hence the theorem.

Theorem 2.12: $E^+_s(P_{3n}) = \emptyset$, where $m = 3n + 1, n \geq 2, 3n + 2, n \geq 1$.

**Proof:** Case i: Let $G = P_{3n}$. Suppose $e_i \notin E^+_s(G)$, where $1 \leq i \leq 3n + 1$. Therefore $\gamma_{srd}(G - e) < \gamma_{srd}(G)$.

**Subcase ia:** Suppose $e_i = e_i = e_{3n+1}$. Thus $G - e = P_i \cup P_{3n-1}$. Hence $\gamma_{srd}(G - e) = n + 2$ and $\gamma_{srd}(G - e) = n + 3$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \in E^+_s(G)$.

**Subcase ib:** Suppose $e_i = e_i = e_{3n+1}$. Suppose $e = e_i = e_{3n+1}$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \notin E^+_s(G)$.

**Subcase ic:** Suppose $e_i = e_i = e_{3n+1}$. Suppose $e = e_i = e_{3n+1}$. Therefore $\gamma_{srd}(G - e) = n + 5$ and $\gamma_{srd}(G - e) = n + 6$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \neq e_i = e_{3n+1}$.

**Subcase id:** Suppose $e_i = e_i = e_{3n+1}$. Suppose $e = e_i = e_{3n+1}$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \neq e_i = e_{3n+1}$.

Result 2.13: Let $G = P_i \cup P_{3n}$. Suppose $e_i = e_{3n+1}$. Then $\gamma_{srd}(G - e) = 3 = \gamma_{srd}(G)$ and $\gamma_{srd}(G - e) = n + 3$. Therefore $\gamma_{srd}(G - e) = \gamma_{srd}(G)$, a contradiction. Therefore $e_i \notin E^+_s(G)$.

Result 2.14: Let $G = P_i \cup P_{3n}$. Suppose $e_i = e_{3n+1}$. Then $\gamma_{srd}(G - e) = 4 = \gamma_{srd}(G)$, a contradiction. Therefore $e_i \in E^+_s(G)$.

Theorem 2.15: Let $G = C_{3n}, m \geq 3$. Then $E^+_s(G) = E(G)$.

**Proof:** Case i: Let $m = 3n, n \geq 1$. Let $e = e_i \in E(G)$. Then $\gamma_{srd}(G - e) = n$, $G - e = P_i \cup P_{3n}$. Hence $\gamma_{srd}(G - e) = n + 2$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$. Hence $e_i \in E^+_s(G)$.

Case ii: Let $m = 3n + 1, n \geq 1$. Let $e = e_i \in E(G)$. Then $\gamma_{srd}(G - e) = n + 1$. $G - e$ is a path $P_{3n+1}$ and $\gamma_{srd}(G - e) = n + 3$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$. Hence $e_i \in E^+_s(G)$.

Case iii: Let $m = 3n + 2, n \geq 1$. Let $e = e_i \in E(G)$. Then $\gamma_{srd}(G - e) = n + 2$. $G - e$ is a path $P_{3n+2}$ and $\gamma_{srd}(G - e) = n + 4$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$. Hence $e_i \in E^+_s(G)$.

Theorem 2.16: Let $G = K_{1,n}$. Suppose $e_i = e_{3i+1}, e_{3i+2} \in E(G)$, $n \geq 2$.

**Proof:** Let $e = e_i \in E(G)$. Then $\gamma_{srd}(G - e) = n + 1$. $G - e = K_{1,n-1}$. Therefore $\gamma_{srd}(G - e) = n + 1$. Therefore $\gamma_{srd}(G - e) = \gamma_{srd}(G)$. Hence $e_i \in E^+_s(G)$. Therefore $E^+_s(G) = E(G)$. Hence the theorem.

Theorem 2.17: Let $G = D_{r,s}, r, s \geq 1$. Then $E^+_s(D_{r,s}) = E(G)$.

**Proof:** Let $e = e_i \in E(G)$. Then $\gamma_{srd}(G - e) = r + s + 2$. Suppose $G = D_{r,s-1}$. Therefore $\gamma_{srd}(G - e) = r + s + 2$. Therefore $\gamma_{srd}(G - e) = E(G)$. Hence $e_i \in E^+_s(G)$. Therefore $E^+_s(G) = E(G)$. Hence the theorem.

Theorem 2.18: Let $G = K_n, n \geq 2$. Then $E^+_s(K_n) = E(G)$.

**Proof:** Let $e = e_i \in E(G)$. Then $\gamma_{srd}(G - e) = 1$. $G - e$ has at least 3 full degree vertices, by result 1.11, $\gamma_{srd}(G - e) = 1$. Therefore $\gamma_{srd}(G - e) = \gamma_{srd}(G)$. Hence $e_i \in E^+_s(G)$. Therefore $E^+_s(G) = E(G)$. Hence the theorem.

Result 2.19: Let $G = K_4$. Suppose $e_i = e_{3i+1}, e_{3i+2} \in E(G)$. Then $\gamma_{srd}(G - e_i) = n + 1$. $G - e_i$ has at least 3 full degree vertices, by result 1.11, $\gamma_{srd}(G - e_i) = 1$. Therefore $\gamma_{srd}(G - e_i) = \gamma_{srd}(G)$. Hence $e_i \in E^+_s(G)$. Therefore $E^+_s(G) = E(G)$. Hence the theorem.

Theorem 2.20: Let $G = W_n, n \geq 5$. Then $E^+_s(W_n) = E(G)$.

**Proof:** Let $V(G) = \{v_i, v_j, 1 \leq i \leq n\}$, $E(G) = \{e_1 = v_i v_{i+1} / 1 \leq i \leq n - 2 \} \cup \{e_{n-1} = v_n v_1 \} \cup \{e_{n+1} = v_1 v_i / 1 \leq i \leq n - 1\}$ and $\gamma_{srd}(W_n) = 1$. Suppose $e_i, e_{3i+1}, e_{3i+2} \in E_s(G)$. Then $\gamma_{srd}(G - e_i) < \gamma_{srd}(G), \gamma_{srd}(G - e_i) < \gamma_{srd}(G)$, and $\gamma_{srd}(G - e_i) < \gamma_{srd}(G)$, a contradiction.

Case i: $G - e_1 \in K_{2,3}$. Suppose $e_i \in E_s(G)$, a contradiction. Therefore $e_i \in E_s(G)$, a contradiction. Therefore $e_i \in E_s(G)$, a contradiction. Therefore $e_i \in E_s(G)$, a contradiction.

Case ii: $G = G - e_1 = P_{n+1}$ and $e_1 \in E_s(G)$. Hence $e_i \in E_s(G)$, a contradiction.

Case iii: $G = G - e_1 = P_{n+1}$ and $e_1 \in E_s(G)$. Hence $e_i \in E_s(G)$, a contradiction.

Case iv: $G = G - e_1 = P_{n+1}$ and $e_1 \in E_s(G)$. Hence $e_i \in E_s(G)$, a contradiction.
\( \gamma_{str}(G) \). Hence \( c_{i \times n - 1} \in E_{\gamma_{str}}(G) \), a contradiction. From cases (i) and (ii), there is no edges belong to \( E_{\gamma_{str}}(G) \). Therefore \( E_{\gamma_{str}}(G) = \emptyset \). Hence the theorem.

**Result 2.21:** Let \( G = W_n \). Let \( e \in E(G) \). \( \gamma_{str}(G) = 1 \). \( G - e \) has two full degree vertices, by theorem 1.10, any strong restrained dominating set of \( G \) consists of two full degree vertices and there is no vertex to strongly dominate the remaining two vertices, they also belong to strong restrained dominating set of \( G \). Hence \( \gamma_{str}(G - e) \geq 4 \). Therefore \( \gamma_{str}(G - e) > \gamma_{str}(G) \). Hence \( e \in E_{\gamma_{str}}(G) \). Therefore \( E_{\gamma_{str}}(G) = E(G) \).

**Theorem 2.22:** Let \( G = W_4 \). Let \( e \in E(G) \), \( \gamma_{str}(G) = 1 \). \( G - e \) has two full degree vertices, by theorem 1.10, any strong restrained dominating set of \( G \) contains two full degree vertices and there is no vertex to strongly dominate the remaining two vertices, they also belong to strong restrained dominating set of \( G \). Hence \( \gamma_{str}(G - e) \geq 4 \). Therefore \( \gamma_{str}(G - e) > \gamma_{str}(G) \). Hence \( e \in E_{\gamma_{str}}(G) \). Therefore \( E_{\gamma_{str}}(G) = E(G) \).

**Remark 2.23:** Suppose \( m = n = 2 \), \( \gamma_{str}(K_{2, 2}) = 2 \). Since \( K_{2, 2} - e = P_4 \), \( \gamma_{str}(K_{2, 2} - e) = 4 \). Hence \( \gamma_{str}(K_{2, 2} - e) > \gamma_{str}(K_{2, 2}) \). Therefore \( e \in E_{\gamma_{str}}(K_{2, 2}) \). Hence \( E_{\gamma_{str}}(K_{2, 2}) = E(K_{2, 2}) \).

### 3. CONCLUSION

In this paper, the authors studied changing and unchanging strong restrained domination number of a graphs. Similar studies can be made on this type.

**REFERENCES**


