Study of Forward, Backward and Evanescent Waves in a Periodic Array

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Abstract : A periodic geometries constructed by repetition of a unit cell or building block in one, two or three dimensions in the electromagnetic band gap structure. In our daily life, microwave filter, antenaa etc. are designed by the use of EBG structure. As these apparatus are the array of infinitely periodic the plant electromagnetic wave is partially reflected back forth and finally attenuates. The attenuation wave is the evanescent wave. In the light of this, we study here the forward, backward and evanescent wave using floquet's theorem.

Keywords : Attenuation, Evanescent wave, Band gap, Microwave.

I. Introduction

One dimensional (1D) periodic structure, i.e. structures in which repetition of the cell occurs in one dimension, are often utilized as microwave filters. Two dimensional (2D) periodic structures, i.e. repetition exists along two directions, are the most popular tpe of EBG Structures. The forward wave and backward wave behaviours of electromagnetic waves in two dimension photonic crystals were studied in[10]. They have been utilized in the design of simultaneous switching noise (SSN) in electronic circuit [3] and [4]. Three dimensional (3D) EBG structures, i.e. unit cell repeats along all three spatial dimensions, have been studied [5].

The 2D geometries can be broadly classified into two categories : Textured type and Uniplanar structures. Uniplanar or patterned EBG structures can be realized as a grid/meshed plane[8], a 2D stepped impedance structure [6], interconnected slotted patches [7], metal patches connected by meander lines [9].

The geometry of threse EBG structures can be altreed in order to obtain the bandgap in the desired frequency region. In simplified cases an EBG structure can be approximated with a surface impedance. This enables prediction of the bandgap using analytical approaches.

II. Floguet's Theorem

The fundamental expression underlying the analysis of periodic structures is known as Floquet theorem which has been discussed in[1]. According to this theorem, we have a periodic continuous function $Q(x)$ with a minimum period Π. Such that

$$Q(x+\Pi)=Q(x) \qquad \text{............ (2.1)}$$

then the second order first degree differential equation

$$y''+Q(x)y=0 \qquad \text{............. (2.2)}$$

has two continuously differentiable solution $y_1(x)$ and $y_2(x)$. Consider a characteristic equation is given by

$$\rho^2-\left[y_1(\Pi)+y_2^{'}(\Pi)\right]\rho+1=0 \qquad \text{............. (2.3)}$$

The above equation has two Eigen values in $\ell_1=e^{ie\Pi}$ and $\rho_2=e^{-ie\Pi}$ respectively. Since both eigen values are different from each other, then (2.3) has two linearly independent solutions,

$$f_1(x) = e^{iex} p_1(x) \qquad \text{.............. (2.4)}$$

$$f_2(x) = e^{-iex} p_2(x) \qquad \text{.............. (2.5)}$$

where $p_1(x)$ and $p_2(x)$ are periodic functions with period.

In the analysis of periodic structures in microwave engineering often Bloch-Floquet theorem is used. According to this theorem, there exists a correlation between the fields at a point in an infinite periodic structure and the field at a point period a away and they are found to differ from each other by a prospagating factor e^{ra}. Where γ is the propagation constant in the direction of propagation.

Based on this theorem, for the structure shown in Fig. (2.1) depicting a one dimensional periodic structure with a period *d*, the voltage and current relationships are given by

$$V_{n+1} = V_n e^{-\gamma d} \qquad \text{............. (2.6)}$$

$$I_{n+1} = I_n e^{-\gamma d} \qquad \text{............ (2.7)}$$

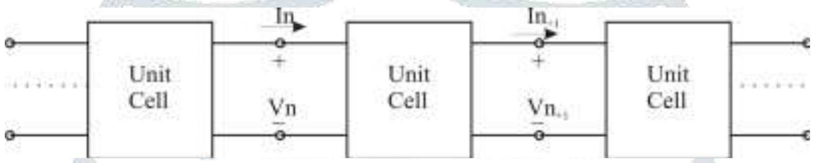

Fig. (2.1) One dimensional representation of a periodic structure

The most important derivation from Floquet's theorem is the possibility of expressing fields in a periodic geometry by restricting the analysis of a unit cell of the periodic structure. Once the field solution F of a particular point is determined, it is possible to predict the field solutions at a period *ma*, away by the following relationship,

$$F(x, y, z + ma) = e^{-\gamma ma} f(x, y, z) \qquad \text{................. (2.8)}$$

Thus Bloch-Floquet theorem be applied to a unit cell of the EBG structure.

III. FORWARD, BACKWARD AND EVANESCENT WAVES IN A PERIODIC ARRAY :

A 2D array of dipoles in fig. (3.1) is considered. The array is infinitely periodic along X- and Z- directions with a periodic D_x and D_z respectively. The array is excited by plane wave propagating in the direction described by

$$\hat{\varsigma} = \hat{x}\varsigma_x + \hat{y}\varsigma_y + \hat{z}\varsigma_z \qquad \text{............... (3.1)}$$

where ς_x, ς_y and ς_z are referred to as the direction cosines of the unit direction vector $\hat{\varsigma}$ of the plane wave.

Since it is an infinite periodic array, the current in each element obeys Floquet's theorem and is given by

$$I_{qm} = I_{o,o} e^{-j\beta_q D_x S_x} e^{-j\beta_m D_z S_z} \qquad \text{............. (1.3.2)}$$

where q and m refer to the column and row of an element, and $I_{o,o}$ refers to the current in the reference element in the array. In accordance with Ohm's law, the voltage in the reference element is -

$$V^{o,o}\left[Z + Z^{o,o}\right] I_{o,o} \qquad \text{.............. (3.3)}$$

where

$$Z^{o,o} = \sum_{q=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} Z_{o,qm} e^{-j\beta_q D_x S_x} e^{-j\beta_m D_z S_z} \qquad \text{.............. (3.4)}$$

is known as the scan impedance [2] and is defined as the array mutual impedance of the reference element. It is composed of numerous elemental mutual impedances referred to as $Z_o, q_m Z_L$ is the load impedance on the reference element.

When we consider an array element arbitrarily oriented along $\hat{P}^{(1)}$ with the reference element of the array at $\bar{R}^{(1)}$ and an external element oriented along $\hat{P}^{(2)}$ and having a reference element at $\bar{R}^{(2)}$ (illustrated in figure (3.2), then the mutual array impedance, based on (3.1) and (3.2) is given in [2] as -

$$Z^{2,1} = -\frac{V^{2,1}}{I_{o,o}} = \frac{Z_0}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{\bar{e}^{j\beta}\left(\bar{R}^{(2)} - \bar{R}^{(1)}\right)\hat{r}}{r_y}$$

$$\left[{}_1P^{(1)} + P^{(2)t} + 11^{P(1)} 11^{p(2)t}\right] \quad \text{................ (3.5)}$$

$$\hat{r} = \hat{x}\left(S_x + k\frac{\lambda}{D_x}\right) \pm \hat{y} r_y + \hat{z}\left(S_z + n\frac{\lambda}{D_z}\right) \text{for } y \neq 0 \quad \text{................ (3.6)}$$

$$r_y = \sqrt{1 - \left(S_x + k\frac{\lambda}{D_x}\right)^2 - \left(S_z + n\frac{\lambda}{D_z}\right)^2} \quad \text{................ (3.7)}$$

$$\frac{1}{11} P^{(1)} \cong \hat{P}^{(1)} \frac{1}{11} \hat{n} P^{(1)} \quad \text{................ (3.8)}$$

$$\frac{1}{11} P^{(2)t} \cong \hat{P}^{(2)} \frac{1}{11} \hat{n} P^{(2)t} \quad \text{................ (3.9)}$$

$$P^{(1)} = \frac{1}{I_0^{(1)}(R^{(1)})} \int_{-\ell_1}^{\ell_1} I_0^{(1)}(\ell) e^{-j\beta\ell\hat{p}^{(2)},\hat{r}} dl \quad \text{............... (3.10)}$$

$$P^{(2)t} = \frac{1}{I_0^{(2)t}(R^{(2)})} \int_{-\ell_2}^{\ell_2} I_0^{(2)t}(\ell) e^{-j\beta\ell\hat{p}^{(2)},\hat{r}} dl \quad \text{............. (3.11)}$$

In the above expression, $P^{(1)}$ and $P^{(2)t}$ are the pattern factors, ${}_1\hat{n}$ & ${}_{11}\hat{n}$ are the unit vectors perpendicular and parallel to the directional vector $\hat{r}$. (3.6) and (3.7) are the governing expressions for determining the nature of the mode of propagation associated with the given array. The exponential term in (3.5) is affiliated to a family of plane waves emanating from $\bar{R}^{(1)}$ and $\bar{R}^{(2)}$ propagating in the direction $\hat{r}$.

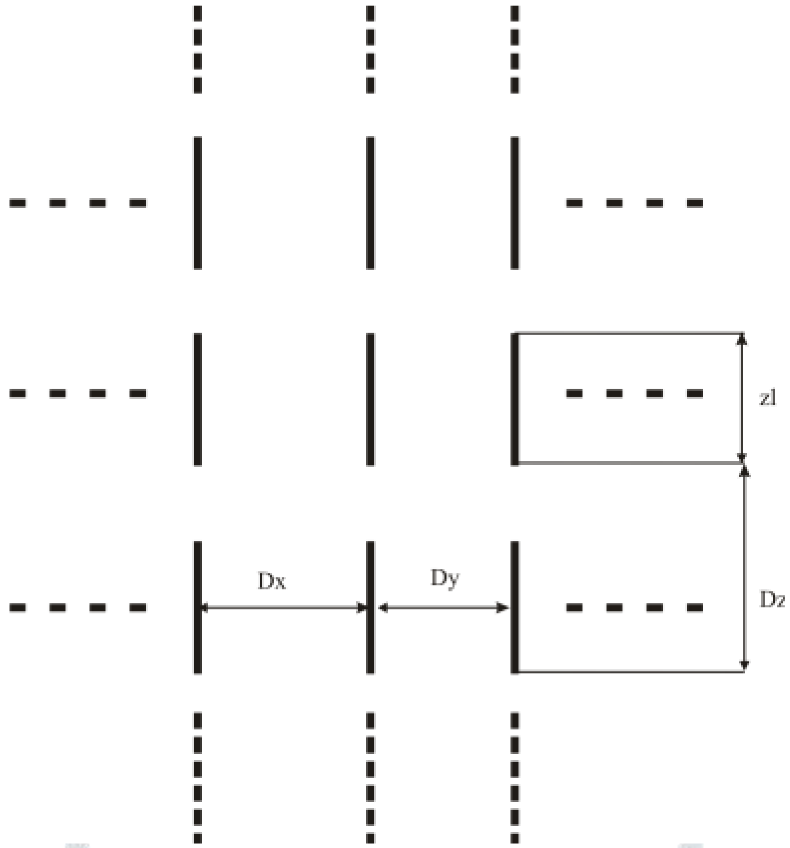

Fig. (3.1) An infinite x infinite dipole array with inter-element spacing of D_x and D_z and element length of 2ℓ .

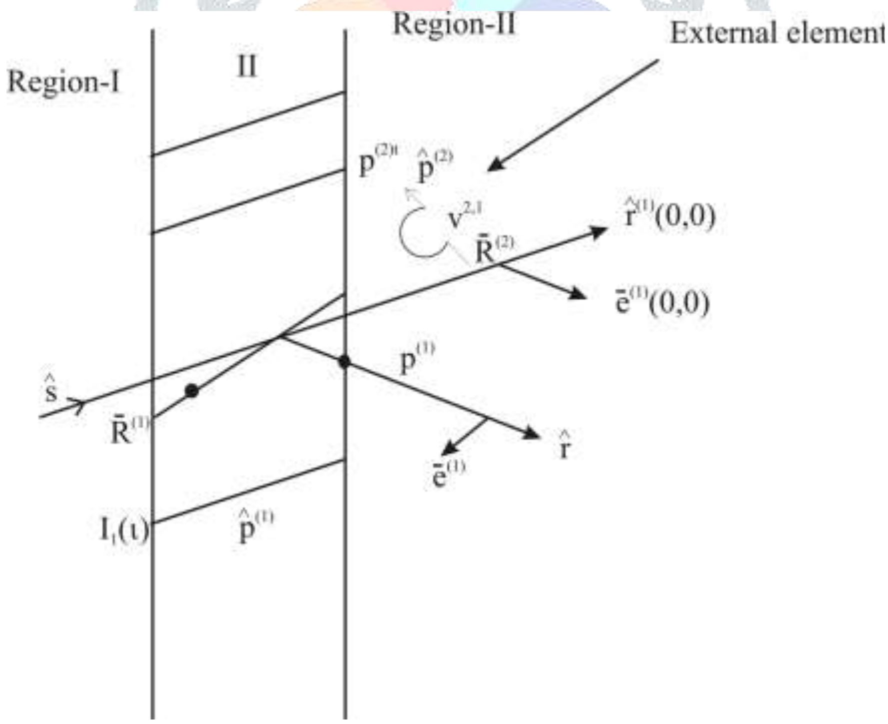

Fig. 3.2 : Array mutual impedance $Z^{2,1}$ between an array with element orientation $\hat{P}^{(1)}$ and an external element with orientation $\hat{P}_2^{(2)t}$ is obtained from the plane wave expression.

The direction and nature of these plane waves are strongly dependent upon the summation indices *k* and *n*. For the event when *k* = *n* = 0, the plane wave direction expression can be written as -

$$\hat{r} = \hat{x} S_x + \hat{y} s_y + \hat{z} s_z \qquad \text{.................. (3.12)}$$

$$\hat{r} = \hat{x} S_x - \hat{y} s_y + \hat{z} s_z \quad \text{.................. (3.13)}$$

(3.12) refers to the fact that plane waves follow the direction of propagation of the incident wave, $\hat{s} = \hat{x} S_x + \hat{y} s_y + \hat{z} s_z$ and are hence termed forward scattering waves.

Similarly (3.13) signify that the plane waves associated with them are in a direction opposite to the direction of the incident wave and are referred as bistatic reflected propagating waves. The nature of the waves depends upon the period of the array too :

If D_x and D_z are large, then (3.6) will result in real values of r_y, which implies that propagation is possible along this / these directions as well. As D_x and D_z tend to ∞. (3.6) yields imaginary values of r_y which results in the attenuation of the propagating waves. These waves are known as "evanescent waves".

IV. Discussion

An interesting property of the evanescent waves is discussed for their phase velocity along the direction of periodicity i.e. along r_x and r_z where

$$r_x = \left(S_x + k \frac{\lambda}{D_x} \right) \text{ and } r_z = \left(S_z + n \frac{\lambda}{D_z} \right)$$

respectively. However, the phase in the *y*-direction for imaginary remains unchanged. Hence, the magnitudes of these waves are inconsequential when compared to those of the propagating modes as one tends to move further away from the array, but are extremely strong as the other one (evanscent) moves closer to the array.

Another interesting phenomenon is the onset of grating lobes which occurs when $r_y = 0$ and (3.6) modifies to

$$\left(S_x + k \frac{\lambda}{D_x} \right)^2 + \left(S_z + n \frac{\lambda}{D_z} \right)^2 = 1 \quad \text{......}$$

This equation represents a family of circles with their centres at $k \frac{\lambda}{D_x}, k \frac{\lambda}{D_z}$.

V. Conclusion

As periods in X- & Z- direction i.e. D_x and D_z tends to infinity, equation (3.6) yields imaginary value of which results in the attenuation of the propagating waves termed as evancescent waves. The phase velocity of these waves remains real in the direction of periodicity. But for $r_y = 0$ (3.6) yields a family of circle with their centres at $\frac{K\lambda}{D_x}, \frac{n\lambda}{D_z}$.

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