Linear Programming Model for Optimal Portfolio Selection

Abstract

The study is based on the selection of optimal portfolio for investment out of stocks and mutual funds. Two shares each from Pharmaceutical sector and IT sector along with two large cap Mutual funds as well as their rate of return and investment requirements were analyzed. With the obtained information, a mathematical model was set up for using simplex method in which the problem was converted into its standard form of linear programming problem. Simplex method of optimization was used in determining the optimal portfolio and return. The LPP model results gave an optimal portfolio of ₹4,00,000, ₹4,00,000, ₹1,00,000, ₹1,00,000 for Sun Pharma, TCS, Infosys and Axis BlueChip fund respectively. Further using the sensitivity report the reasons for 0 investment in Cipla and ICICI Prubluechip fund were analyzed. With Simplex method the values to be invested in stocks and mutual funds were obtained at once instead of obtaining them separately. We further concluded that a combination of stock and mutual funds can optimally give the best return considering with the fact that diversification of investment is highly encouraged to reduce loss.

Portfolio is a popular investment technique and is often regarded as investment combination of stocks, bonds, commodities, mutual funds, exchange traded funds and closed funds. This technique has seen a very fast spread among urban elites and has experienced a continuous increase in demand during the last few years. It has seen a diversification into non-publicly tradable securities from real estate, art, and private investments. Investors hold these portfolios but are managed by financial professionals. Maximizing the returns and Minimizing the risk are the major objectives of every investor. Investment is driven up or down by Risk. This risk has to be managed and the long term potential of an investment depends on the investor being willing and prepared for the risk. One has to be ready for the risk and also has to decide the potential loss that can be afforded, (Litterman 2003). Diversification is the method adopted for reducing risk. It essentially results in the construction of portfolios.

The proper goal of portfolio construction would be to come up with a portfolio that gives the very best return and therefore the lowest risk. Such a portfolio would be referred to as the optimal portfolio. The method of finding the best portfolio is delineated as portfolio choice. The abstract framework and analytical tools for determining the best portfolio in disciplined and objective manner were provided by Harry Markowitz in his pioneering work on portfolio analysis delineated in 1952 Journal of Finance article and later book in 1959. His methodology of portfolio choice has come back to be referred to as the Markowitz model. In fact, Markowitz 's work marks the beginning of what is known today as modern portfolio theory. Considering the above, this study intends to carry out simplex optimization with a view to determining the optimal portfolio for an investor.

GCE Mbah and LC Onwukwe (2016) discussed portfolio selection and optimal financial investment in a developing economy using Linear Programming method. They particularly considered the Nigerian economy and got the values for liquidity, dividend and risk from five establishment used from the record of the Central bank of Nigeria. They took the ratio of each company's dividend, risk and liquidity to the total for each of these parameters and then formulated the effectiveness function and then the constraint equations. Thus they obtained a linear programming problem which was solved to get the three best stocks to invest in and the capital required for such investment.

Sukono and Y. Hidayat (2018) studied the investment portfolio optimization using linear programming model based on genetic algorithms. They assumed that the portfolio risk is measured by absolute standard deviation, and each investor has a risk tolerance on the investment portfolio. To complete the investment portfolio optimization problem, they arranged the issue into a linear programming model. Furthermore, determination of the optimum solution for linear programming was done by using a genetic algorithm.

Faith Konak and Bugra Bagci (2016) explained the fuzzy environment technique in linear programming problem and used Warner's Method for creating suitable portfolios and to determine how much to invest in various stocks. Moreover, it helps in achieving an optimal solution from a varying combination of returns and risks to calculate the expected return on investment. Two groups of data were chosen for portfolio management analysis: 33 securities over 72 months and 63 securities over 120 months. Four models were used, Konno's, Markowitz's, Cai's and Teo's. It was found that the risks were higher for Konno's and Markowitz's Models than for Cai's and Teo's model. Konno's Mean Absolute Deviation (MAD) Model was developed after Markowitz's, to improve on the quadratic programming problem of the latter by converting it to linear Programming. From the analysis of five BIST Mining companies, they found that the MAD model gives results based not only on high returns but also low risk, thus only KOZAL and PRKM were recommended.

Methodology: -

Portfolio Selection involves various methods and techniques, In this research as a numerical illustration, we analyzed the Rate of Return of Top 2 companies in terms of market capitalization in Pharma and IT sector respectively and two large cap mutual funds based on CRISIL MF Ranking. The data used is shown as: -

Investment	Rate of Return in 1 Year(%)
Pharmaceuticals –Sun Pharma	21.31
Pharmaceuticals-Cipla Ltd	10.23
IT- TCS	75.08
IT- Infosys	59.50
MF-Axis Bluechip fund D(G)	17.80
MF-ICICI PruBluechip fund D(G)	16.30

We have taken into consideration the following requirements for formulating the LPP Model: -

- Total amount-₹10,00,000.
- Amount in shares of a sector no larger than 50% of total available.
- Amount in shares of Sun Pharma less or equal to 80% of total amount in Sun Pharma and Infosys.
- Amount in shares of TCS less than or equal to 80% of the IT sector's total amount.
- Amount in Cipla shares less than or equal to 10% of the whole share amount.
- Amount in mutual funds less or equal to 25% of the amount in pharmaceutical shares.

Based on the gathered data the following LPP model is formed: -

Decision Variables: -
x_1 = invested amount in share of the Sun Pharma.
x_2 = invested amount in share of Cipla Ltd.
x_3 = invested amount in share of TCS.
x_4 = invested amount in share of Infosys.
x_5 = invested amount in Axis Bluechip fund.
x_6 = invested amount in ICICI PruBlue Chip fun.

Objective Function: - Max $Z=0.2131x_1+0.1023x_2+0.7508x_3+0.5950x_4+0.1780x_5+0.1630x_6$

Subjective Constraints: -
1. $x_1+x_2+x_3+x_4+x_5+x_6<=1000000$
2. $x_1+x_4<=500000$
3. $x_3+x_4<=500000$

4. $0.2x_1-0.8x_4<=0$
5. $0.2x_3-0.8x_4<=0$
6. $-0.1x_1+0.9x_2-0.1x_3-0.1x_4<=0$
7. $-0.25x_1-0.25x_2+x_5+x_6<=0$

Non-Negative Condition: - x_1, x_2, x_3, x_4, x_5, $x_6>=0$

To solve the mathematical set up model shown above using the simplex method, it requires standard form of linear programing problem: -

- All constraints should be expressed as equations by adding slack variables or surplus variables.
- The right-hand side of each constraint should be made non-negative if it is not already. This should be done by multiplying both sides of the resulting constraints by -1.

To apply the simplex/iterative method, it is necessary to state the problems in the form in which the inequalities in the constraints have been converted to equalities because it is not possible to perform arithmetic calculations upon an inequality: -

- Setup the inequalities describing the problem.
- Convert the inequalities to equations by adding slack variables.
- Enter the inequalities in a table for initial basic feasible solutions with all slack variables as basic variables. The table is called simplex table.
- Compute C_j and Z_j values for this solution where C is objective function coefficient for variable j and Z_j represents the decrease in the value of the objective function that will result if one unit of variable corresponding to the column is brought into the solution.
- Determine the entering variable (key column) by choosing the one with the highest C_j-Z_j value.
- Determine the key row (outgoing variable) by dividing the solution quantity values by their corresponding solution quantity values by their corresponding key column values and choosing the smallest positive quotient. This means that we compute the ratios for rows where elements in the key column are greater than zero.
- Identify the pivot element and compute the values of the key row by dividing all the numbers in the key row by the pivot element. Then change the product mix to the heading of the key column.
- Compute the values of the other non-key rows.
- Compute the Z_j and C_j- Z_j values for this solution.
- If the column value in C_j - Z_j row is positive, return to step (vi). If there is no positive C_j - Z_j, then the final solution has been reached.

  For the formulated LPP model Simplex will applied using Excel Solver Add-in.

Results and Discussion: -

The formulated LPP model is solved using Excel Solver and Sensitivity Report is generated. It provides a classical sensitivity analysis info for each linear and nonlinear programming issues, as well as twin values (in each cases) and vary info (for linear issues only).

Figure 1:-Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B2	x1 Amount (in Rs)	400000	0	0.2131	0.00385	0.07185
B3	x2 Amount (in Rs)	0	-0.07185	0.1023	0.07185	1E+30
B4	x3 Amount (in Rs)	400000	0	0.7508	0.2874	0.0154
B5	x4 Amount (in Rs)	100000	0	0.595	0.0154	0.2874
B6	x5 Amount (in Rs)	100000	0	0.178	2.8794	0.00385
B7	x6 Amount (in Rs)	0	-0.015	0.163	0.015	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B12	Value	1000000	0.17492	1000000	125000	0
B13	Value	400000	0	500000	1E+30	100000
B14	Value	500000	0.57588	500000	0	125000
B15	Value	0	0.19475	0	0	320000
B16	Value	0	0	0	1E+30	0
B17	Value	-90000	0	0	1E+30	90000
B18	Value	0	0.00308	0	0	125000

The Optimal Solution is represented by the final values i.e. x_1=400000, x_2=0, x_3=400000, x_4=100000, x_5=100000 and x_6=0.So the optimal objective function value can be found by plugging the optimal solution Max Z=0.2131*400000+0.1023*0+0.7508*400000+0.595* 100000+0.178*100000+0.163*0 = Rs406028.The allowable increase and decrease specify how much the objective coefficients can change before the optimal solution will change For example since the allowable increase for x_1 is 0.00385 if we increase the objective coefficient of x_1 from 0.2131 to any value, up to an upper limit of 0.21695 the optimal solution will not change. For the allowable decrease same can be think of as if we decrease the objective coefficient of x_1 from 0.2131 to any value, up to a lower limit of 0.14124 the optimal solution will not change. For x_2 the upper limit will be 0.17415 and the lower limit will be negative infinity. So what will happen to the optimal solution if the expected rate of return on x_2 (that is the coefficient) increase by 0.2. We can see the allowable increase is 0.07185. Therefore, the optimal solution will change if we increase it by 0.2. That is the final values will no longer be optimal. Now let's discuss the reduced cost for x_2. The reduced cost of -0.07185 here represents the amount by which return will be reduced if we invest a rupee in x_2. In essence product x2 is not attracting enough return to warrant it's inclusion in the portfolio mix. To include the product x_2 (or to make x_2 positive) it's expected rate of return needs to improve by at least 0.07185 but at its current value of 0.1023 making x_2 positive in the optimal solution will bring a reduction of 0.07185 to return per rupee invested in x2. In the same way limits for remaining objective coefficients can be determined. For x6 the reduced cost is -0.015 which shows the amount by the return would decrease if if we invest a rupee in x_6. To include product x_6 in the optimal solution it's expected rate of return needs to improve by at least 0.015.

Now, the bottom part of the table titled Constraints addresses the range of feasibility that is the range for the Right Hand Side of a constraint where the shadow price remains unchanged. For example, as long as the RHS of constraint 1 is between 1000000 and 1125000 the shadow price of 0.17492 will apply. Shadow Price here refers to the amount of change in optimal objective function value, per unit increase in the RHS of a constraint. So what will happen to optimal profit if the RHS of Constraint 1 increases by 100000.Notice that the allowable increase for the constraint 1 is 125000.So an increase of 100000 is allowable. Therefore, the optimal profit will change by 100000 (the amount of change) times 0.17492 (the shadow price) to give 17492. Since this is positive profit will increase by 17492 from ₹406028 to become ₹423520 and what will happen if the RHS of constraint 2 decreases to 300000.Notice here that the allowable decrease for constraint 2 is 100000.The current value is 500000, so decreasing it to 300000 is a proposed decrease of 200000.Since the proposed decrease exceeds the allowable decrease the shadow price is no longer valid. As a result, we cannot tell what will happen to optimal return based on this output. Next how will the objective function change if the RHS of constraint 3 changes to 400000.The current RHS value for constraint 3 is 500000.Changing it to 400000 represents a decrease of 100000 which is less than the allowable decrease of 100000.Therefore the shadow price applies and the total profit changes by 0.57588*-100000 to give -57588.Since the value is negative, optimal profit will actually decrease by ₹57588 to become ₹348440.For slack and surplus we simply take the differences between the Final Values(that is the left side of the constraint) and the RH sides. Slack Values for constraints 2 and 6 are 100000 and 90000 respectively and for remaining are 0. Lastly Binding constraints are the ones that have Final Value equal to RH Side that is the ones that have 0 slack or surplus values so constraints 1,3,4, 5 and 7 are binding and 2 and 6 are non-binding.

In this work, we took return, and investment requirements for forming the equation and generated the sensitivity report using Excel Solver. This is to establish the effect of each investment on the other and ultimately on the optimal result. We also used the simplex method because it will allow us choose the best return portfolio to go for among the six investment options which we are to choose from. It also tells us how much we can invest in each to optimize return. In making the first choice, we have that investing of ₹400000 in Sun Pharma gives us a maximum yield of ₹85240 In making the second choice, we have that investing of 0 because the sensitivity analysis report showed us that every single rupee invested in x2 the optimal return would decrease by 0.07185. In making the third choice, we have that investing of ₹400000 in TCS will give us a maximum yield of ₹300320. In making the fourth choice, we have that investing of ₹100000 in Infosys which will give us a maximum yield of ₹59500. Now for the mutual funds we have an investing of ₹100000 in Axis Blue Chip fund which will give us a maximum yield of ₹17800 and an investing of 0 in ICICI PruBlue Chip fund because the sensitivity analysis report showed us that for every single rupee invested in x6 the return would decrease by 0.015.Hence the required optimal portfolio selected out of stocks of Sun Pharma , Cipla ,TCS , Infosys and MF of Axis and ICICI is Sun Pharma Ltd---₹4,00,000 TCS---₹4,00,000 Infosys---₹1,00,000 Axis Bluechip fund ---₹1,00,000.

In conclusion, the results provided a process to determine the optimal portfolio. This method can be used for any number of selected stocks from where an investor wants to select the required portfolio that he wishes to hold. This is actually a new method of portfolio selection which takes into consideration the liquidity of the organization and goes beyond not only selecting the portfolio but also tells how much should be invested in each stock for optimal return. In this work, we can see that the Simplex method gave the values required to be invested on each of the stocks at one time rather obtaining the respective values separately. Reason for this behavior is clear and we can conclude that a combination of stocks can optimally give the best dividend coupled with the fact that diversification of investment is highly encouraged to reduce loss. Thus, this is a good method to find stocks to invest and also at the same time how much to invest in them.

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