SEQUENCE OF MULTIVALUED MAPPINGS AND FIXED POINT THEOREM IN HAUSDORFF METRIC

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ABSTRACT: In this article, derived a general case study by using Hausdorff metric for corroborating a fixed point result with sequence of multivalued mappings . Outcome of this article is the combination, extension and infer several comparable reviewed results of researchers, analyzers and others in this existing note.

KEY WORDS AND PHRASES. Hausdorff Metric; Cluster Point; Multivalued Mappings; Fixed Point.

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1. INTRODUCTION

In general the notion of Hausdorff metric or in other words Hausdorff distance is used for measuring how far two subsets of a metric space are from each others . Using this concept Sayyed et.al. [19] demonstrated a fixed point theorem for multivalued mappings which was generalization of Wong's [27] result. Bonsal [2] proved continuity of fixed point of contraction mapping , subsequently Nadler [13],Chartjee [4] Mizoguchi and Takahashi [12], Bose & Mukharjee [3] ,Kelly [7], Khan[8],Khan & Imdad [9] Sayyed & Badshah [20] ,Liu Et. Al [11],Gornicki [6],Dhamjanovic & Dragan-Dori [5], Lateef , Sayyed & Bhatacharya [10] , Yadav ,Sayyed & Badshah [28] , Sayyed [21],Pourmoslemi [14],Rakotch[15],Rus [18],Semenov [22], Singh[23,24], Vyas & Sayyed [26] , Abdou [1],Suhas &Dolhare [25],Younisa et. al [29] Rasham, Shoaib, Arshad and Khan [16,17] proceeding in the same manner and established the result of fixed point theory.

2.PRELIMINARIE

For achieving goal ,going to be needed about to be mentioned definitions and results,

DEFINITION 2.1. Kelley [4] : Let $H(A, B) = \text{Inf}\{\varepsilon \mid A C\, N(\varepsilon,B) \text{ and } B C\, N(\varepsilon, A)\}$ for $A,B \in CB(X)$, where $N(\varepsilon , C) = \{ x \in X \mid d(x,c) < \varepsilon \text{ for some } c \in C\}$, where $\varepsilon > 0$ and $C \in CB(X)$. The function H is said to be a metric on CB(X) and is called hausdroff metric. The metric H depends on the metric d of X and two equivalent metrics on X' may not generate equivalent Hausdorff metrics for CB(X).

DEFINITION 2.2 : Let (X,d_1) and (Y,d_2) be two metric spaces. Let $F:(X,d_1)\rightarrow CB(Y)$. F is said to be a multivalued contraction mapping if and only if $H(Fx,Fy) \leq kd_1(x,y)$, $x,y \in X$, where $0 \leq k < 1$, is a fixed real number .

DEFINITION 2.3: Let (X,d) be a complete metric space. A mapping $F : X\rightarrow X$ is said to be of generalized another type if

$$[d(Fx,Fy)]^2 \leq \alpha[d((x,Fx)d(y,Fy) + d(x,Fy)d(y,Fx)] + \beta [d(x,Fx)d(x,Fy) + d(y,Fy)d(y,Fx)]$$

where α and β are non negative numbers such that $0\leq \alpha +\beta < 1$.

Let CB(X) denote the set of non empty closed bounded subset of X. The following is a simple consequence of the definition of Hausdorff metric H. Let $A,B \in CB(X)$ and $a \in A$. If $\eta >0$, then there exists $b \in B$ such that $d(a,b)\leq H(A,B) + \eta$, i.e., $d(a,b) \leq PH(A,B)$ where $P>1$. If $A, B \in C(X)$ and $a \in A$, then there exists $b \in B$ such that $d(a,b)\leq H(A,B)$.

THEOREM 2.1 BOSE AND MUKHERJEE[3] : Let {Fn} be a sequence of self mapping of X having at least one fixed point x_n each and let {Fn} converge uniformly to F_0 , a mapping of generalized Kannan - Reich type. Let x_0 be the unique fixed point of F_0 then $x_n\rightarrow x_0$.

THEOREM 2.2 BOSE AND MUKHERJEE[3]: Let {Fn} be a sequence of mappings of generalized Kannan- Reich type and let {Fn} converges pointwise to F, a generalized Kannan - Reich type mapping. Let x_n and x_0 be fixed points of Fn and F respectively. Then $x_n\rightarrow x_0$.

3.MAIN RESULT:

THEOREM 3.1: Let {Sn} be a sequence of multivalued mappings of X into CB(X) satisfying the following condition:

$$H(S_nx, S_ny)\leq b_1[d(S_nx, x)+d(S_ny, y)] +b_2[d(S_nx, y)+d(S_ny,x)]+b_3[d(x,y)]$$

for all x, y ∈ X, where b_1, b_2 and $b_3 \geq 0$ and $0 \leq b_1+3b_2+b_3 \leq 1$. Let {Sn} converges to S_0 pointwise and let x_n be the fixed point of Sn. If x_0 is any cluster point of the sequence $\{x_n\}$, then $x_0 \in S_0x_0$.

PROOF: Let $x_{ni} \to x_0$, for simplicity of notation, we write in place of n_i,

Then

$$D(x_0, S_0x_0) \leq d(x_0,x_i)+D(x_i, S_0x_0)$$

$$\leq d(x_0,x_i)+H(S_ix_i, S_0x_0)$$

$$\leq d(x_0,x_i)+ b_1[D(S_ix_i,x_i)+D(S_0x_0,x_0)]+ b_2[D(S_ix_i,x_0)+D(S_0x_0,x_i)]+b_3d(x_0,x_i)$$

$$\leq d(x_0,x_i)+ b_1 D(S_0x_0,x_0)]+ b_2[d(x_0 , x_i) + D(S_0x_0,x_i)] + b_3d(x_0,x_i)$$

By using triangle inequality, and simplified, then

$$D(x_0, S_0x_0) \leq \frac{1+2b_2+b_3}{1-b_1-b_2} d(x_0,x_i)$$

Taking limit as $i\to\infty$, we have $D(x_0,S_0x_0) \leq 0$. Since S_0x_0 is closed ,we have $x_0 \in S_0x_0$.

. ***COROLLARY 3.1***: Let (X, d) be a complete metric space and let $S_i :X\to X$, i=1,2 be mappings satisfying the following condition :

$$d(S_1u_1, S_2u_2) \leq b_1[d(S_1u_1,u_1)+d(S_2u_2,u_2)]+b_2[d(S_1u_1,u_2)+d(S_2u_2,u_1)]+b_3 d(u_1,u_2)$$

for any u_1,u_2 in X, where $b_1,b_2,b_3\geq 0$ and $0\leq b_1+3b_2+b_3\leq 1$. Then $S_1$and S_2 have a common fixed point.

COROLLARY 3.2: Let (X,d) be a complete bounded metric space and let $S:X\to CL(X)$ be multivalued mapping satisfying the following condition.

$$H(Su_1,Su_2) \leq b_1[d(Su_1,u_1)+d(Su_2,u_2)]+b_2[d(Su_1,u_2)+d(Su_2,u_1)]+b_3d(u_1,u_2)$$

for any u_1,u_2 in X where $b_1,b_2,b_3\geq 0$ and $0\leq b_1+3b_2+b_3\leq 1$. Then S has a fixed point.

4.CONCLUSION

In this paper, proving a unique fixed point theorem with contractive type inequality for multivalued mappings in Hausdorff metric. These results can be extended to any directions and can also be extended to fixed point theory of multi-valued mappings, compatible, weakly compatible, ordered and many mappings with enhancement by some properties and inequalities .

5.CONFLICTS OF INTEREST

The author declares that he has no conflicts of interest.

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