The Analysis of Central Tendency of Grouped and Ungrouped Frequency Distributions

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Abstract : *Central tendency of grouped and ungrouped frequency distributions has been discussed in this article. Using the geometric mean, arithmetic mean and harmonic mean, we solved grouped, and ungrouped frequency distribution. There are four important "measures of central tendency". This article promotes critical and creative thinking while providing readers with the fundamental concepts needed for research in central tendency.*

Index Terms **–frequency distribution, geometric mean, arithmetic mean, harmonic mean, mode.**

I. INTRODUCTION

In this article we discussed about central tendency of grouped and ungrouped frequency distribution. A variable is a quantitative property that differs with units. As a result, variables include income, expenditure, Imports, and exports. Every variable has a range of variations, whether continuous or discrete. The set of all possible values for a variable is called its range. The variable's assumed value determines the classification of the units. As a result, such a classification will contain a set of values for the variable the and a set of units that are related to these values. The frequency of a given value in the variable is the number of units linked. The phrase "frequency distribution" refers to the variable's values and associated frequencies. A frequency table is a tabular display of the frequency distribution. There are two types of frequency distributions: grouped and ungrouped. A frequency distribution in which class intervals are grouped according to given data is called a grouped frequency distribution. Otherwise, it is called ungrouped frequency distribution.

II. OBJECTVES

The main objective is to find central tendency of grouped and ungrouped frequency distributions.

III. METHODOLOGY

Using the geometric mean, arithmetic mean and harmonic mean we solved grouped and ungrouped frequency distribution. Examples of grouped and ungrouped frequency distributions that are given below.

Number of workers below 23 years (Variable)	Number of Companies (Frequency)
2	21
1	18
3	8
6	15
7	12
8	4
Total	**78**

Table 1. An ungrouped frequency distribution

Monthly Rentals (Class Interval)	Number of Apartments (Frequency)
5000 – 6000	5
6000 – 7000	8
7000 – 8000	3
8000 – 9000	4
9000 – 10000	2
10000 – 11000	3
Total	**25**

Table 2. A grouped frequency distribution

Measures of Central Tendency

A central value is usually the centre of attention for the observations in a frequency distribution. Central tendency is the characteristic of the observations being concentrated around a central value. The term "measure of central tendency" refers to the central value that is concentrated around. Here we discuss about the geometric mean, harmonic mean, mode, median, arithmetic mean, median and mode.

Definitions:

(i) Geometric Mean (GM): The m^{th} root of the product of the values yields the geometric mean of the m values.
(ii) Arithmetic Mean (AM): The sum of the m values divided by the total number of values is the arithmetic mean of the m values.
(iii) Harmonic Mean (HM): The multiplicative inverse of the arithmetic mean of the multiplicative inverses of the given m values is called harmonic mean.
(iv) The middle most value in an ascending order of magnitude(array) is called the median of a set of values.
(v) The value with the maximum frequency is called the mode.

Formulas for finding GM, AM and HM for a raw data:

If $a_1, a_2, a_3, \dots, a_m$ is a given set of values then GM = $\sqrt[m]{a_1 a_2 a_3 \dots a_m}$ or using common logarithm

GM = antilog $\left(\frac{\log_{10} a_1 + \log_{10} a_2 + \log_{10} a_3 + \dots + \log_{10} a_m}{m}\right)$ ...(1)

AM = $\frac{a_1 + a_2 + a_3 + \dots + a_m}{m}$ ...(2)

HM = $\frac{m}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_m}}$ ...(3)

Example: If the weights of four oranges are 126, 130, 158 and 165 grams respectively, then by (1), (2) and (3),

GM = antilog $\left(\frac{2.1004 + 2.1139 + 2.1987 + 2.2175}{4}\right)$ = antilog (2.1576) = 143.7

AM = $\frac{579}{4}$ = 144.75 and HM = $\frac{4}{\frac{1}{126} + \frac{1}{130} + \frac{1}{158} + \frac{1}{165}} = \frac{4}{0.0079 + 0.0077 + 0.0063 + 0.0061} = \frac{4}{0.028}$ = 142.86

Formulas for finding GM, AM and HM of Grouped and Frequency Distribution:

If $f_1, f_2, f_3, \dots f_m$ and $M = f_1 + f_2 + f_3 + \dots + f_m$, then GM = $\sqrt[M]{f_1 a_1 f_2 a_2 f_3\, a_3 \dots f_m a_m}$ or using common logarithm

GM = antilog $\left(\frac{f_1 \log_{10} a_1 + f_2 \log_{10} a_2 + f_3 \log_{10} a_3 + \dots + f_m \log_{10} a_m}{M}\right)$ ...(4)

AM = $\frac{f_1 a_1 + f_2 a_2 + f_3\, a_3 + \dots + f_m a_m}{M}$ ...(5)

HM = $\frac{M}{\frac{f_1}{a_1} + \frac{f_2}{a_2} + \frac{f_3}{a_3} + \dots + \frac{f_m}{a_m}}$ ...(6)

Geometric Mean, Arithmetic Mean and Harmonic Mean for Ungrouped Frequency Distribution

Using the Table 1 mentioned above calculate $f_i a_i$, $\frac{f_i}{a_i}$, $\log_{10} a_i$ and $f_i \log_{10} a_i$

Number of workers below 23 years (a_i)	Number of Companies (f_i)	$\log_{10} a_i$	$f_i a_i$	$\frac{f_i}{a_i}$	$f_i \log_{10} a_i$
2	21	0.3010	42	10.5	6.321
1	18	0	18	18	0
3	8	0.4771	24	2.6667	3.8168
6	15	0.7782	90	2.5	11.673
7	12	0.8451	84	1.7143	10.1412
8	4	0.9031	32	0.5	3.6124
Total	***M* = 78**	**3.3045**	**290**	**35.881**	**35.5644**

By (4), (5) and (6),

GM = antilog $\left(\frac{35.5644}{78}\right)$ = antilog (0.4560) = 2.858; AM = $\frac{290}{78}$ = 3.7179 and HM = $\frac{78}{35.881}$ = 2.1739

Geometric Mean, Arithmetic Mean and Harmonic Mean for Grouped Frequency Distribution

Using the Table 2 mentioned above calculate a_i, $f_i a_i$, $\frac{f_i}{a_i}$, $\log_{10} a_i$ and $f_i \log_{10} a_i$.

Monthly Rentals (in Rupees)	No. of Apartments (f_i)	Mid -value (a_i)	$\log_{10} a_i$	$f_i a_i$	$\frac{f_i}{a_i}$	$f_i \log_{10} a_i$
455000– 6000	5	5500	3.7404	27500	0.0009	18.702
6000 – 7000	8	6500	3.8129	52000	0.0012	30.5032
7000 – 8000	3	7500	3.8751	22500	0.00039	11.6253
8000– 9000	4	8500	3.9294	34000	0.00048	15.7176
9000 – 10000	2	9500	3.9777	19000	0.00022	7.9554
10000 – 11000	3	10500	4.0212	31500	0.00003	12.0636
Total	***M* = 25**	**48000**	**23.3567**	**186500**	**0.00322**	**96.5671**

By (4), (5) and (6)

GM = antilog ($\frac{96.5671}{25}$) = antilog (3.8627) = 7290; AM = $\frac{186500}{25}$ = 7460 and HM= $\frac{25}{0.00322}$ = 7763.98.

Formula for finding median Md for a raw data:

Md = $\left(\frac{m+1}{2}\right)^{th}$ value in the array.

Example: If the weights of four oranges are 126, 130, 158 and 165 grams respectively, then in the array

Md = $\left(\frac{4+1}{2}\right)^{th}$ value = $(2.5)^{th}$ value = 2^{nd} value + 0.5(3^{rd} value-2^{nd} value) = 130 + 0.5(158−130) = 144.

Formula for finding median Md for Ungrouped Frequency Distribution

Md = $\left(\frac{M+1}{2}\right)^{th}$ value in the array.

Example: arranging the values of Table 1 in an ascending order of magnitude (array)

Number of workers below 23 years (a_i)	Number of Companies (f_i)	< cumulative frequency
1	18	18
2	21	39
3	8	47
6	15	62
7	12	74
8	4	78
Total	***M* = 78**	

Md = $\left(\frac{78+1}{2}\right)^{th}$ value = $(39.5)^{th}$ value = 39^{th} value + 0.5(40^{th} value-39^{th} value) = 2 + 0.5(3−2) = 2.5.

Formula for finding median Md for Grouped Frequency Distribution

Md = L + $\left[\left(\frac{M}{2} - n\right) \times \frac{w}{f}\right]$, where L= Minimal limit of the median class, M = Total number of frequencies, n < cumulative frequency up to L, w is width and f is the frequency of the median class.

Example: arranging the values of Table 2 in an ascending order of magnitude (array)

Monthly Rentals (in Rupees)	No. of Apartments (f_i)	< cumulative frequency
5000– 6000	5	5
6000 – 7000	8	13
7000 – 8000	3	**16 (n)**
8000– 9000 (Median Class)	**4 (f)**	20
9000 – 10000	2	22
10000 – 11000	3	25
Total	$\boldsymbol{M}$ **= 25**	

Md = 8000 + $\left[\left(\frac{25}{2} - 16\right) \times \frac{1000}{4}\right]$ = 7125.

IV. CONCLUSION

This article promotes critical and creative thinking while providing readers with the fundamental concepts needed for research in central tendency of frequency distribution. It helps to evaluate geometric, arithmetic and harmonic means and medians of raw data, grouped and ungrouped frequency distributions.

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